Chemical freeze-out phenomenology

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- Definition
- Equilibrium $(T = 170MeV, \gamma_{q,s} = 1)$ vs
  explosive $(T = 140MeV, \gamma_{q,s} > 1)$ vs
  Continuus emission+resonance modification vs
  More complicated models (dynamics)
- Yields and fluctuations in SHM
- Sensitive probes with yields <u>and</u> fluctuations
- Analysis of existing (130 and 200 GeV RHIC data)
- Do it yourself: SHARE

  http://www.physics.arizona.edu/∼torrieri/SHARE/share.html

The statistical model:

$$N = \int \mathcal{M} \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

$\mathcal{M} \rightarrow constant$ (dynamics $\rightarrow$ phase space)

$$P_N = \frac{\Omega_N}{\sum_n \Omega_n} \qquad \Omega = \int \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

Observables:

$$< N > \quad , \quad \omega = \frac{< N^2 > - < N >^2}{< N >}, \quad \text{higher comulants}$$

calculable through partition function

Several ways of defining $\delta_{E,Q} \rightarrow$ Ensembles.

Ensembles , or how to deal with conservation laws
$\lim_{V\to\infty}^{N/V=const} < N >$same in $\forall$ ensembles. not ω

Micro-canonical : EbyE conservation

$$\delta_E \delta_Q = \delta\left(\sum_i E_i - E_T\right)\delta\left(\sum_i Q_i - Q_T\right) \quad \omega_E = \omega_Q = 0$$

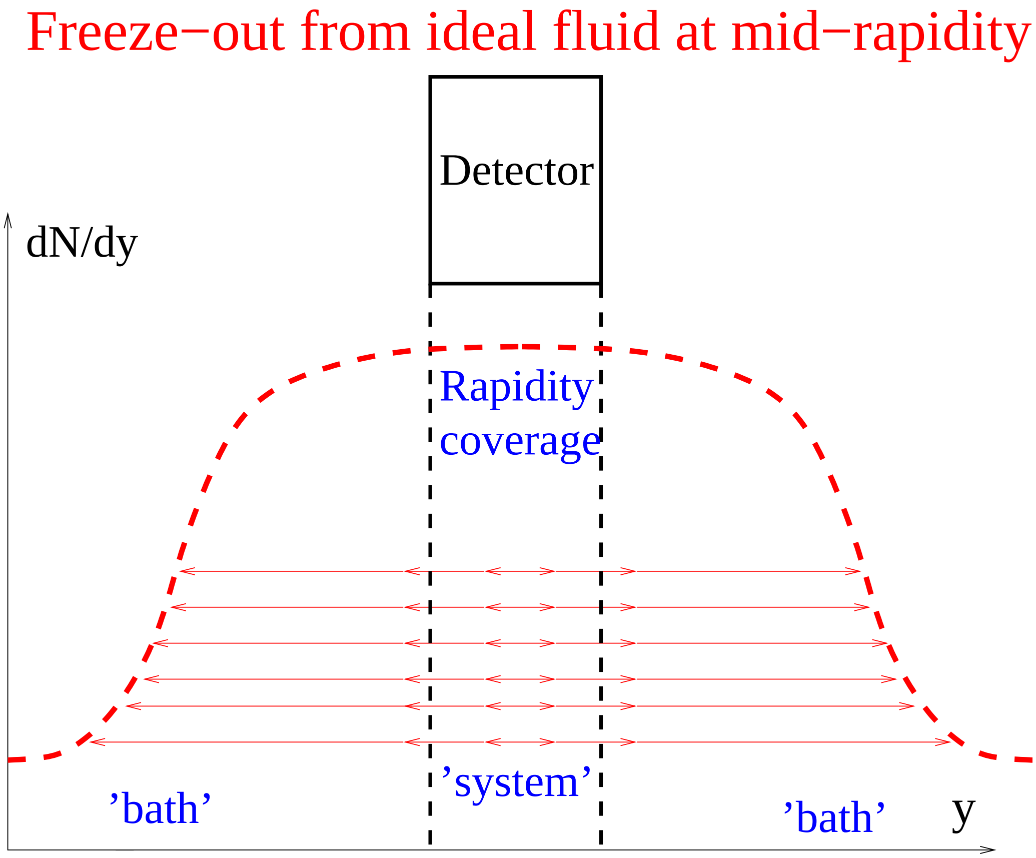
Canonical : Energy conserved on average
Appropriate for system in equilibrium with bath

$$\delta_E \to \delta\left(E_T - < E >\right) \qquad \omega_E \sim 1$$

Grand Canonical : Charge conserved on average

$$\delta_Q \to \delta\left(Q_T - < Q >\right) \qquad \omega_E \sim \omega_Q \sim 1$$

Appropriate for detector sampling part of a fluid



Boost invariance: Rapidity ⇔configuration space

- Mid-rapidity ⇔system
- Peripheral regions ⇔bath

⇒ Grand Canonical ensemble needs to be used!

Cleymans, Redlich, PRC 60, 054908 (1999):

$$\left[\frac{dN}{dy}\right]_{b.i.} \sim < N >_{4\pi} \quad \left[\frac{d(\Delta N)^2}{dy}\right]_{b.i.} \sim (\Delta N)^2_{4\pi}$$

- All details of flow and freeze-out integrate out

- Up to Normalization, $< N >, \omega$ calculable from Grand Canonical T, λ_i

Ensemble choice forced to be Grand Canonical by $\omega_Q \sim 1$at both SPS and RHIC.

NB: Low multiplicity yields also approximately statistical, if canonical suppression used (Becattini,Redlich,Cleymans,...)

but

linear scaling of fluctuations w.r.t. yields (KNO scaling) very difficoult to explain this way as exact conservation suppresses all fluctuations (Gorenstein, Gazdzicki, Zozulya,...)

$\Rightarrow$

Explaining both yields and fluctuations with same thermodynamic parameters probably only possible in A-A collisions, if at all

Grand canonical statistical hadronization

All particles described in terms of T and $\lambda_{q,s,I3}$.
Detailed balance: $\lambda_{\overline{q}} = \lambda_q^{-1}$Integral can be done in rest-frame wrt flow using Bessel function K_2

$$N_i = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} m_i^2 T K_2 \left(\frac{n m_i}{T} \right)$$

$$\Delta N_i^2 = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} \left(\begin{array}{c} 2+n-1 \\ n \end{array} \right) m_i^2 T K_2 \left(\frac{n m_i}{T} \right)$$

$$V' = V(2J_i+1) \frac{4\pi}{(2\pi)^3}$$

Chemical potentials for conserved quantities

$$\lambda_i = \lambda_u^{u-\overline{u}} \lambda_d^{u-\overline{u}} \lambda_s^{s-\overline{s}}$$

Non-equilibrium through phase space occupancies

$$\lambda_i \rightarrow \lambda_i^{\mathrm{eq}} \gamma_u^{u+\overline{u}} \gamma_d^{u+\overline{u}} \gamma_s^{s+\overline{s}} \qquad \gamma^{\mathrm{eq}} = 1$$

Resonance feed-down

$$N_i = N_i^{direct} + \sum_j b_{j\to i} N_j$$

$$\Delta N_i^2 = \Delta N_i^2 + \sum_j \left[b_{j\to i} \left(1 - b_{j\to i}\right) N_j + b_{j\to i}^2 \Delta N_j^2) \right]$$

Widths

Decay $M \to m_1, m_2, ...$ width Γ_i, total width Γ_T
relative angular momentum l, threshold mass M_{th}

$$N(M,\lambda) \to \frac{\int_{M_{Th}}^{\infty} \rho(m) N(m,T,\lambda) dm}{\int_{M_{Th}}^{\infty} \rho(m) dm}$$

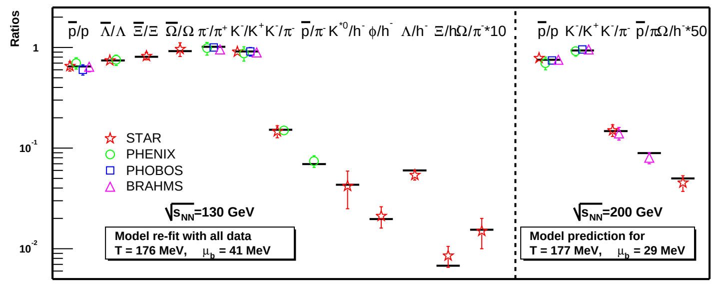
Where

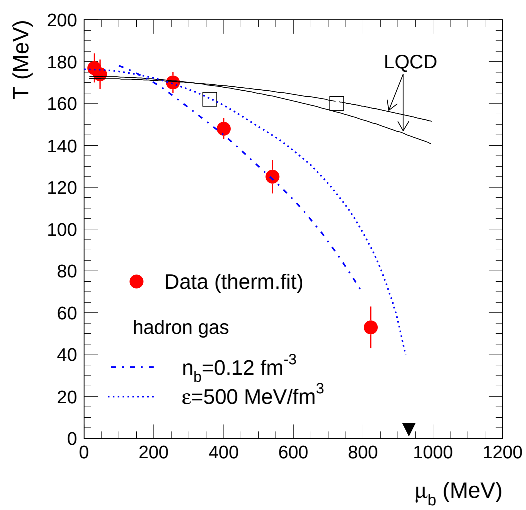
$$\rho(m) = \frac{\Gamma_T \Gamma(m)}{(M-m)^2 + \Gamma^2(m)/4}$$

$$\Gamma(m) = \Gamma_i \left[1 - \left(\frac{M_{Th}}{m} \right) \right]^{l/2}$$

Model I:Equilibrium statistical mechanics

(Braun-Muntziger,Magestro,Florkowski,Broniowski,Redlich,...)

