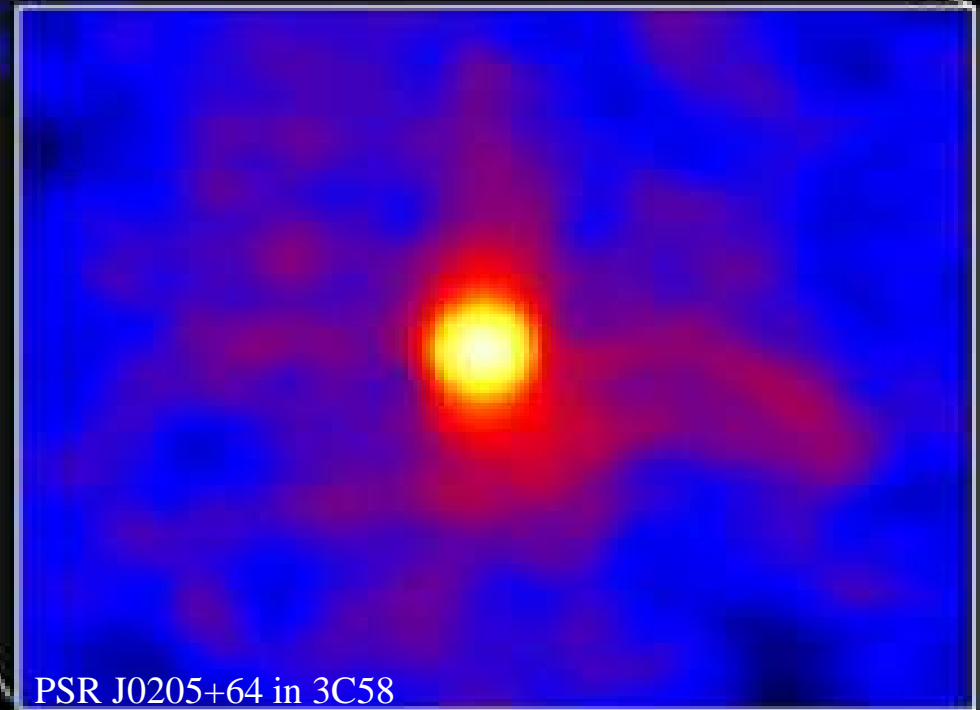
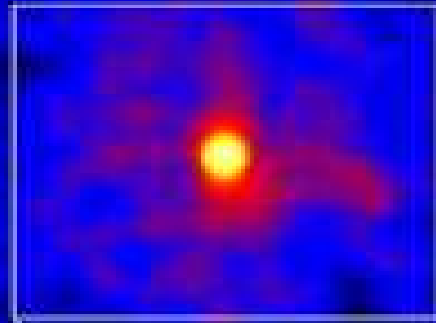


Cooling of Neutron Stars with Color Superconducting Quark Cores

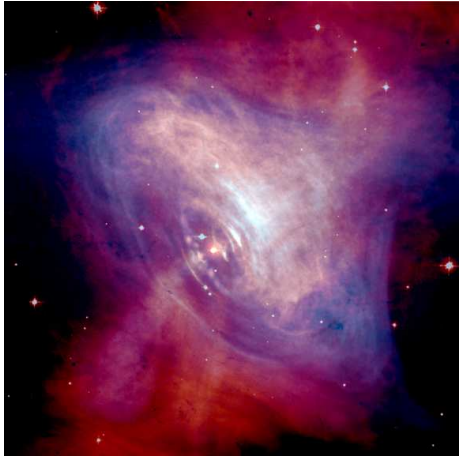
David Blaschke
GSI Darmstadt & JINR Dubna



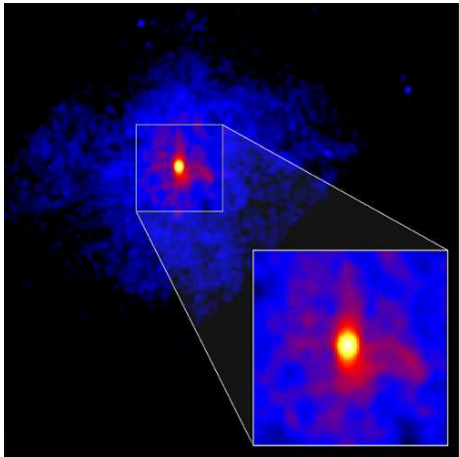
Compact Star Cooling - Introduction

1. Introduction
2. Hadronic Cooling
3. Quark Matter Phase Diagram
4. Hybrid Star Cooling
5. Conclusions

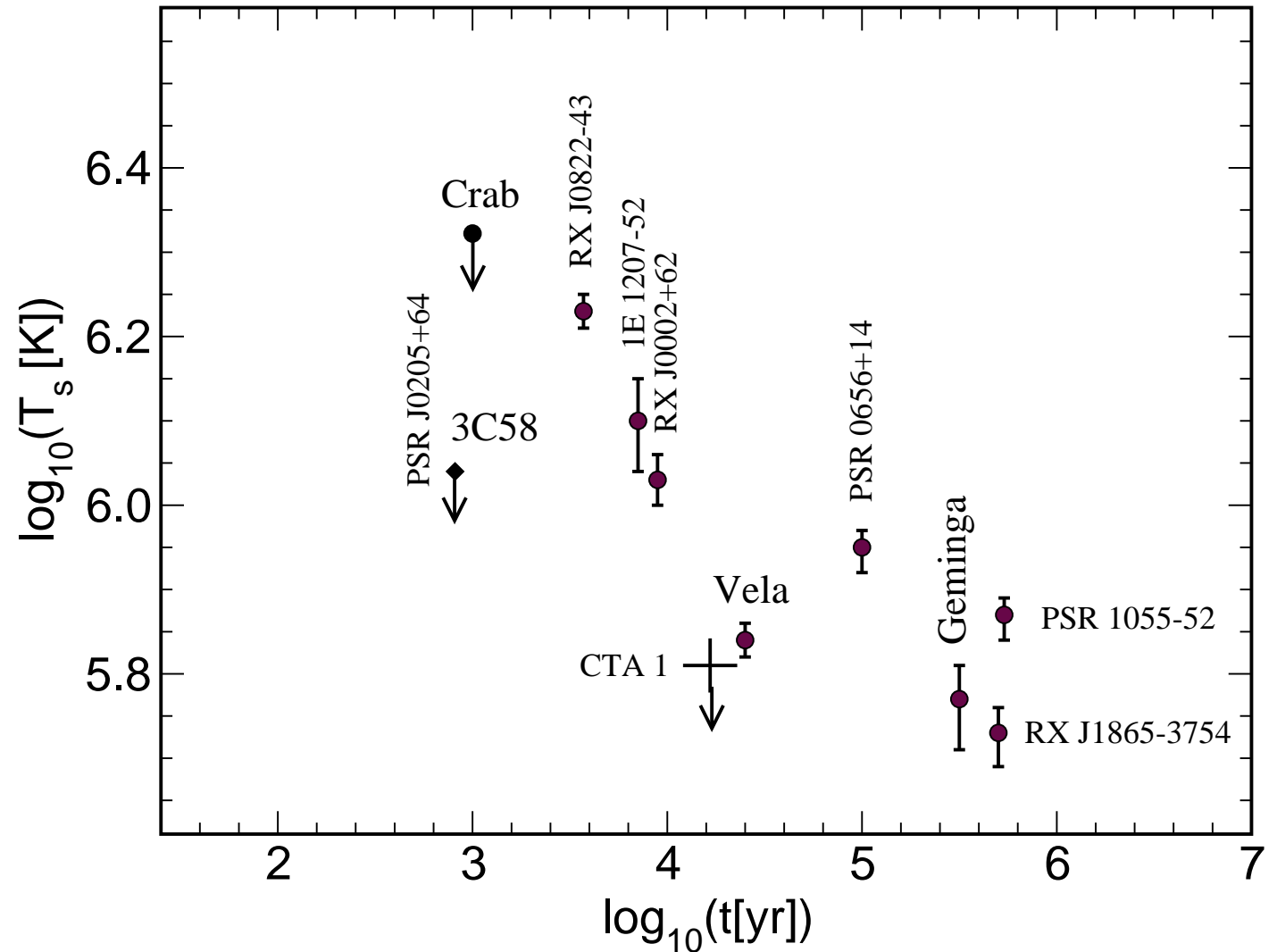
Pulsars in SN remnants:
1054 - Crab



1181 - 3C58



Temperature - age plot: characterizes compact star matter properties



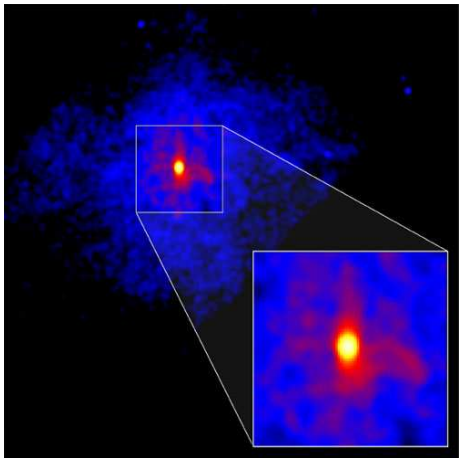
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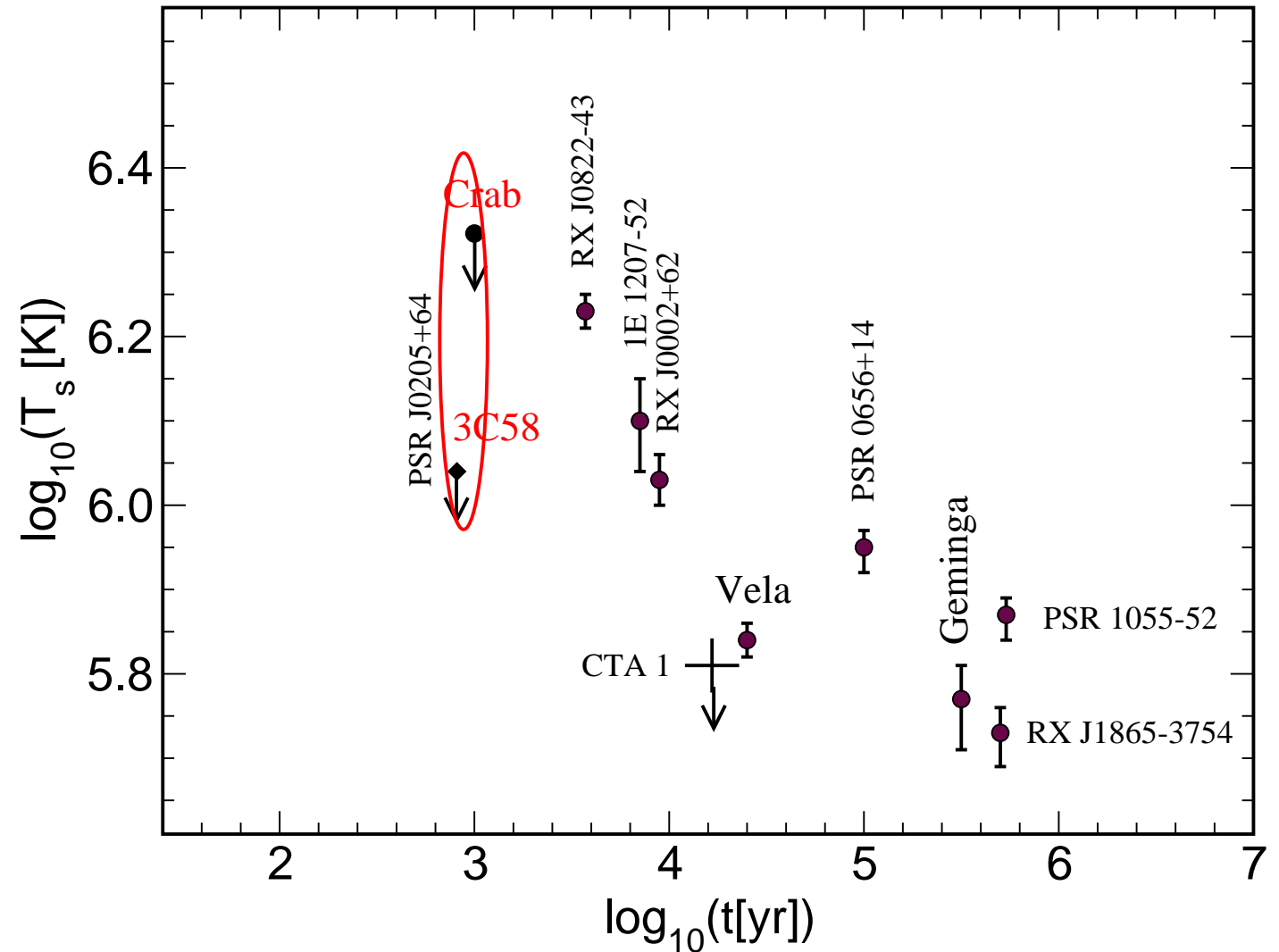
Pulsars in SN remnants:
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1181 - 3C58