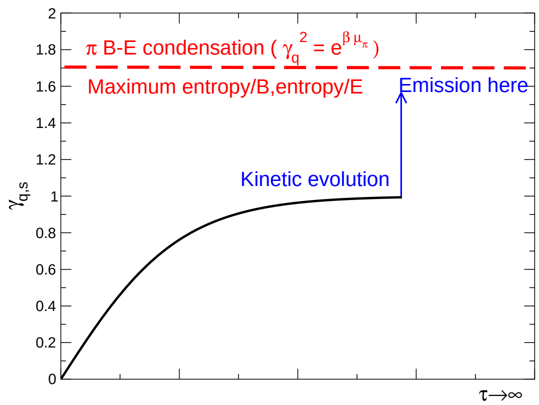




$$\frac{dT}{d\sqrt{s}} > 0 \qquad \frac{d\mu_B}{d\sqrt{s}} < 0 \qquad \frac{E}{V} \sim 1GeV$$

Resonances not well described (Rescattering?)

Alternatively... Supercooling+oversaturation
(chemical Non-equilibrium)

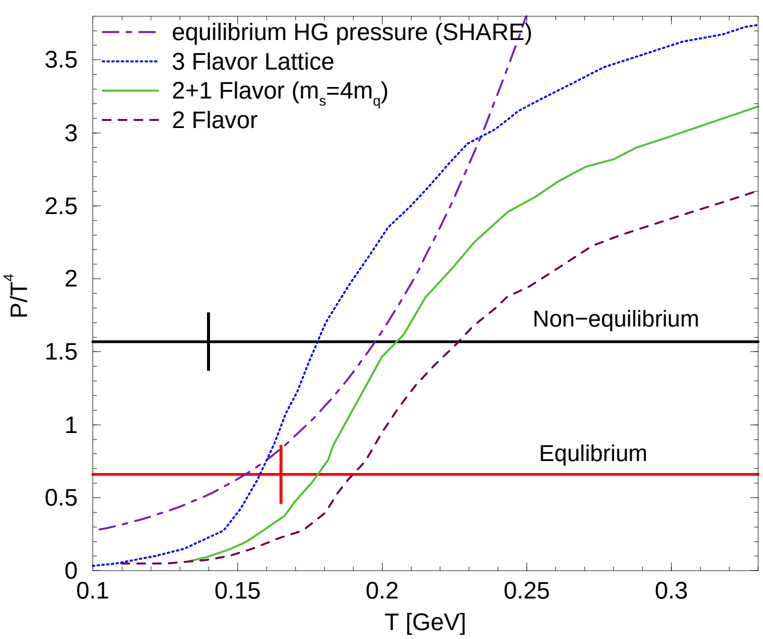


inaccessible by kinetic evolution
(if put "in a box", this system would heat)
but accessible in a fast phase transition
from a high entropy phase $\gamma_q > 1, \gamma_s/\gamma_q > 1$.

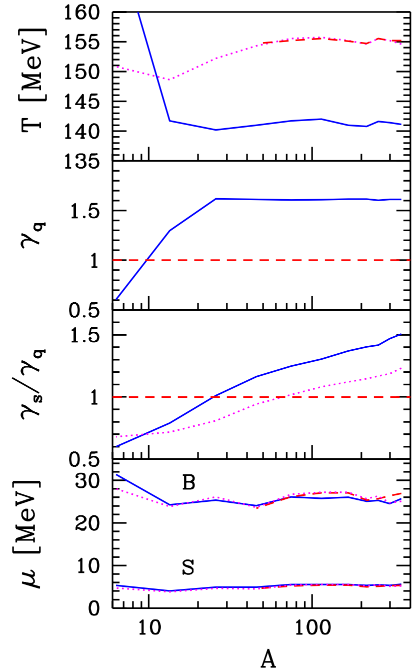
J. Rafelski, J Letessier, PRL 85:4695-4698,2000:
Explosive hadronization from supercooled QGP

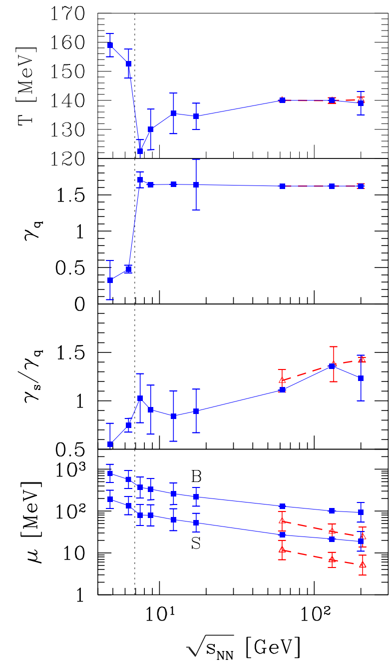
$$P_{vacuum} = P_{QGP} \qquad S_{HG} = S_{QGP} \qquad V \sim \frac{2}{3} V_{equilibrium}$$

$$T = 140 MeV, \qquad \gamma_q \sim 0.9 e^{m_\pi / 2T} \sim 1.6$$

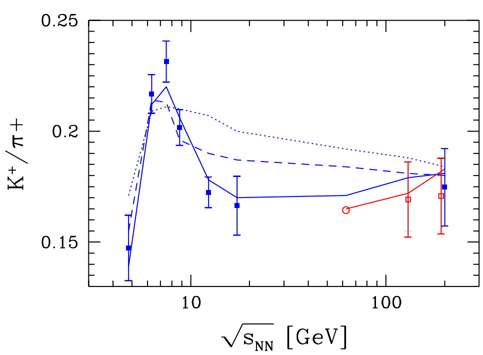


$\gamma_q > 1, T \rightarrow \sim 140 MeV$ @ critical $\sqrt{s}, N_{part}$



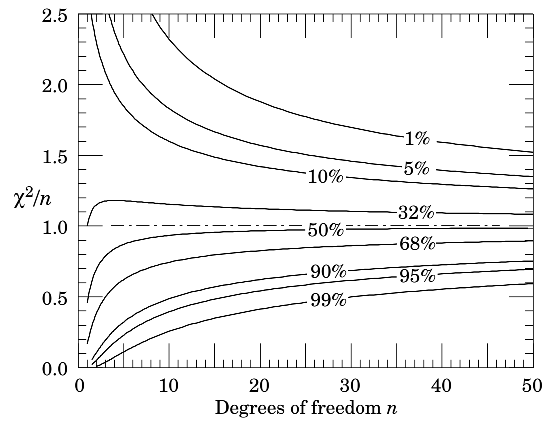


“Horn” explained by this critical point



Smoking gun 4 deconfinement?? Perhaps...

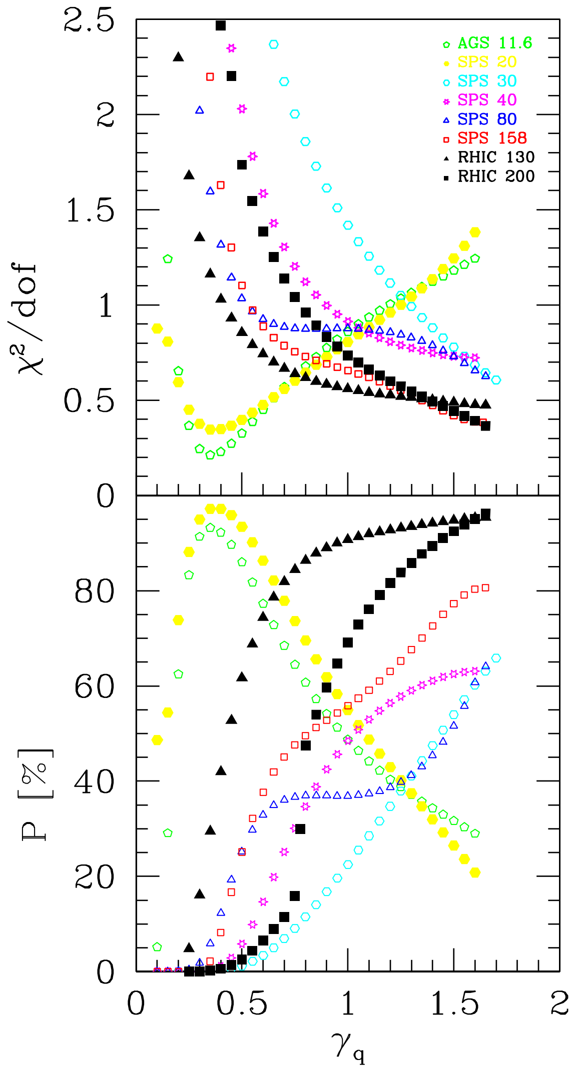
Equilibrium and non-equilibrium models have a different number of parameters.
Comparison standard: Statistical significance



- Statistical significance, the probability of getting χ^2 with n DoF given that "your model is true", is a quantitative measure of your fit's goodness

- With few DoF, "nice" looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won't get a good statistical significance.