Too cool for its age: **Quark matter in PSR J0205+64** ? (NASA 2002)



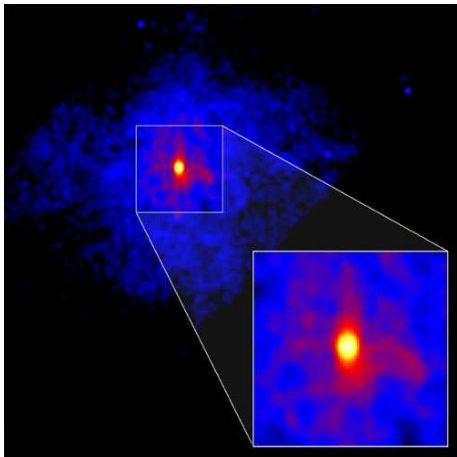
Compact Star Cooling - Phenomenology

1. Introduction
2. Hadronic Cooling
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5. Conclusions

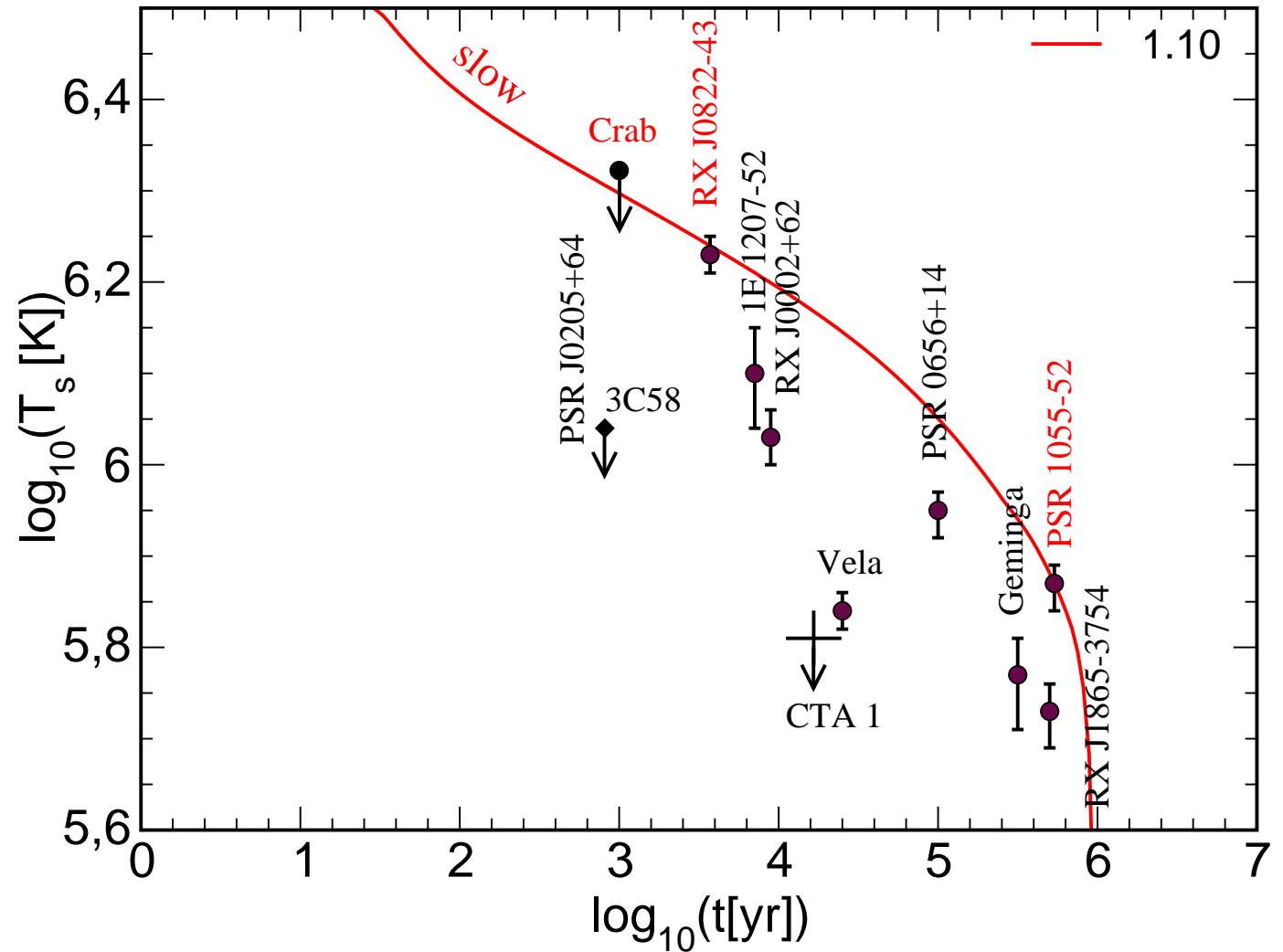
Pulsars in SN remnants:
1054 - Crab



1181 - 3C58



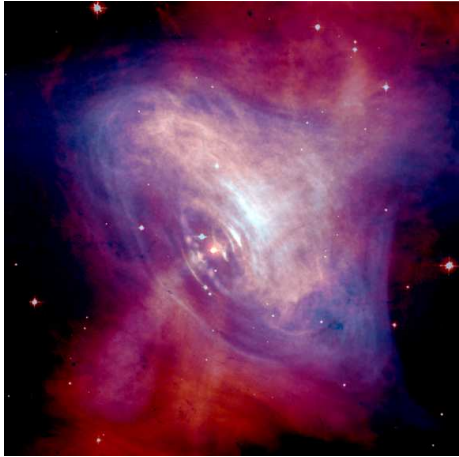
Temperature - age plot: characterizes compact star matter properties



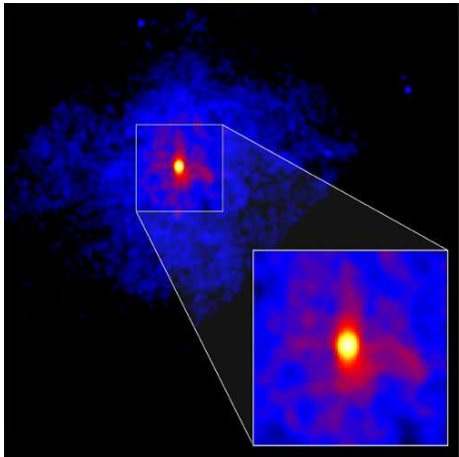
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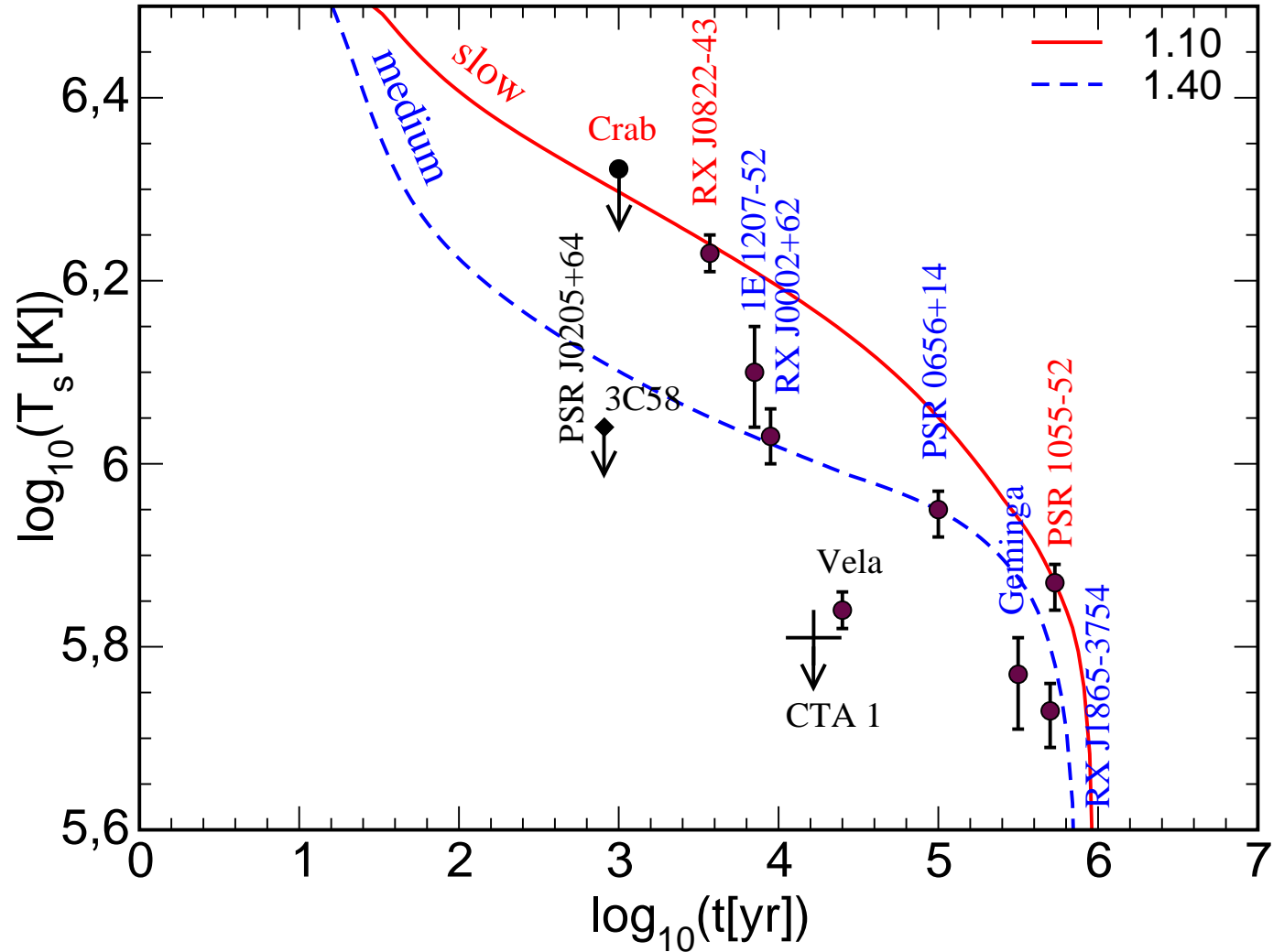
Pulsars in SN remnants:
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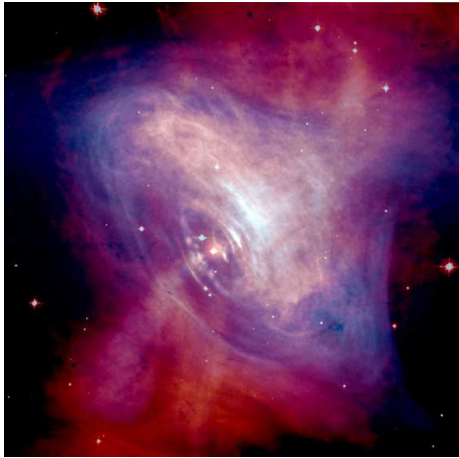
Temperature - age plot: characterizes compact star matter properties



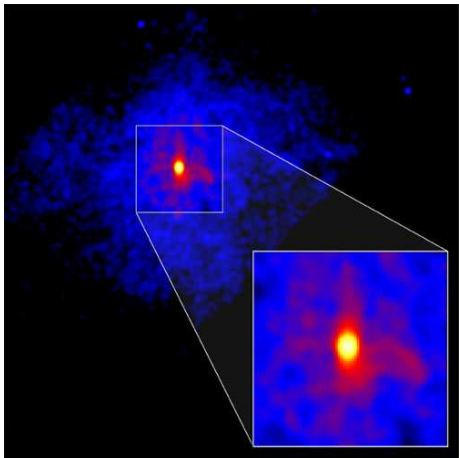
Compact Star Cooling - Introduction

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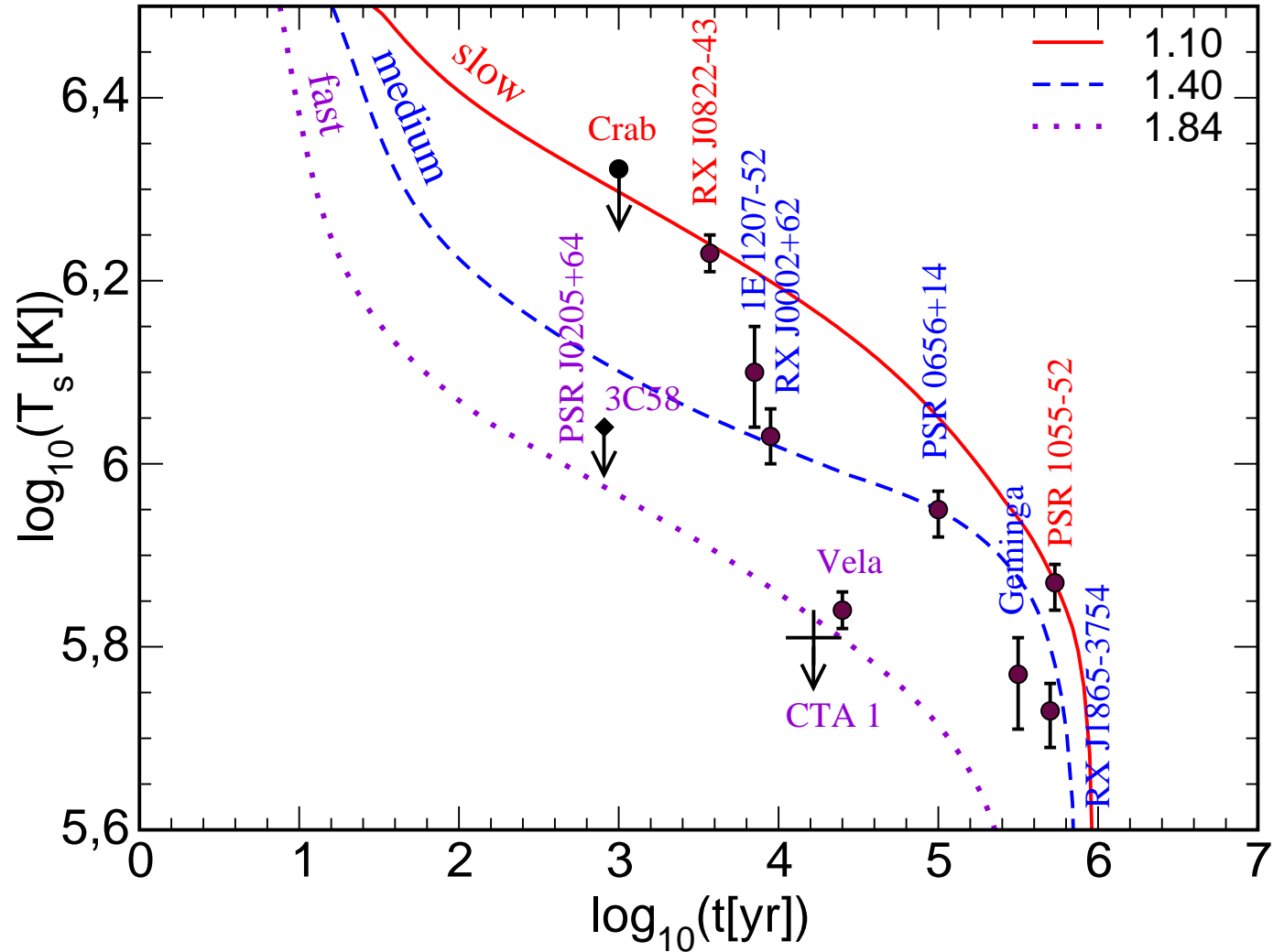
Pulsars in SN remnants:
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1181 - 3C58



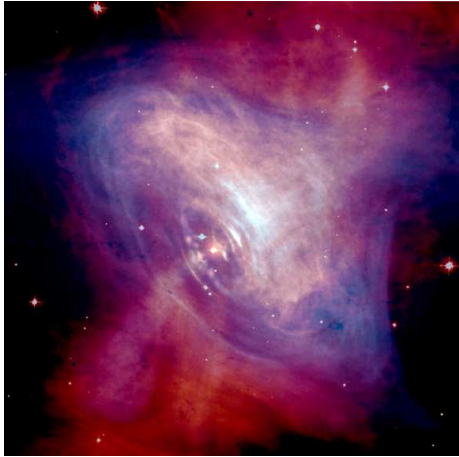
Classification of cooling compact stars



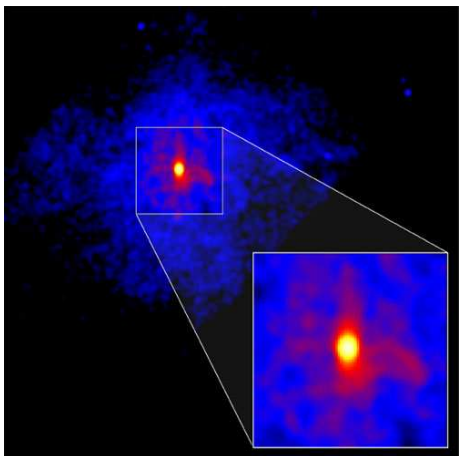
Compact Star Cooling - Hadronic Scenario