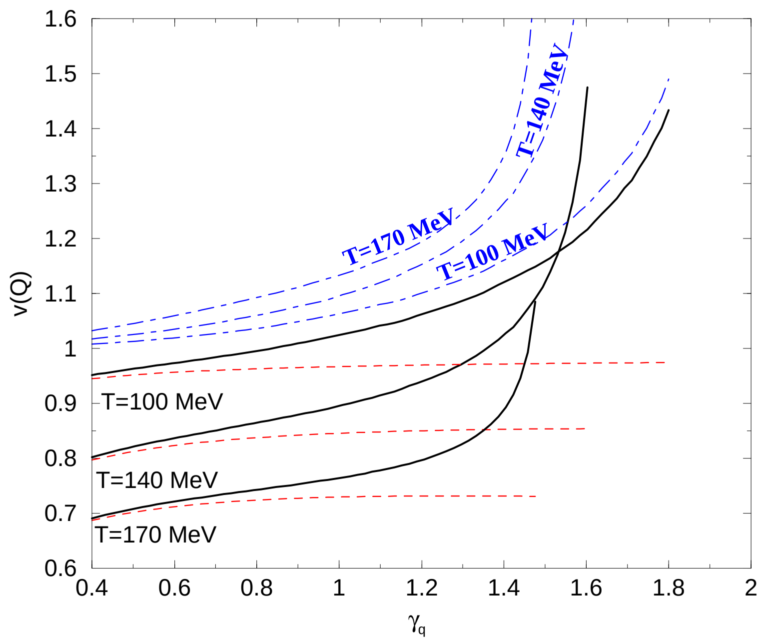


Maximum for SPS and RHIC is at $\gamma_q > 1$, but equilibrium not ruled out!

Need further data capable of determining γ_q.

Resonances and fluctuations

Diagnostics with Yields and fluctuations



T increase $\Rightarrow \pi$ Fluctuations decrease because of enhanced resonance production

over-saturation ($\gamma_q > 1$) $\Rightarrow \pi$ Fluctuations increase because of BE corrections

$\gamma_q > 1$, unlike resonances (detectable,rescattered,in-medium modified,...) affects fluctuation terms rather than correlations

Suitable:

Yields Independent of γ_s, λ_s,volume

- Λ/K^-, better
  - Λ corrected for $\Xi, \Omega \rightarrow \Lambda$
  - K^- corrected for $\phi \rightarrow K^- K^+$
- Ξ/ϕ

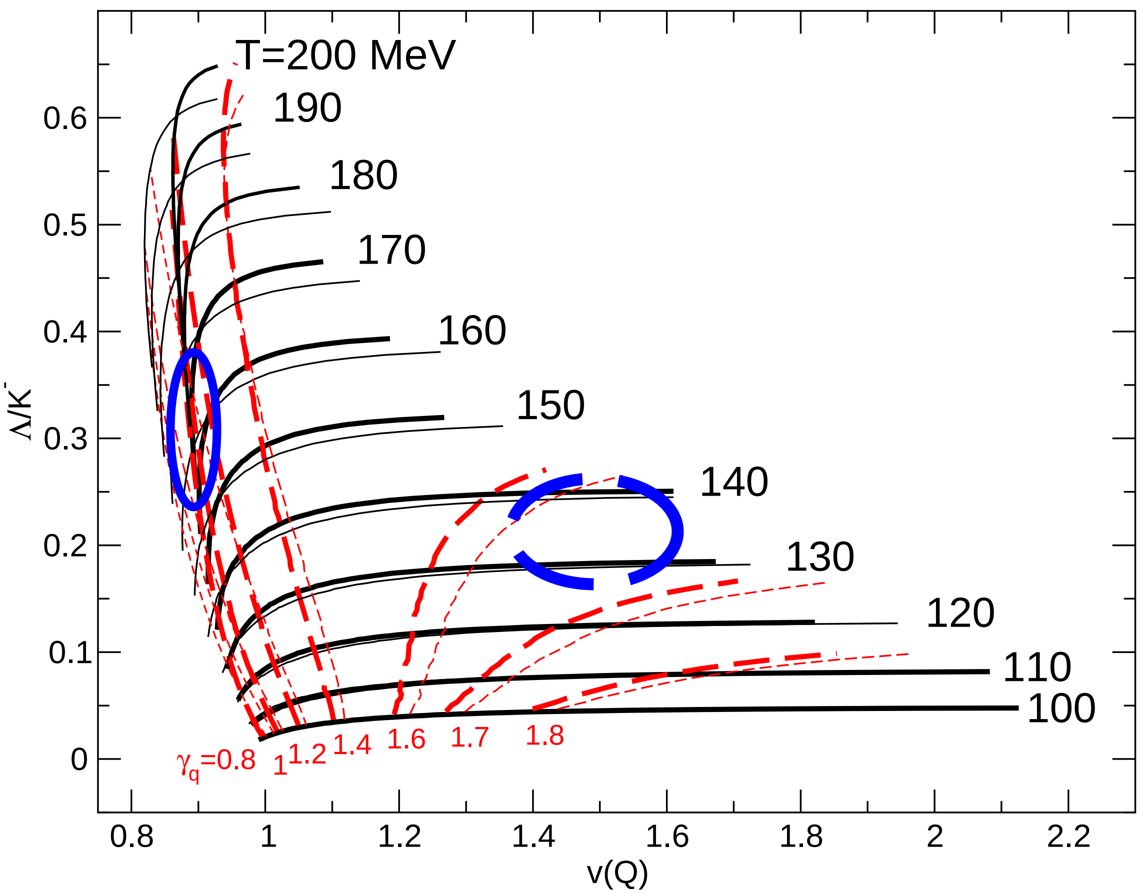
Fluctuations $v(Q)$,σ_{π^+/π^-}

$$v(Q) = \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{N_{ch}}$$

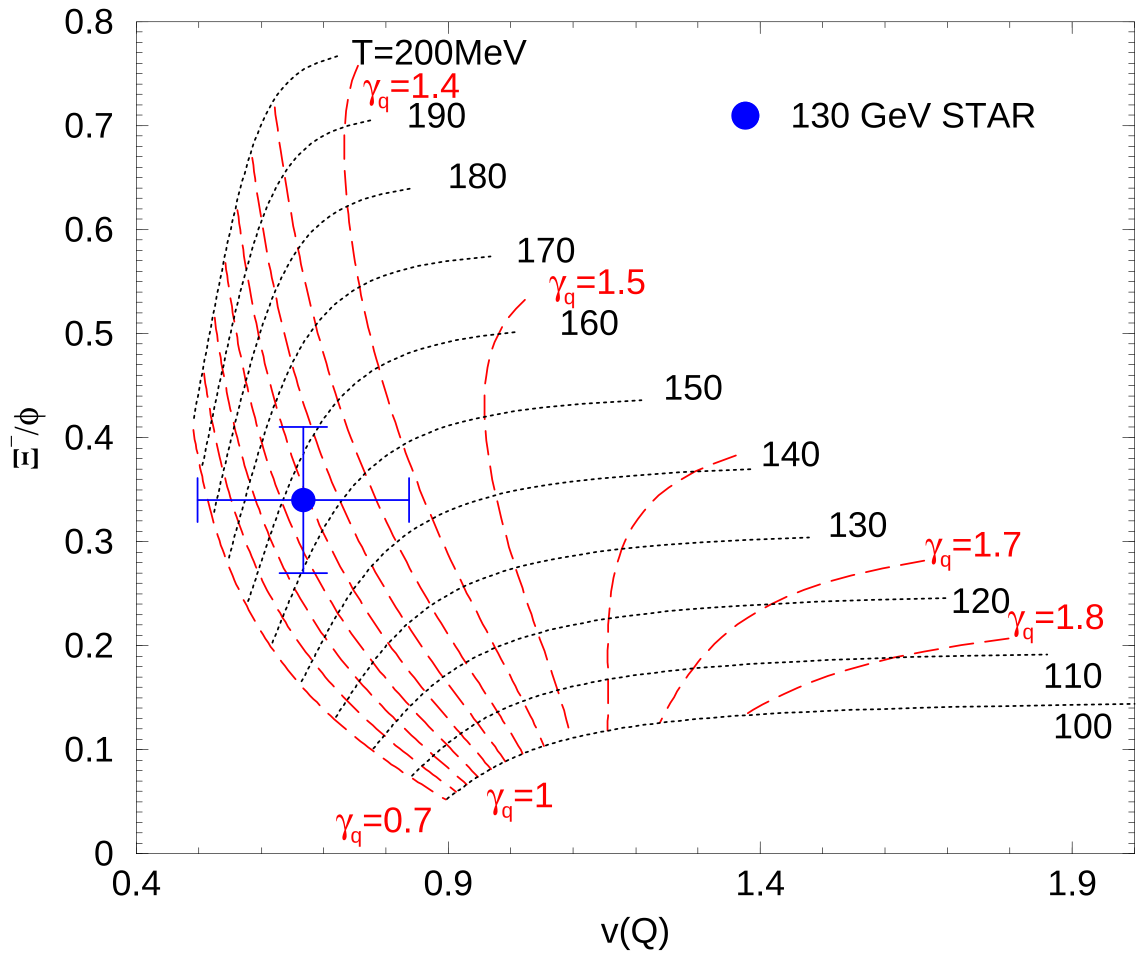
$$\sigma_{N_1/N_2} = \frac{\omega_{N_1}}{\langle N_1 \rangle} + \frac{\omega_{N_2}}{\langle N_2 \rangle} - 2\frac{\langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

σ_{π^+/π^-}can also be used to probe ρ, f_0 mass modificationdue to $\pi^+ - \pi^-$ correlations
undetectable or rescattered resonances also contribute to fluctuations!!!