1. Introduction
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Pulsars in SN remnants:
1054 - Crab

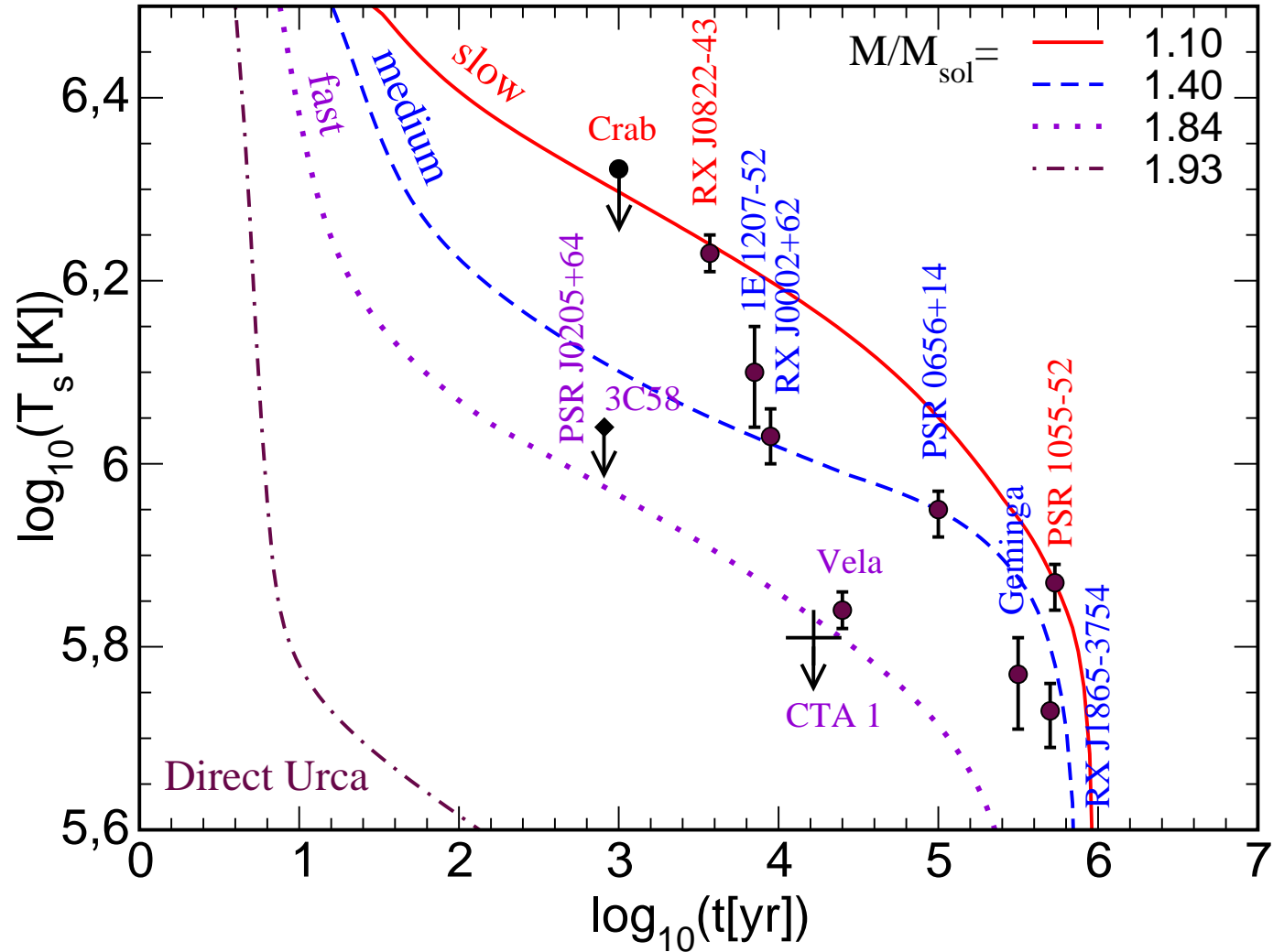


1181 - 3C58



Classification of cooling compact stars: **parameter - mass**

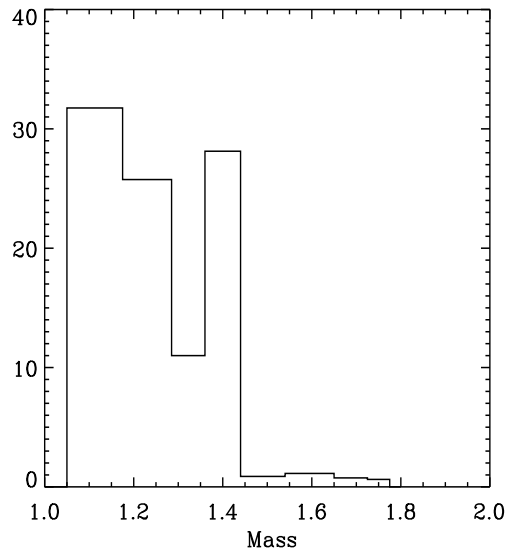
D.B., Grigorian, Voskresensky, A& A 424, 979 (2004)



Compact Star Cooling - Hadronic Scenario

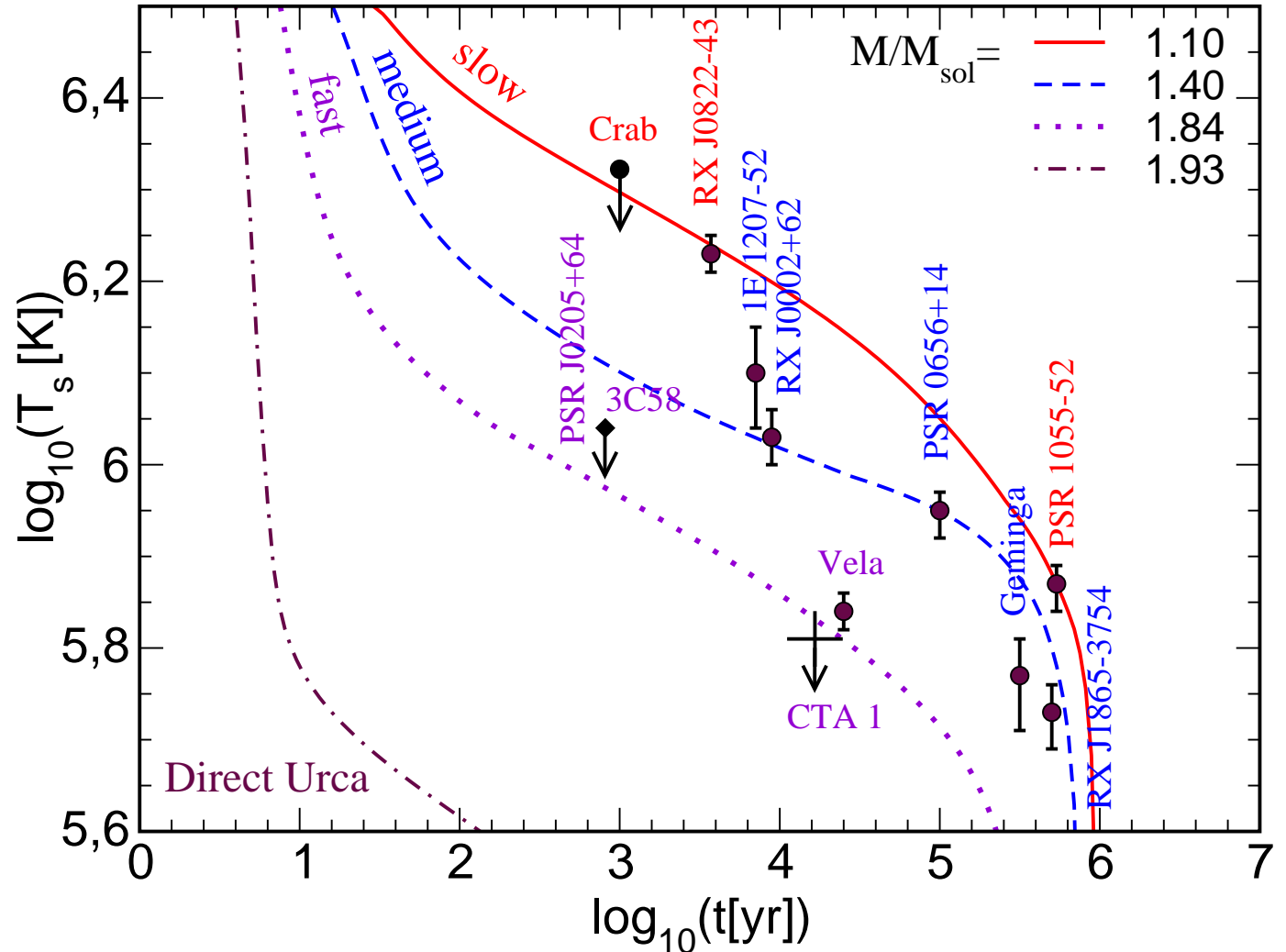
1. Introduction
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Mass distribution from population synthesis models for the solar vicinity



Popov et al: astro-ph/0411618
 Radiopulsar masses ($1.4 M_{\odot}$) not reliable for isolated pulsars
 - evolutionary effects in binaries

Classification of cooling compact stars: **parameter - mass**



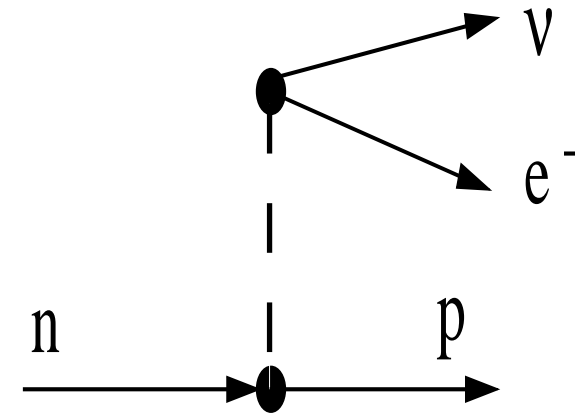
Caution: Beware of the direct Urca process!

1. Introduction
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Casino DE URCA today ...

First studied by Gamov and Schönberg
(≈ 1940)



$$\varepsilon_{\nu}[DU] \sim 10^{27} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1}$$

Huge emissivity \rightarrow **cools** the star **too fastly!!**

Direct Urca process constraint

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DU process w/o neutrino trapping ($\lambda_\nu \gg R, \mu_\nu = 0$):

β -Equilibrium: $\mu_n = \mu_p + \mu_e$

Charge neutrality: $n_p = n_e + n_\mu \Leftrightarrow p_{F,p}^3 = p_{F,e}^3 + p_{F,\mu}^3$

Momentum conservation:

$\vec{p}_{F,n} = \vec{p}_{F,p} + \vec{p}_{F,e} \Leftrightarrow |\vec{p}_{F,n}| \leq |\vec{p}_{F,p}| + |\vec{p}_{F,e}|$

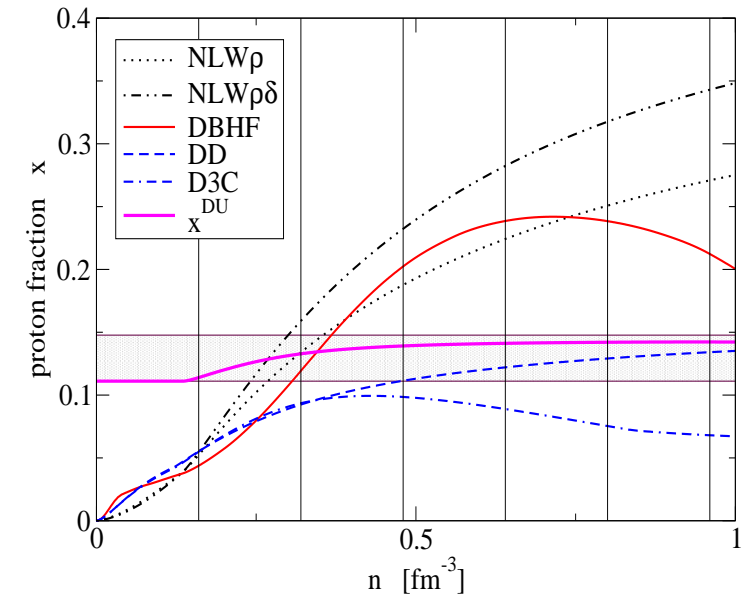
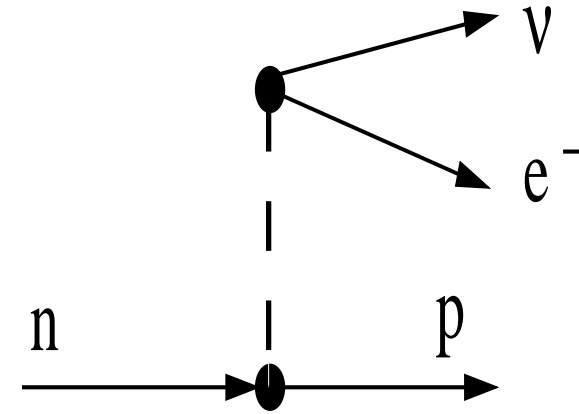
$p_{F,n} \leq p_{F,p} [1 + (1 - n_\mu/n_p)^{1/3}] \Rightarrow n_n \leq 8 n_p - 4 n_\mu$

$$\Rightarrow \frac{n_p}{n_p + n_n} = x_p \geq \frac{1}{9} + \frac{4}{9} x_\mu$$

Luminosity:

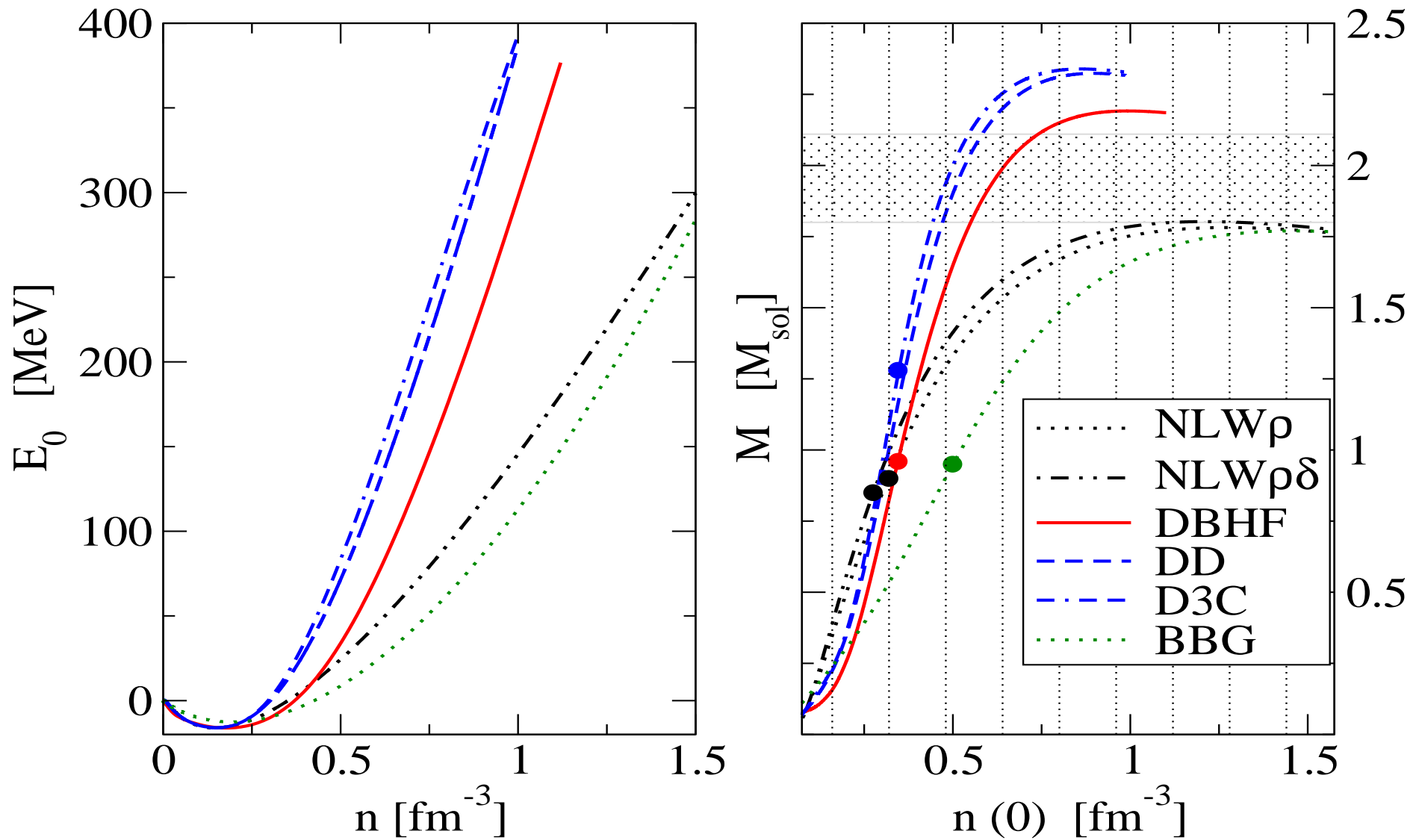
$L_\nu = (2\pi)^4 \int \frac{d^3 p_n}{(2\pi)^3 2E_n} \cdots \int \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} \delta^3(\vec{p}_i) \delta(E_i) |M_{fi}|^2 f_n (1 - f_p) (1 - f_e)$

Emissivity: $\epsilon_\nu = \frac{L_\nu}{V} \sim 10^{27} \left(\frac{m_n^* m_p^*}{m_N^2} \right) \left(\frac{n_e}{n_0} \right)^{1/3} \left(\frac{T}{10^9 K} \right)^6 \frac{\text{erg}}{\text{cm}^3 \text{s}}$



DU constraint eliminates most hadronic EoS

1. Introduction
2. Hadronic Cooling
3. Quark Matter Phase Diagram
4. Hybrid Star Cooling
5. Conclusions



RMF EoS violate DU constraint unless couplings and masses are density dependent !
Kolomeitsev, Voskresensky, [nucl-th/0410063](#); Klähn, et al., in prep.

Quark Matter Phase Diagram

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4. Hybrid Star Cooling
5. Conclusions

$$\begin{aligned}\Omega(T, \mu) &= \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} \\ &- T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left(\frac{1}{T} S^{-1}(i\omega_n, \vec{p}) \right) \\ &+ \Omega_e - \Omega_0.\end{aligned}$$

Inverse propagator of Nambu-Gorkov spinors

$$S^{-1}(i\omega_n, \vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M + \mu\gamma^0 & \hat{\Delta} \\ \hat{\Delta}^\dagger & \gamma_\mu p^\mu - M - \mu\gamma^0 \end{bmatrix},$$

with diquark gaps ($\Delta_{ur} = \Delta_{ds}, \dots$)

$$\Delta_{k\gamma} = 2G_D \langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} q_{j\beta}^C \rangle.$$

as elements of the gap matrix

$$\hat{\Delta} = i\gamma_5 \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} \Delta_{k\gamma}.$$

Fermion determinant ($\text{Tr} \ln \mathbf{D} = \ln \det \mathbf{D}$)

$$\ln \det \left(\frac{1}{T} S^{-1}(i\omega_n, \vec{p}) \right) = 2 \sum_{a=1}^{18} \ln \left(\frac{\omega_n^2 + \lambda_a(\vec{p})^2}{T^2} \right).$$

Result for thermodynamic potential

$$\begin{aligned}\Omega(T, \mu) &= \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} \\ &- \int \frac{d^3p}{(2\pi)^3} \sum_{a=1}^{18} \left(\lambda_a + 2T \ln \left(1 + e^{-\lambda_a/T} \right) \right) \\ &+ \Omega_e - \Omega_0.\end{aligned}$$

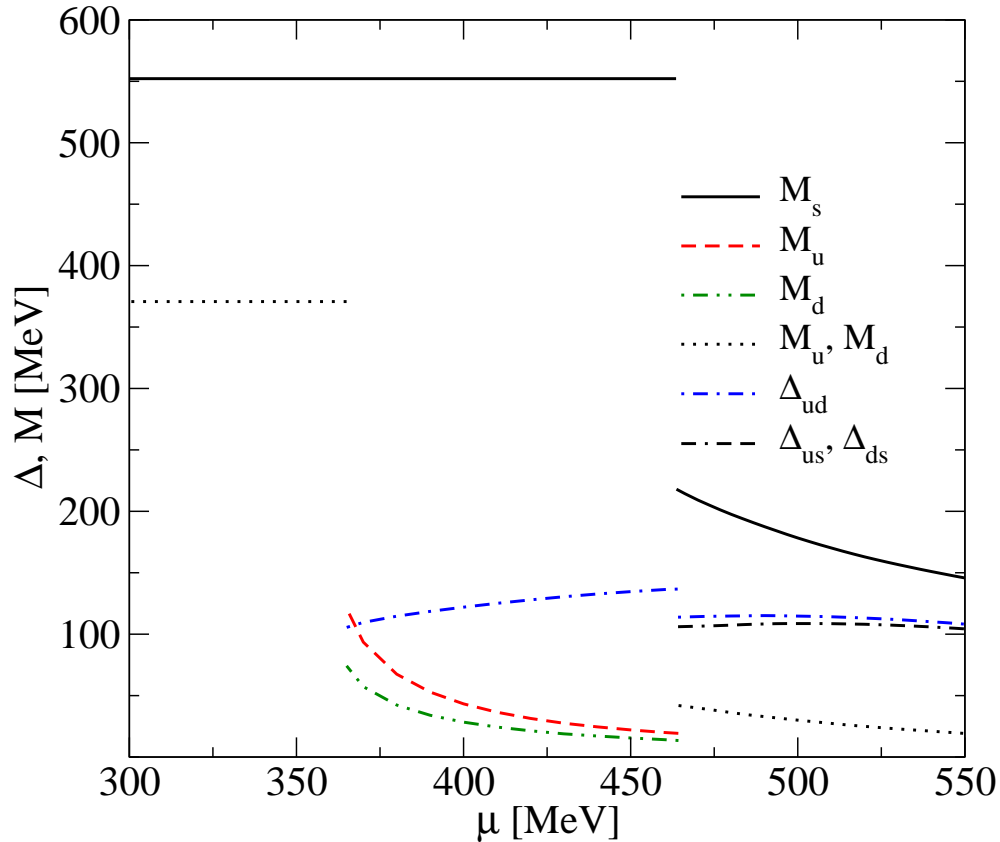
Neutrality conditions: $n_Q = n_8 = n_3 = 0$,