v(Q)vs Λ/K^-

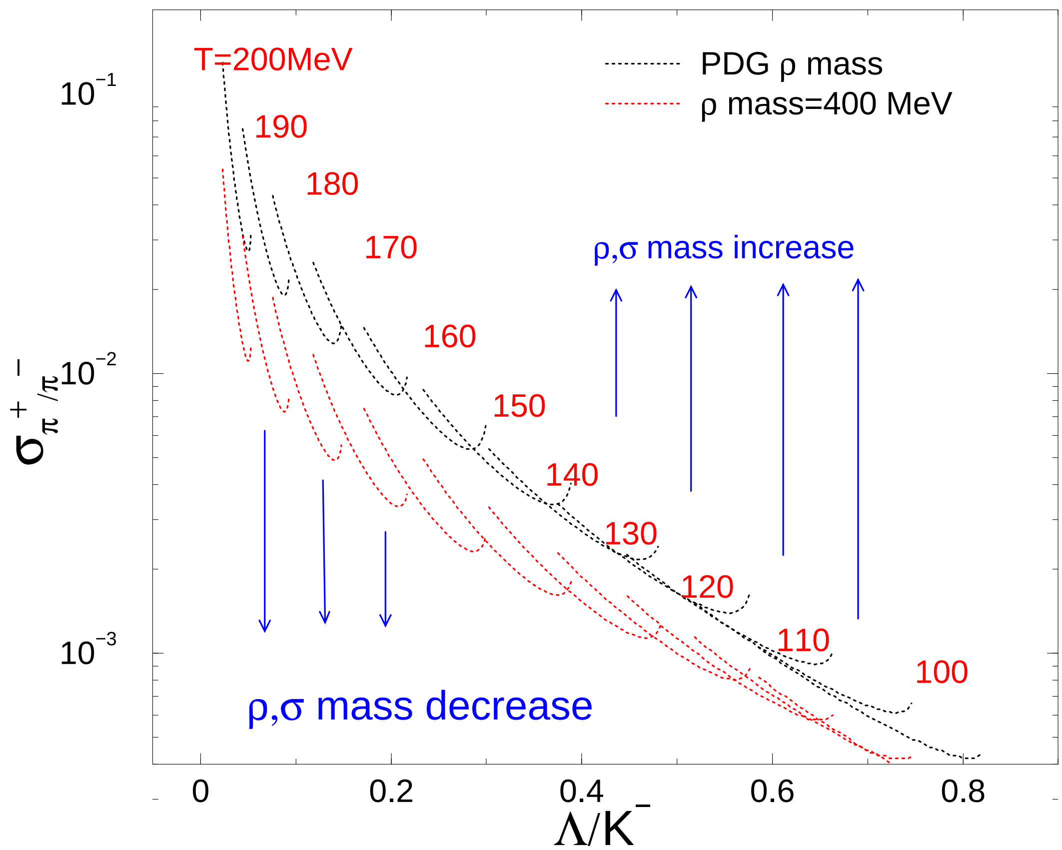


v(Q) vs Ξ^- / ϕ



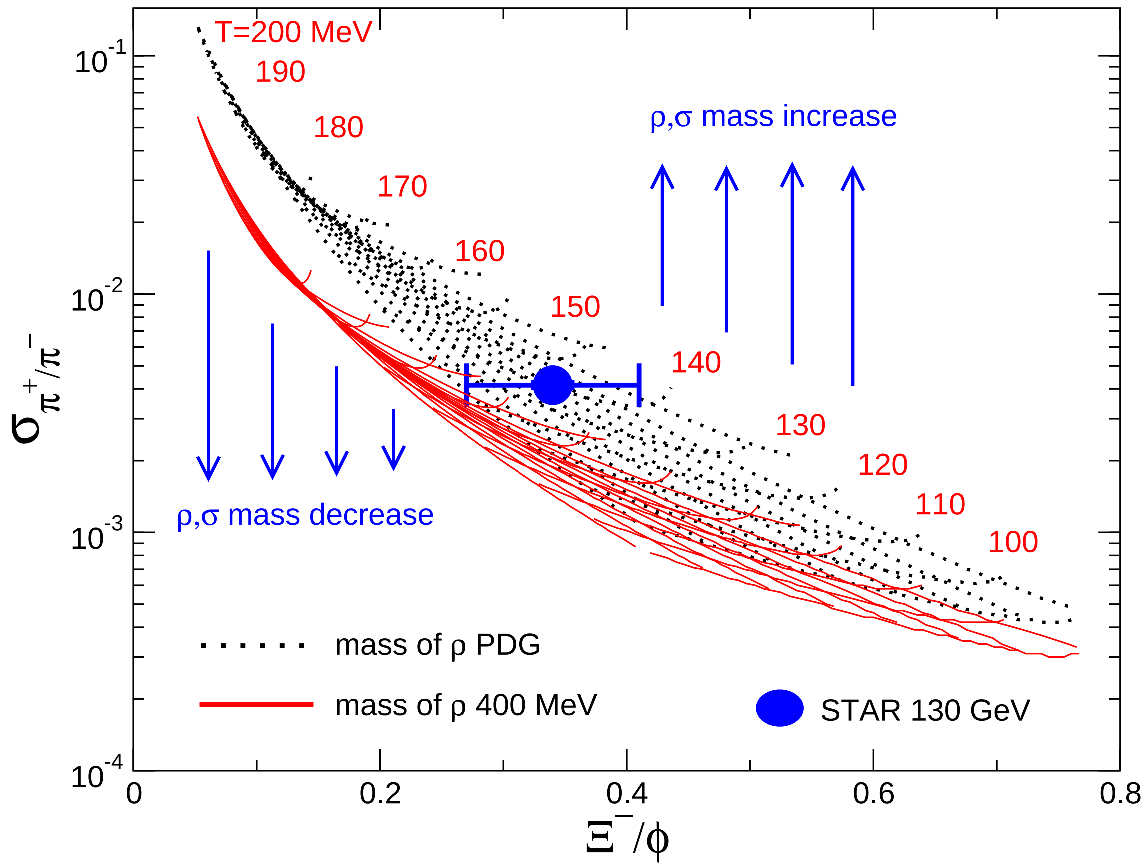
T=200MeV
190
180
170
160
150
140
130
120
110
100
γq=1.4
γq=1.5
γq=1.7
γq=1.8
γq=1
γq=0.7
130 GeV STAR
Ξ⁻/φ
v(Q)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.4
0.9
1.4
1.9

σ_{π^+/π^-} vs Λ/K^-



- σ_{π^+/π^-} probes T independently of (non)equilibrium
- Allowed region <u>narrow</u> $\rightarrow$ mass modification probe

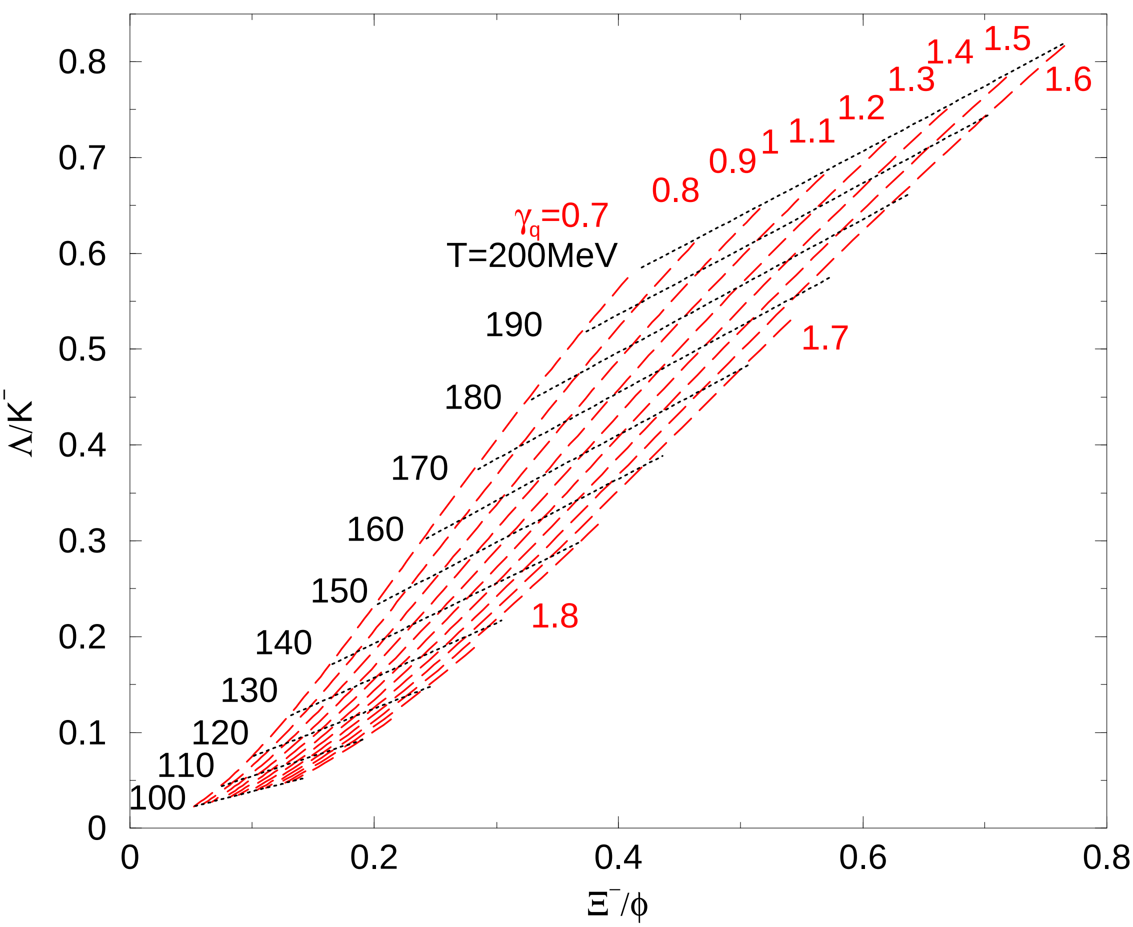
σ_{π^+/π^-}vs Ξ^-/ϕ



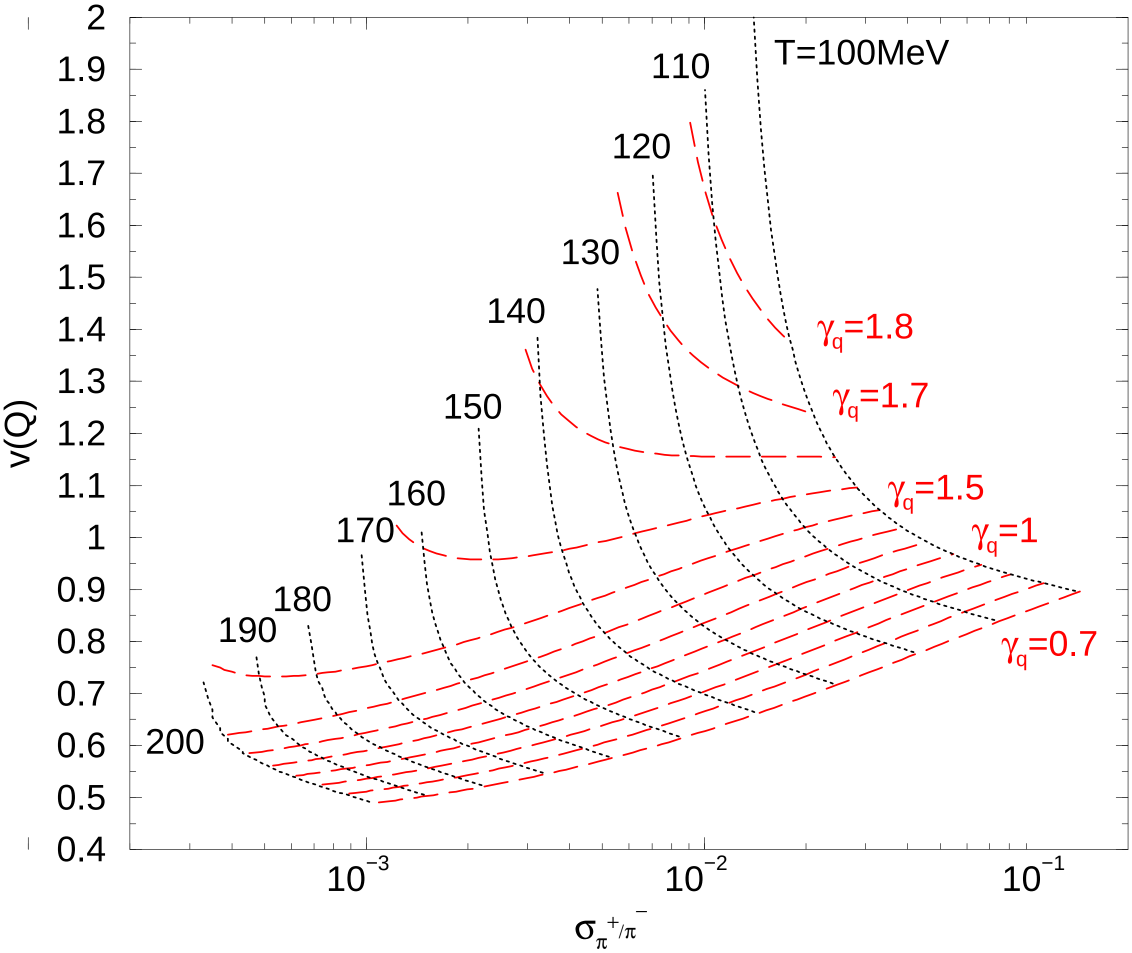
130 GeV within band! Good agreement with non-equilibrium freeze-out T

No evidence of ρ, σmodification

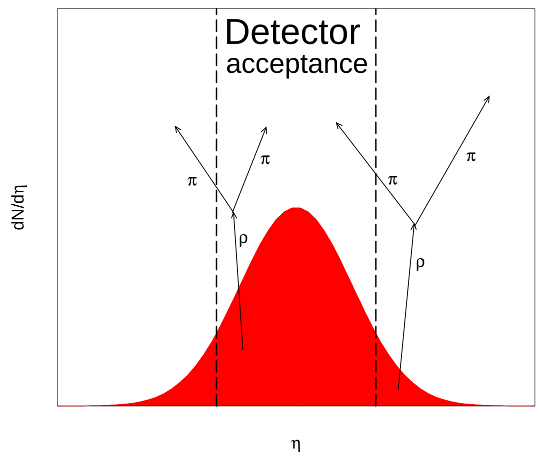
And for consistency... Ξ/ϕvs Λ/K^-



And for consistency... σ_{π^+/π^-}vs $v(Q)$



These diagrams are made with static fluctuations susceptible to detector response effects



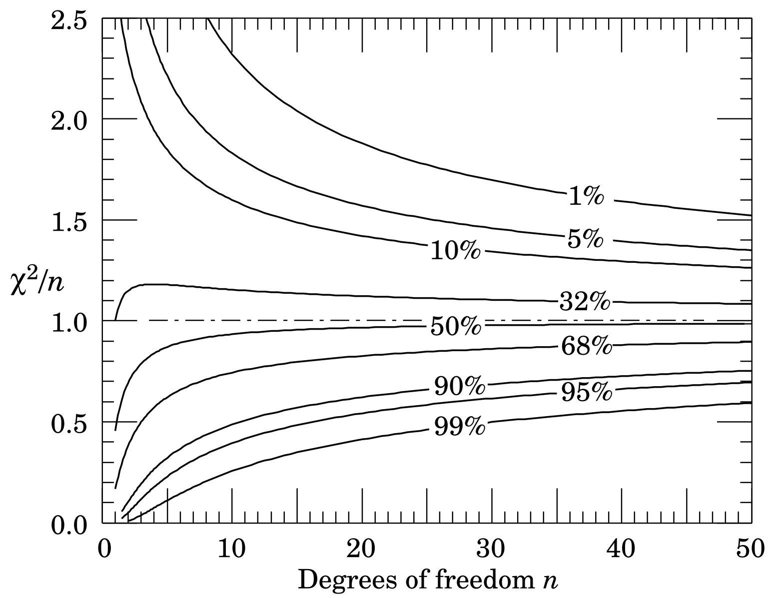
use dynamical fluctuations $\sigma_{dyn} = \sigma - \sigma_{stat}$ Where $\sigma_{stat} \sim \frac{1}{<N_1>} + \frac{1}{<N_2>}$ obtained by mixed event technique

σ_{dyn} robust against detector acceptance but needs more parameters ("volume") to be described $\Rightarrow$ no diagrams. Can use it in fit, including one/more yields at same centrality as σ_{dyn} .

For large acceptance, can hope fitting both σ_{dyn} and $\sigma, \omega, v(Q)$

Fit exp. yields, ratios, ω, σ, $< s >= 0, \frac{Q}{B} = 0.4$ for

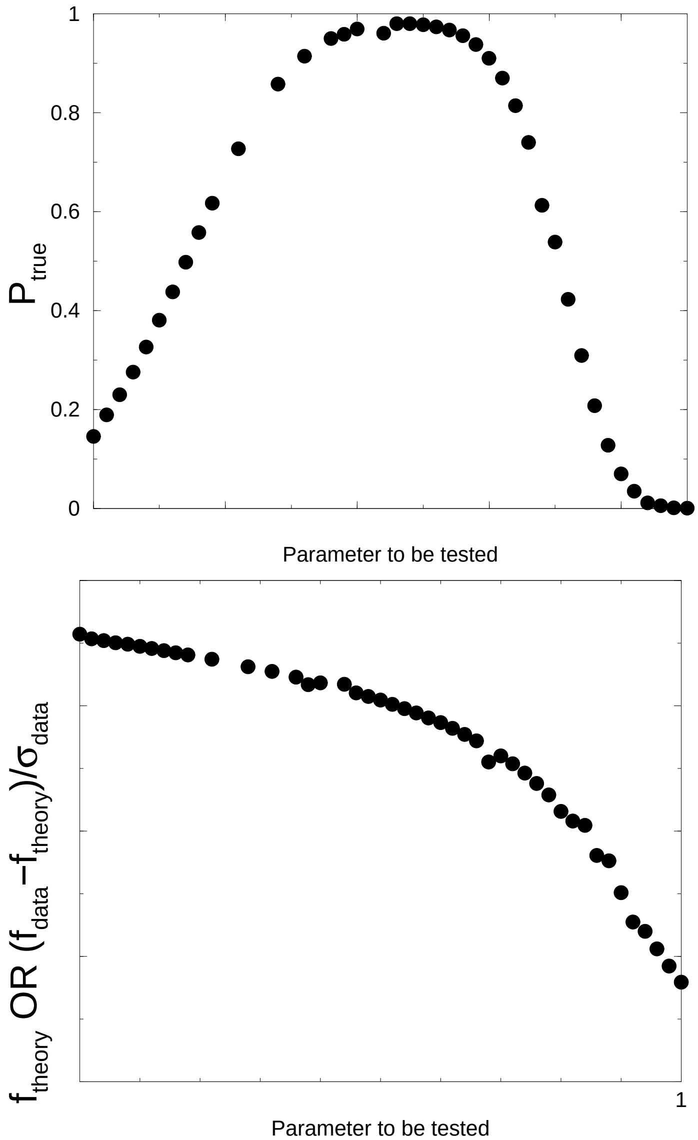
- Equilibrium parameters T, λ_{q,s,I_3}
- Non-equilibrium parameters γ_{q,s,I_3}
- System "volume" dV/dy