$$n_i = -\frac{\partial \Omega}{\partial \mu_i} = 0,$$

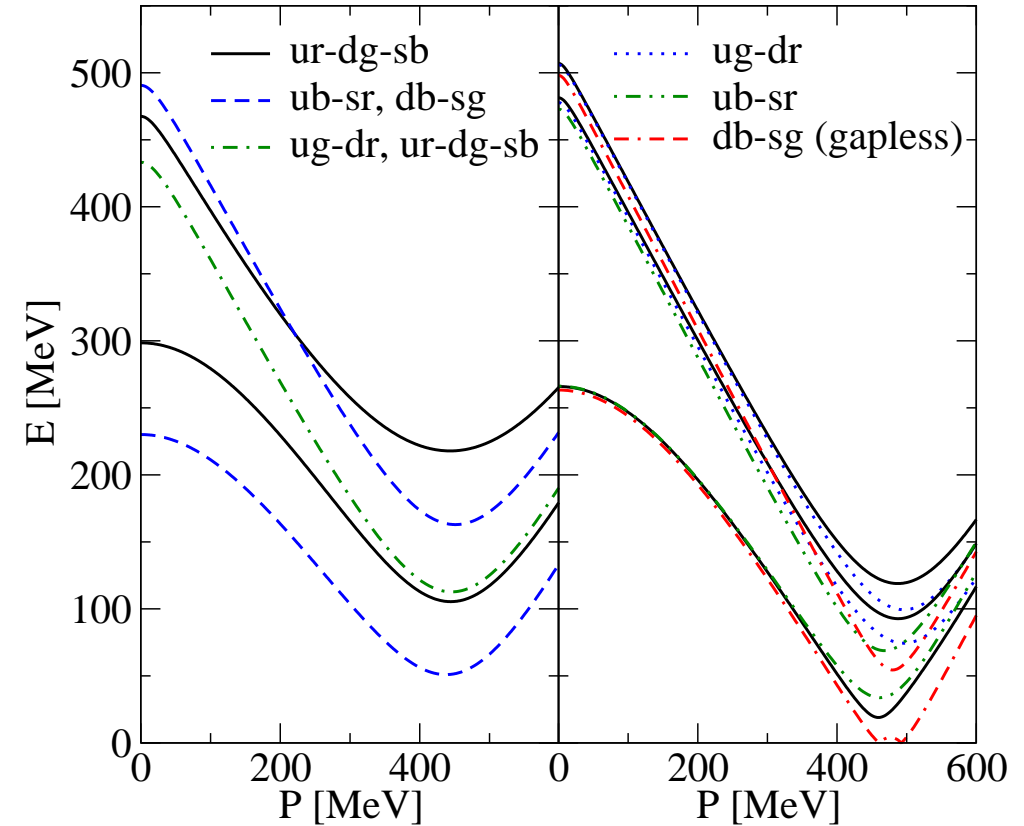
Equation of state: $P = -\Omega$, etc.

Quark Masses, Diquark Gaps, Gapless Modes

1. Introduction
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3. Quark Matter Phase Diagram
4. Hybrid Star Cooling
5. Conclusions



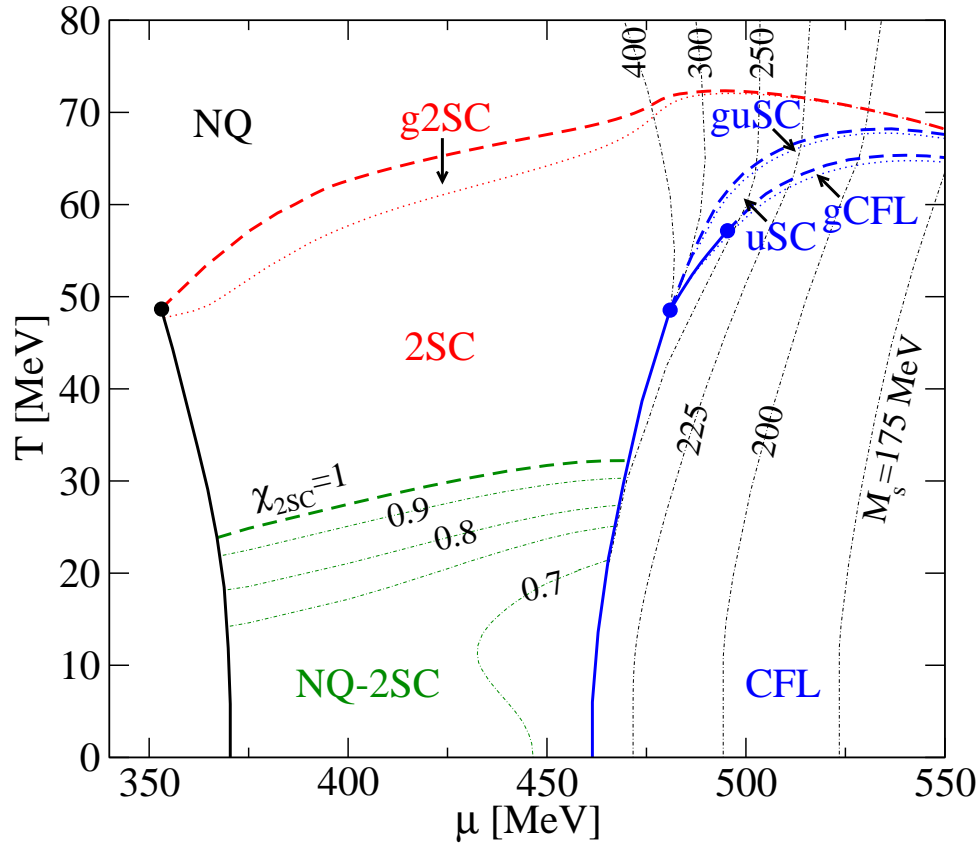
Dynamical quark masses and diquark gaps at $T = 0$ for intermediate diquark coupling $G_D = 0.75 G_S$



Dispersion relations for $G_D = 0.75 G_S, T = 0, \mu = 465$ MeV (left), $G_D = 1.0 G_S, T = 59$ MeV, $\mu = 500$ MeV (right)

Three-flavor Quark Matter Phase Diagram

1. Introduction
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5. Conclusions



Rüster et al: [hep-ph/0503184](https://arxiv.org/abs/hep-ph/0503184)

Blaschke et al: [hep-ph/0503194](https://arxiv.org/abs/hep-ph/0503194)

The phases are:

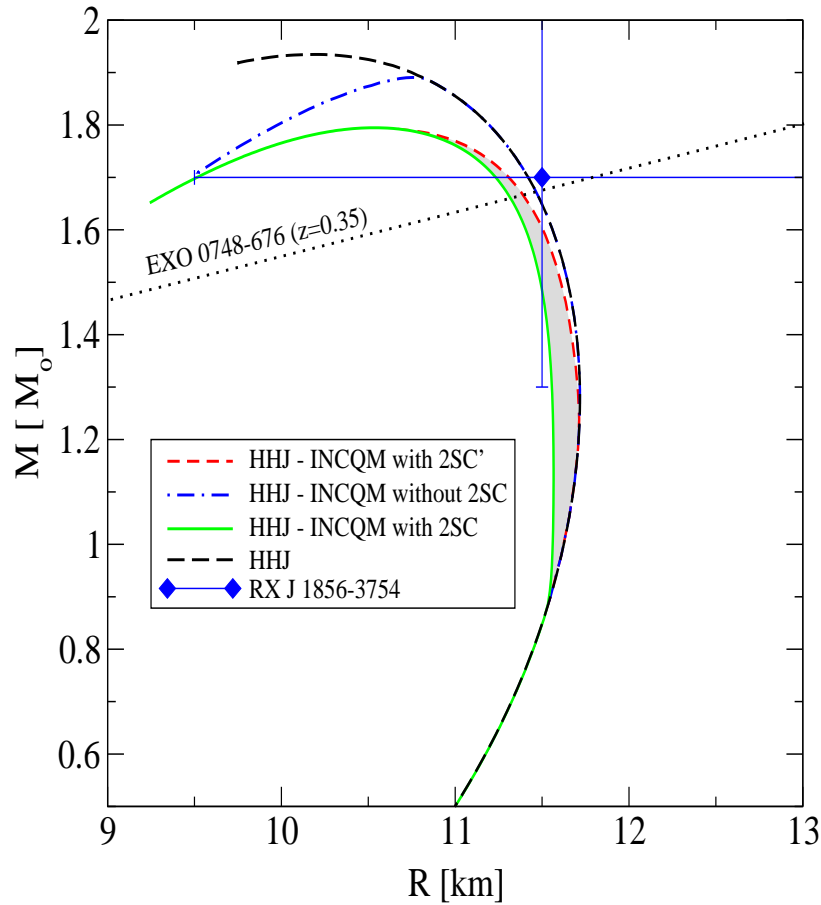
- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0, \Delta_{us} = \Delta_{ds} = 0, 0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0, \Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0, \Delta_{us} \neq 0, \Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0, \Delta_{ds} \neq 0, \Delta_{us} \neq 0$;

Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At $T \leq 25-30$ MeV: mixed NQ-2SC phase,
- Critical point $(T_c, \mu_c) = (48 \text{ MeV}, 353 \text{ MeV})$,
- Strong coupling, $G_D = G_S$, similar, no NQ-2SC mixed phase.

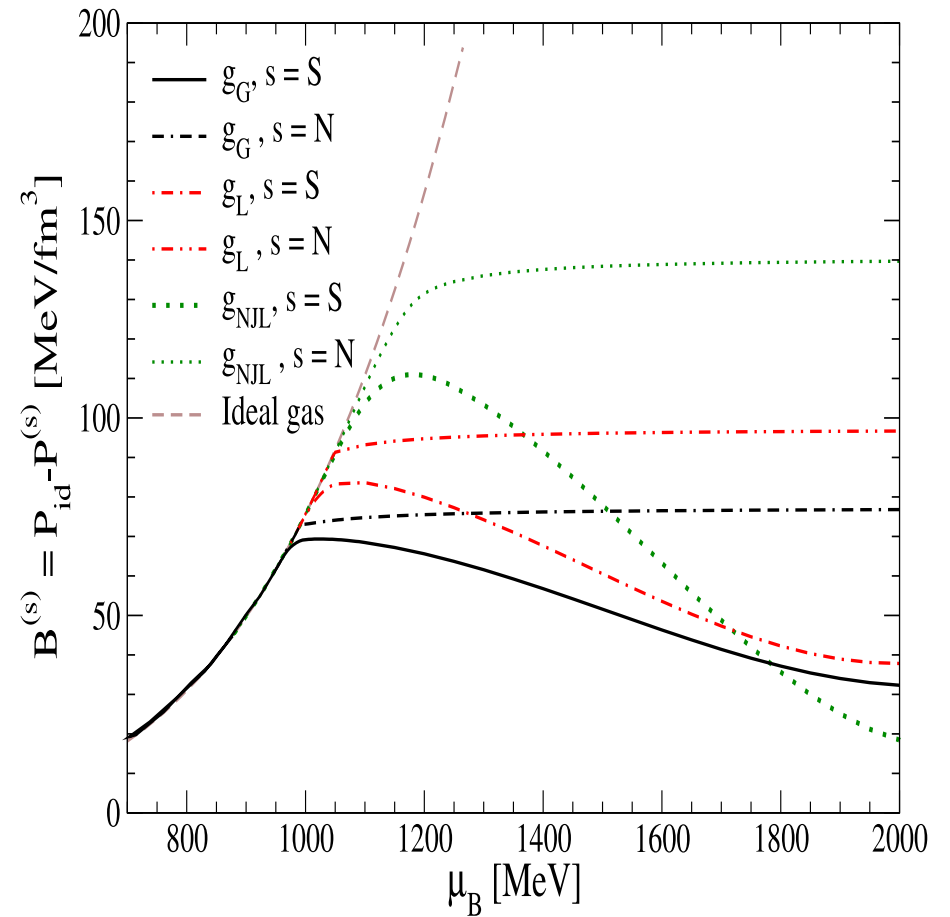
Hybrid Star Configurations

1. Introduction
2. Hadronic Cooling
3. Quark Matter Phase Diagram
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5. Conclusions



Mass-radius relation for hybrid star with HHJ and instantaneous nonlocal chiral quark model.