Different models have a different DoF $\rightarrow$ Use Statistical significance (P_{true}) to judge fit quality

Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance

<u>All other parameters at their best fit value for point in abscissa</u>



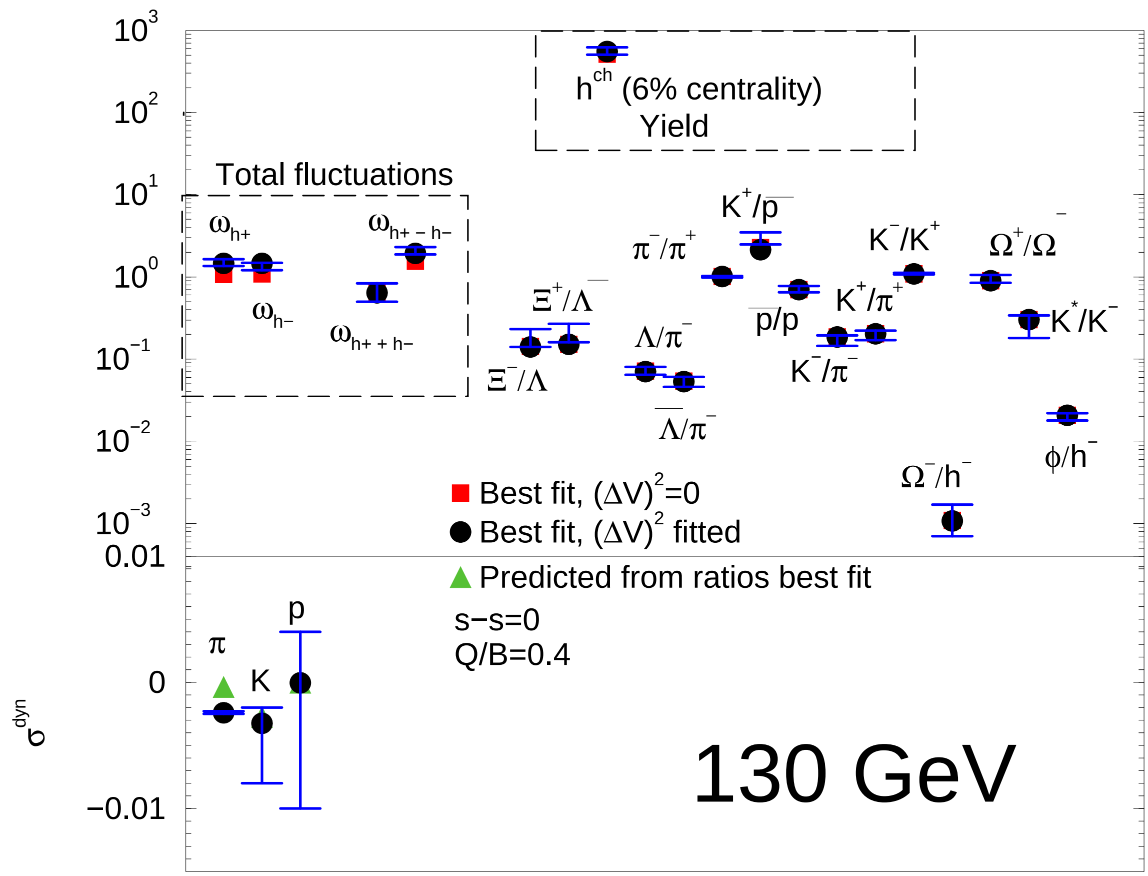
Fits at 130 GeV

fluctuations

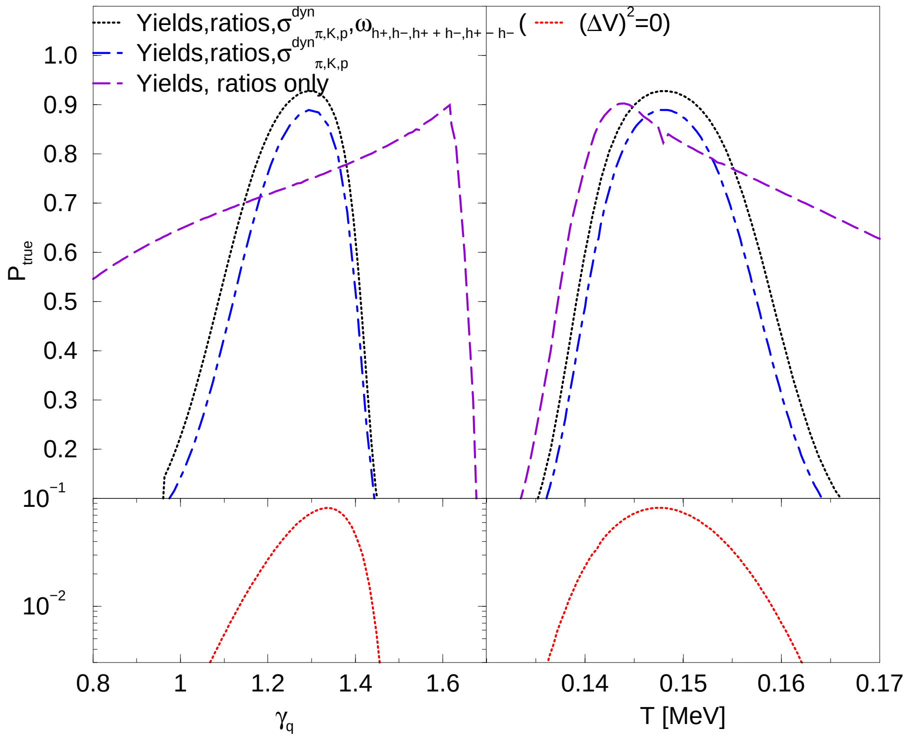
- $\sigma^{dyn}_{\pi^+/\pi^-}, \sigma^{dyn}_{K^+/K^-}, \sigma^{dyn}_{\overline{p}/p}$: STAR
  PRC 68, 044905 (2003), nucl-ex/0307007
  Insensitive to Volume fluctuations but require $< V >$ (6% centrality)
- $\omega_{h^+}, \omega_{h^-}, \omega_{h^+ - h^-}, \omega_{h^- + h^+}$ STAR
  Nucl. Phys. A 698 (2002) 611
  Need $\Delta V/V$, independent of centrality.
  No error bar, assume Ad Hoc 10%

Yields and Ratios

- $h^{charged}$, PHOBOS
  PRL (2003) , nucl-ex/0201005 (6% centrality)
- $\pi, K, p, \Lambda, \Xi, \phi, K^*$ and antiparticles, STAR,
  various (5% centrality)



- When volume fluctuation is used as a fit parameter, both yields and fluctuations fit nicely
- Volume fluctuations needed 4 $\omega_{h+,h-,h+h-}$,not4 $\sigma^{dyn}$and ω_{h+-h-}(as understood by Koch,Jeon).
- Fluctuations shift fitted γ_q, T w.r.t. ratios-only fit



T, γ_q not as well determined as it could be

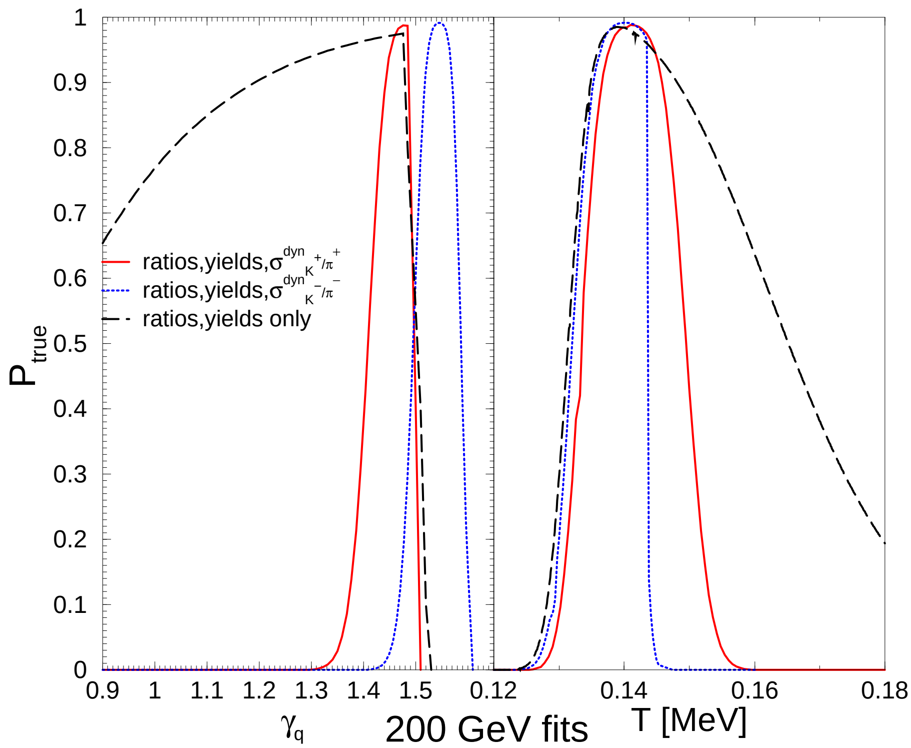
- As we saw earlier, $\sigma^{dyn}_{\pi,K,p}$are not very good constraines on γ_q
- Volume fluctuation <u>essential</u> for fitting $\omega_{h^+}, \omega_{h^-}, \omega_{h^+ \pm h^-}$(if $\Delta V = 0$, $P_{true} < 0.1$ due to disagreement with these data points), <u>but</u> correlates with normalization, γ_q.

Fits at 200 GeV

- $\sigma^{dyn}_{K/\pi}$: Supriya Das et al [STAR]
  nucl-ex/0503023

- Ratios:O. Barannikova et al [STAR]
  nucl-ex/0403014

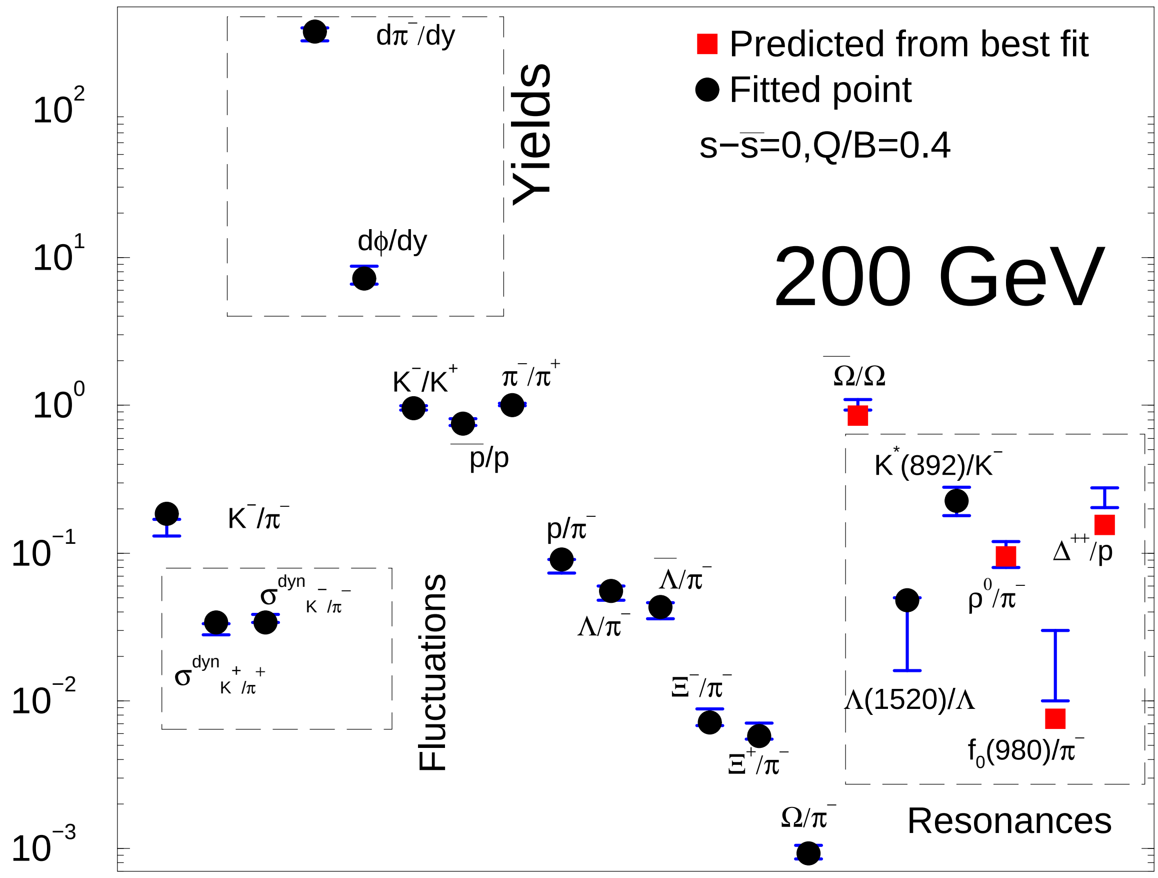
NB: All preliminary



With fluctuations, T, γ_q determined
($K/\pi+$ its fluctuation is very sensitive to γ_q).

- firmly in $\gamma_q > 1, T \sim 140$ MeV,
- T, γ_q consistent with 130 GeV considering all preliminary

but...

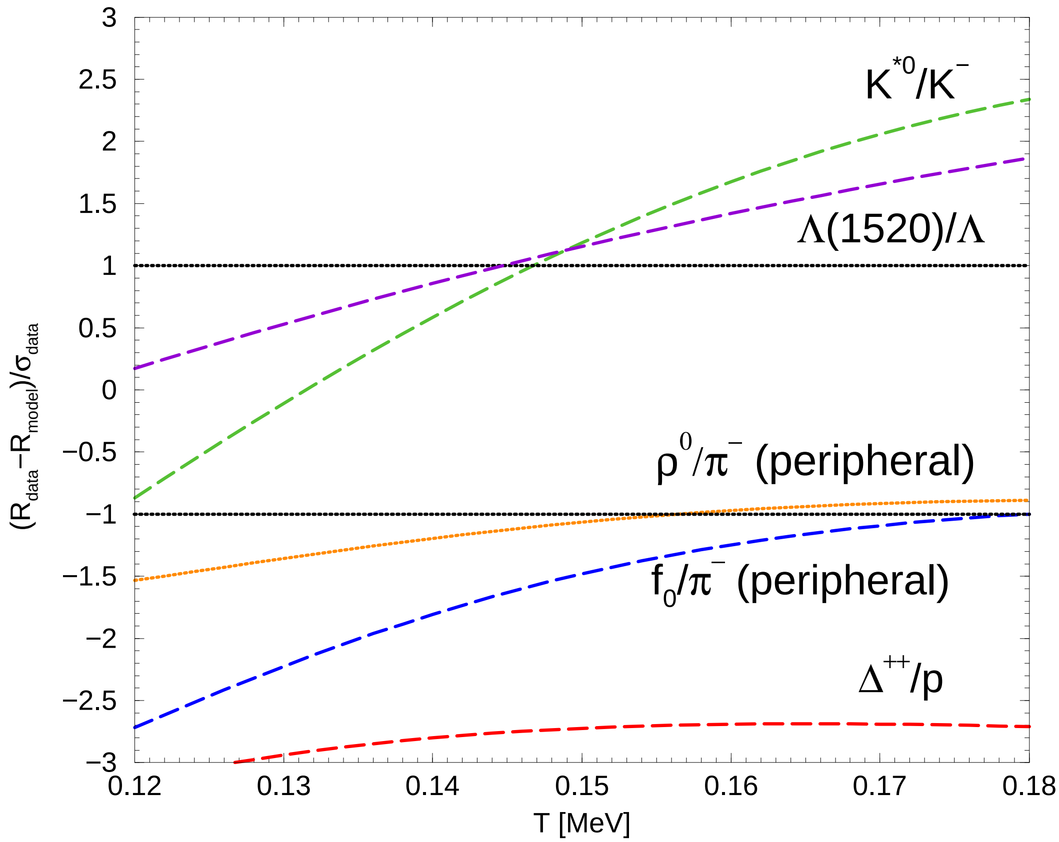


Some datapoints fail

$\sigma_{K+/\pi+}$ vs $\sigma_{K-/\pi-}$:	too different
$\overline{\Omega}/\Omega > 1$	not thermal: Liu,Bleicher,Aichelin
$\Delta^{++}/p \sim 0.25$	way too large

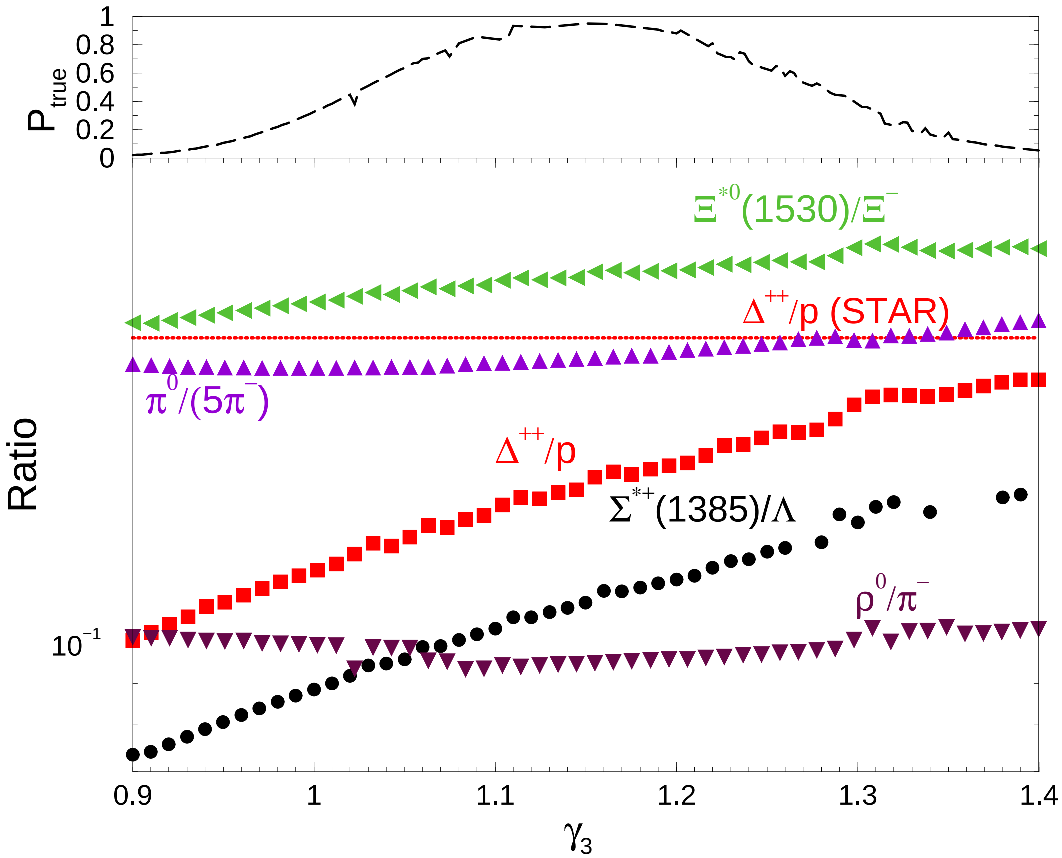
If these fitted, best fit unchanged but $P_{true} < 0.1$