Grigorian et al, PRC 71 (2005)



Density dependent bag constant corresponding to the INCQM for Gaussian, Lorentzian and NJL formfactors.

Grigorian et al, PRC 69 (2004)

General Relativistic Cooling Equations

1. Introduction
2. Hadronic Cooling
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5. Conclusions

The energy flux per unit time $l(r)$ through a spherical slice at distance r from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial(Te^\Phi)}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$

The factor $e^{-\Phi} \sqrt{1 - \frac{2M}{r}}$ corresponds to relativistic corrections of time and distance scales.

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(le^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(Te^\Phi))$$

$$\frac{\partial}{\partial N_B}(Te^\Phi) = -\frac{1}{k} \frac{le^\Phi}{16\pi^2 r^4 n}$$

where $n = n(r)$ is the baryon number density, $N_B = N_B(r)$ is the total baryon number in the sphere with radius r and

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n \left(1 - \frac{2M}{r}\right)^{-1/2}$$

F. Weber: Pulsars as Astrophys. Labs ... (1999); D.B., Grigorian, Voskresensky, A&A 368 (2001) 561.

Neutrino processes in quark matter: Emissivities

1. Introduction
2. Hadronic Cooling
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5. Conclusions

- Quark direct Urca (QDU) the most efficient processes



$$\epsilon_{\nu}^{\text{QDU}} \simeq 9.4 \times 10^{26} \alpha_s u Y_e^{1/3} \zeta_{\text{QDU}} T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1},$$

Compression $u = n/n_0 \simeq 2$, strong coupling $\alpha_s \approx 1$

- Quark Modified Urca (QMU) and Quark Bremsstrahlung (QB)



$$\epsilon_{\nu}^{\text{QMU}} \sim \epsilon_{\nu}^{\text{QB}} \simeq 9.0 \times 10^{19} \zeta_{\text{QMU}} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1}.$$

- Suppression due to the pairing

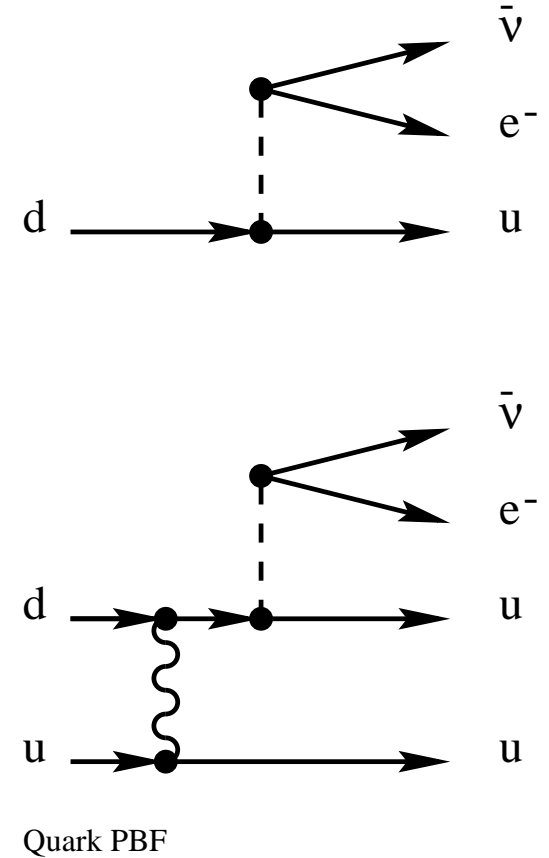
$$\text{QDU} : \zeta_{\text{QDU}} \sim \exp(-\Delta_q/T)$$

$$\text{QMU and QB} : \zeta_{\text{QMU}} \sim \exp(-2\Delta_q/T) \text{ for } T < T_{\text{crit},q} \simeq 0.57 \Delta_q$$

- $e + e \rightarrow e + e + \nu + \bar{\nu}$

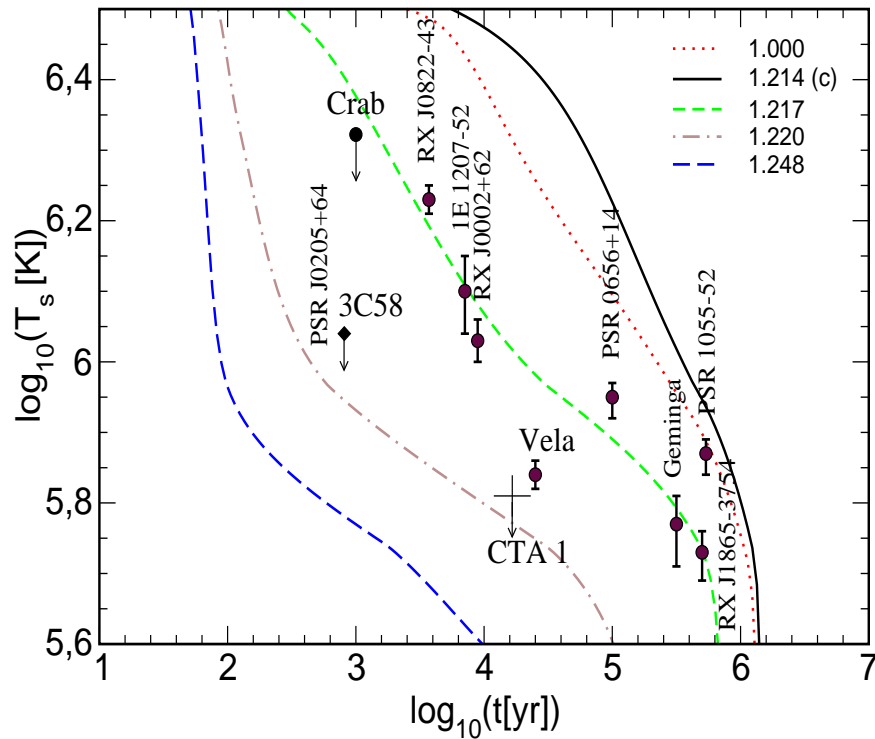
$$\epsilon_{\nu}^{ee} = 2.8 \times 10^{12} Y_e^{1/3} u^{1/3} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1},$$

becomes important for $\Delta_q/T \gg 1$

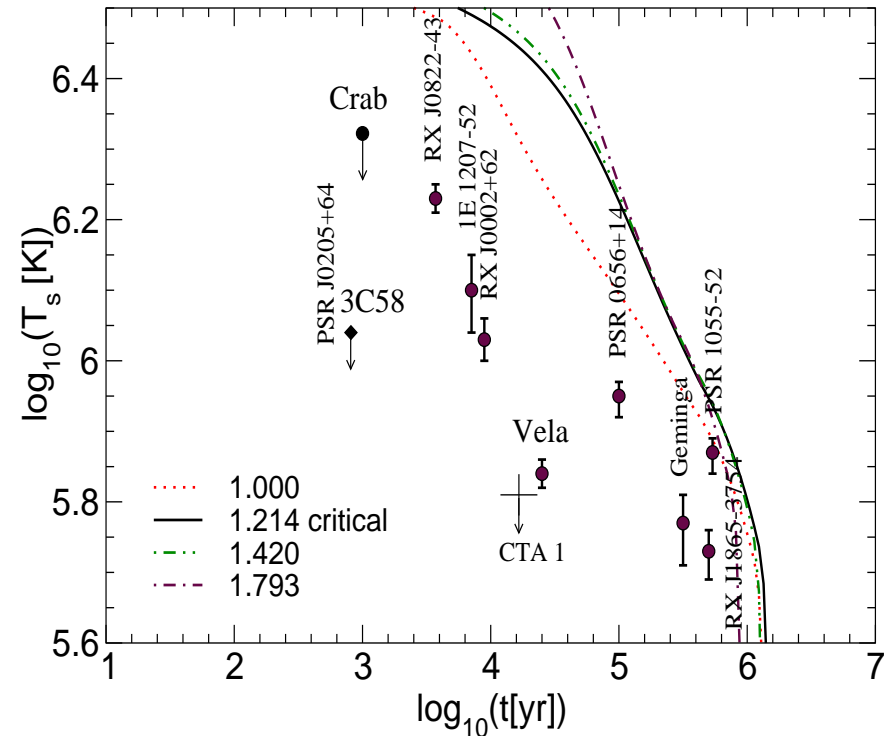


Hybrid Star Cooling with 2SC Quark Matter

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2SC phase - 1 color is unpaired
(mixed superconductivity)