Data-model (dis)agreement for ratios



$\frac{\Lambda^*}{\Lambda}, \frac{K^*}{K^-}$fail @ $\gamma_q = 1$, fit @ $\gamma_q = 1.6, T = 140$ GeV

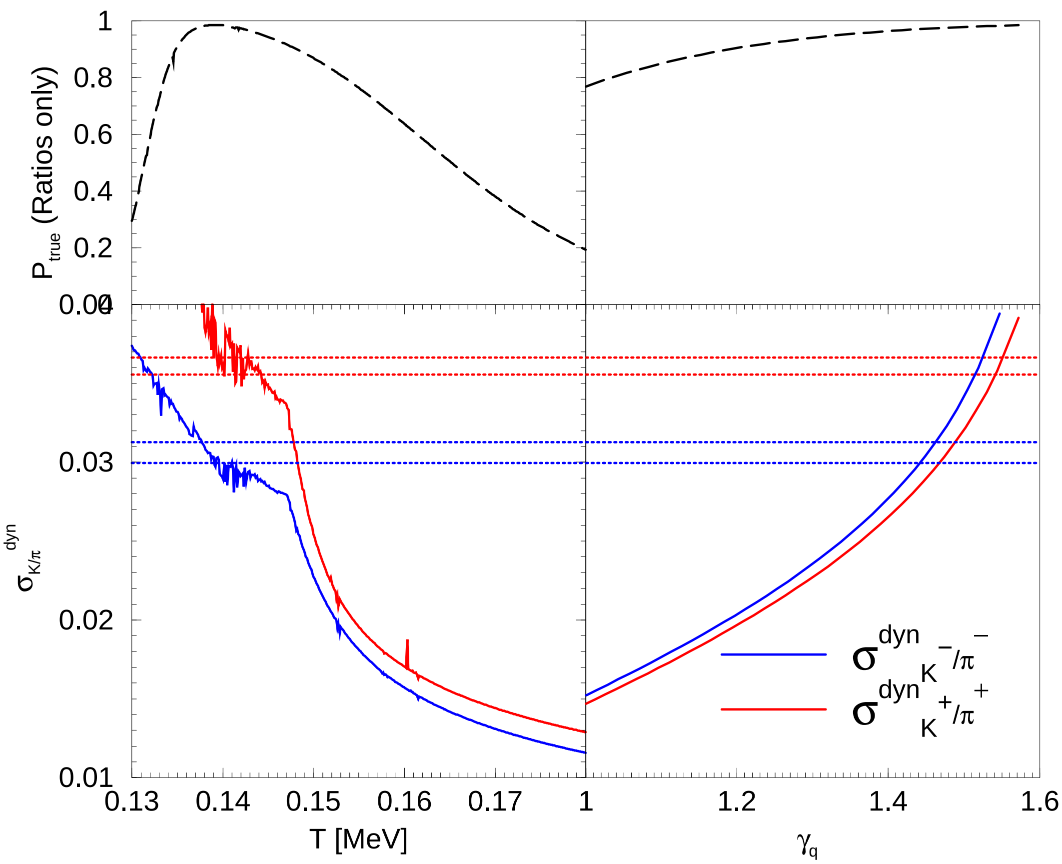
$\frac{\Delta^{++}}{p}$ hopeless (>2 std)



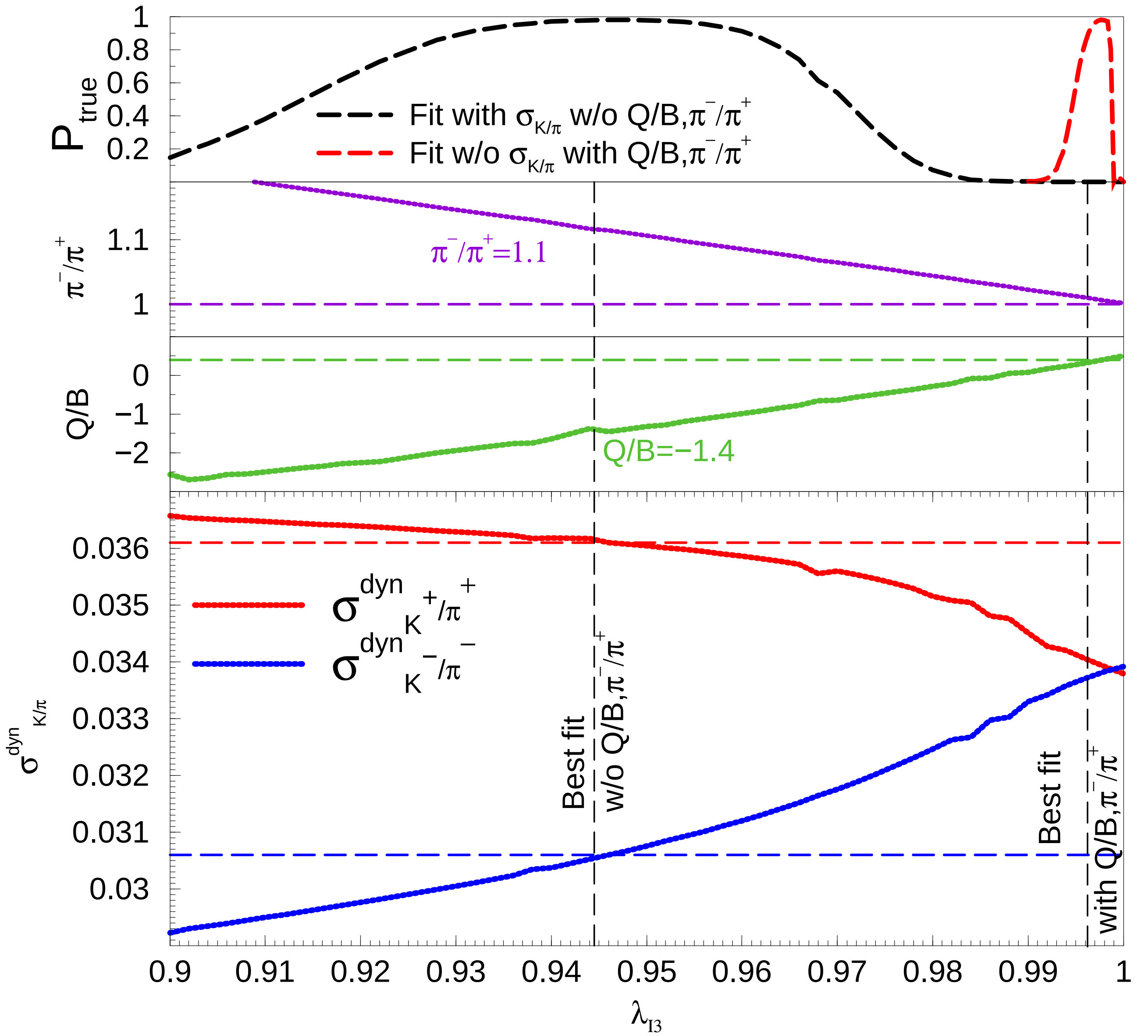
$\gamma_3 > 1$(u/d non-equilibrium) If $\gamma_3 \neq 1$,.

- Helps with Δ^{++}/p
- $\frac{\Sigma^+}{\Lambda}, \frac{\pi^0}{\pi^{\pm}}, \frac{\rho^0}{\pi^{\pm}}, \frac{f_0}{\pi^{\pm}}, \ldots$ enhanced.

Data-model (dis)agreement for $\sigma^{dyn}_{K/\pi}$



either σ_{K^-/π^-} or σ_{K^+/π^+} explained @ $\gamma_q > 1$, but not the difference



Only way in SHM to increase difference is λ_{I3}

necessary λ_{I3} excluded by $\pi^-/\pi^+, Q/B$ constraint

Some tentative conclusions

- Ξ/ϕ vs $v(Q), \sigma_{\pi^+/\pi^-}$fits SHM well, no indication of ρ, σ modification

- Yields, ratios, resonances and fluctuations fit reasonably well provided $\gamma_q > 1$

- very unlikely equilibrium SHM can do it

- $\Lambda(1520), K^*$ well accounted for in non-equilibrium. Something very funny going on with Δ^{++}

Do it yourself: SHARE

http://www.physics.arizona.edu/∼torrieri/SHARE/share.html