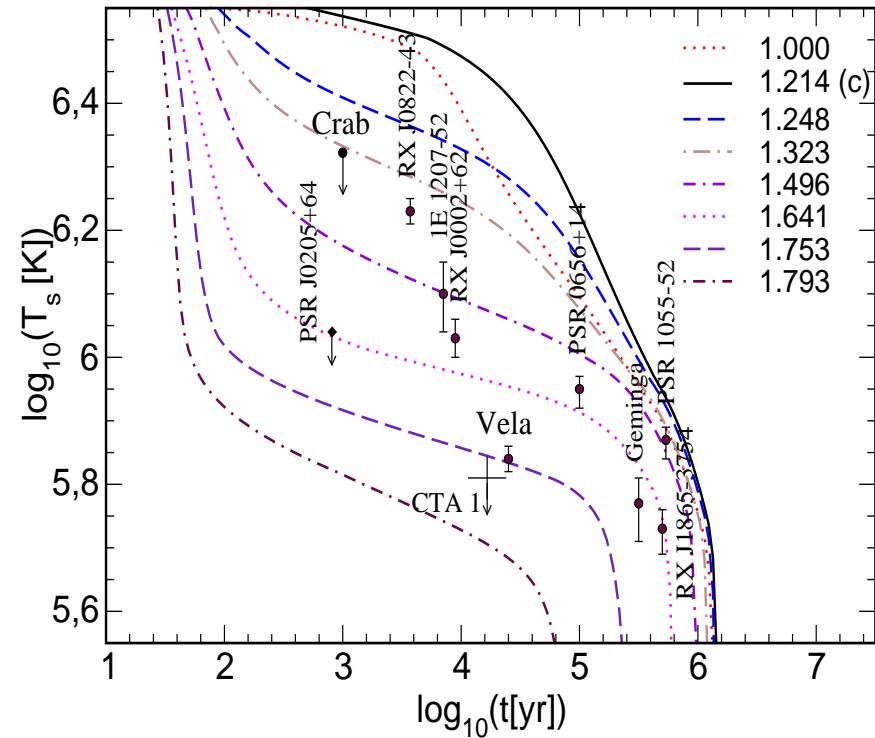
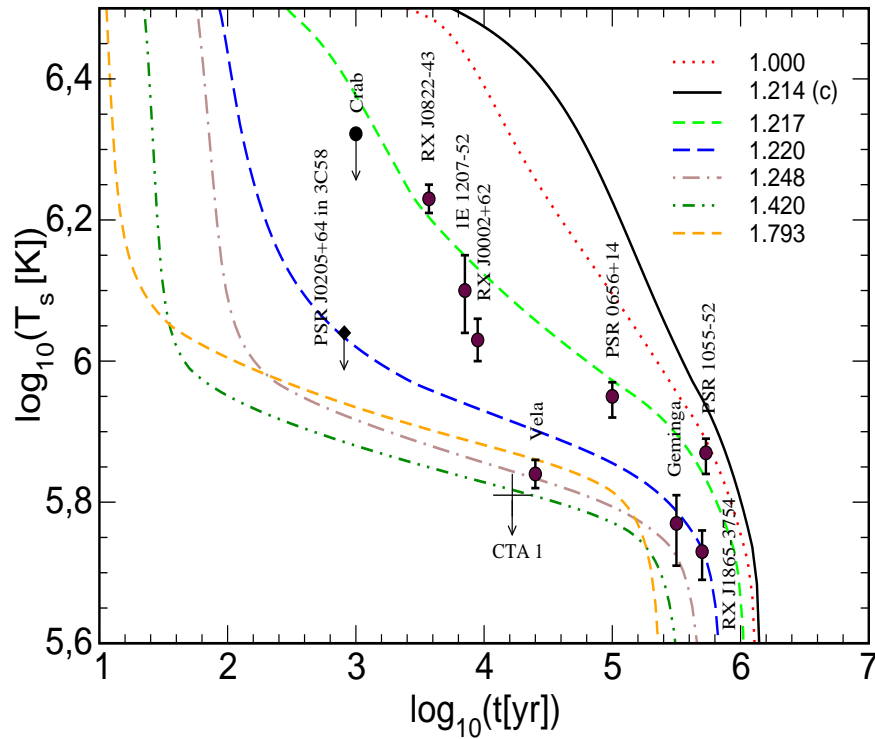


2SC + X phase,
 $\Delta_X = 1 \text{ MeV}$

Grigorian, D.B., Voskresensky, PRC 71 (2005)

Hybrid Star Cooling with 2SC Quark Matter

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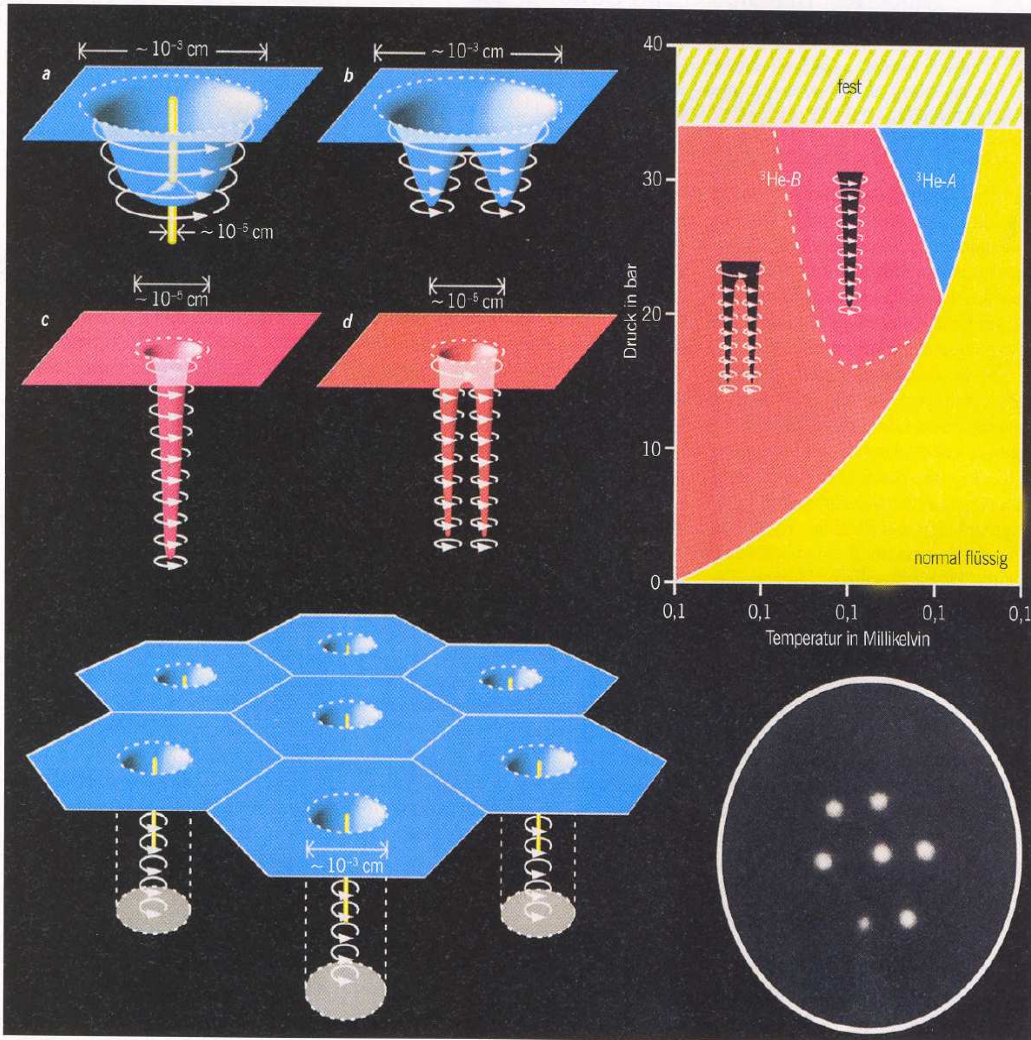
2SC + X phase, $\Delta_X = 30 \text{ keV}$

2SC + X phase, - μ dependent gap
 $\Delta_X(\mu) = 1 \text{ MeV} \exp(10(1 - \mu/\mu_c))$

Grigorian, D.B., Voskresensky, PRC 71 (2005)

Spin-1 (CSL) Phase for Compact Stars

1. Introduction
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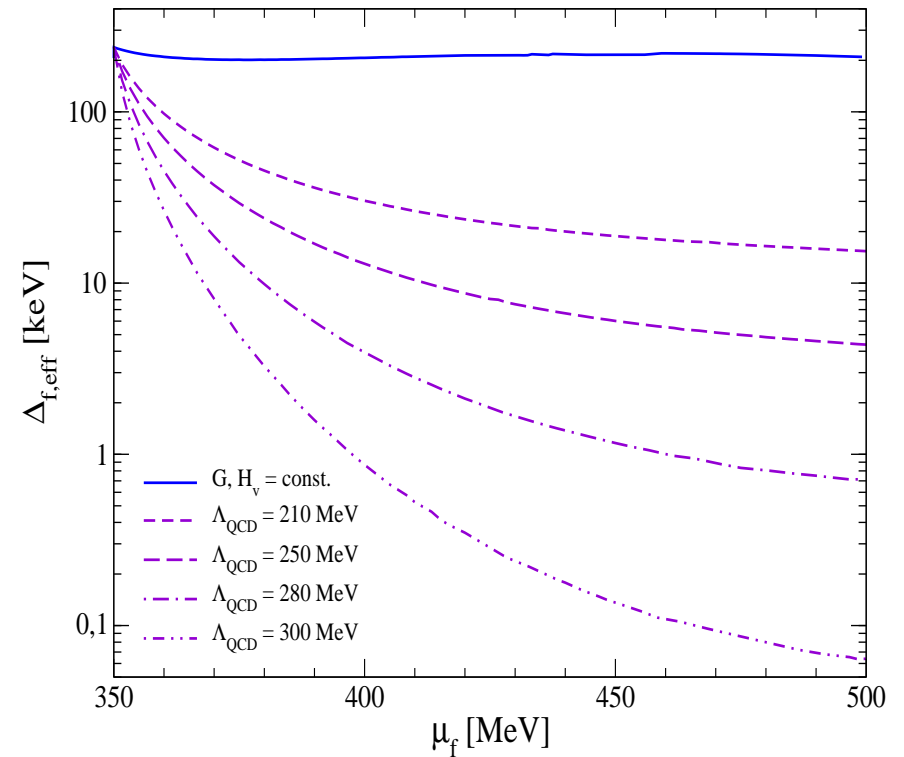
Phases of superfluid ${}^3\text{He}$

Color-spin-locking (CSL) condensate

Aguilera et al., hep-ph/0503288 \rightarrow PRD

$$\langle q_f^T C \gamma^3 \lambda_2 q_f \rangle = \langle q_f^T C \gamma^1 \lambda_7 q_f \rangle =$$

$$\langle q_f^T C \gamma^2 \lambda_5 q_f \rangle \equiv \eta_f ,$$



THEORY OF EOS WITH CONSTRAINTS

- Nuclear Matter - Quark Matter Phase transition depends on details of the modeling, e.g. Coupling Constants, Form Factors \Rightarrow Improve by using **DSE Models**
- Stable Neutron Stars with 2-Flavor Quark Matter Core possible

EXPERIMENTAL OBSERVABLES

- **Observation of Compact Star Configurations:**

- \Rightarrow A very massive star ($M > 2M_{\odot}$) makes quark core configurations implausible
- \Rightarrow A very compact star ($R < 10$ km) makes corresponding hadronic stars very unlikely

- **Cooling Evolution:**

- \Rightarrow Hadronic EoS should not support DU process ($x > 11$ %) for typical star masses $M \leq 1.4 M_{\odot}$
- \Rightarrow Medium effects are required; 3P_2 neutron pairing gap has to be suppressed
- \Rightarrow Quark matter core \Rightarrow all quarks gapped, $\Delta_{\min} \approx 30$ keV, \Rightarrow **2SC+X oder CSL phase?**
- \Rightarrow CFL phase unlikely, critical density (and gaps) too large, gCFL only at $T > 50$ MeV
- \Rightarrow Decreasing $\Delta(n)$ would explain cooling by hybrid stars

THANK YOU

1. Introduction
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COLLABORATION

- Virtual Institute 'Dense Hadronic Matter and QCD Phase Transition' at GSI Darmstadt, with the Color Superconductivity Seminar at Frankfurt and Darmstadt
- Deborah Aguilera, Jens Berdermann, Michael Buballa, Hovik Grigorian, Thomas Klähn, Sergei Popov, Fredrik Sandin, Roberto Tuolla, Valery Yudichev, Dmitry Voskresensky, ...

ECT* WORKSHOP

'The New Physics of Compact Stars', Trento, Sept. 12-16, 2005

Virtual Institute of the Helmholtz-Association Dense Hadronic Matter and QCD Phase Transition

