The Role of Quark Mass in Cold and Dense Perturbative QCD and Quark Stars

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Outline

- Motivation
- Asymptotic freedom and in-medium pQCD
- A simple model for cold QCD at high density
- Cold and dense pQCD with $m_q > 0$ up to $O(\alpha_s)$
- Final comments

Motivation

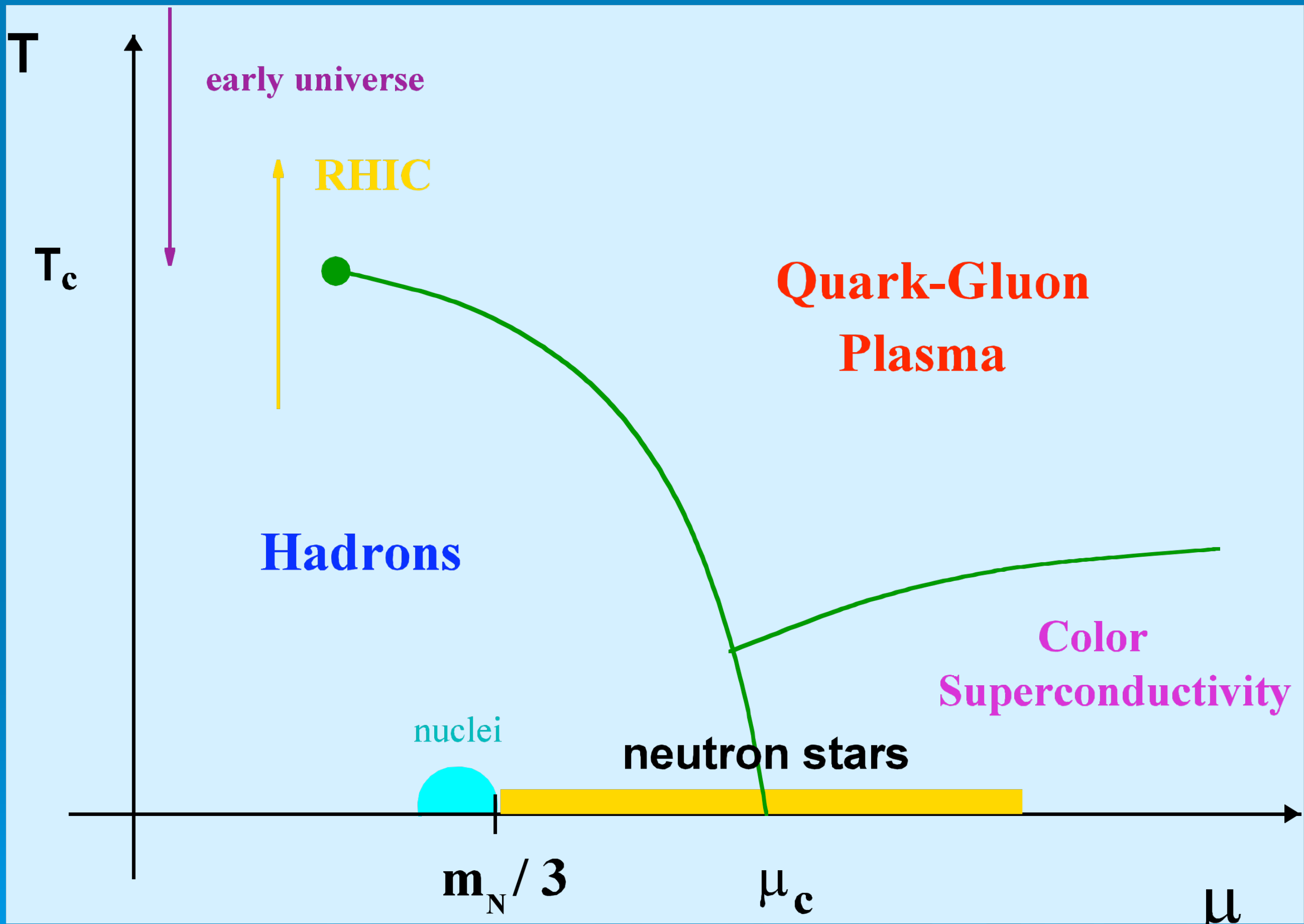
General wisdom: finite quark masses bring negligible corrections to the equation of state at finite density (< 5%) and play a minor role.

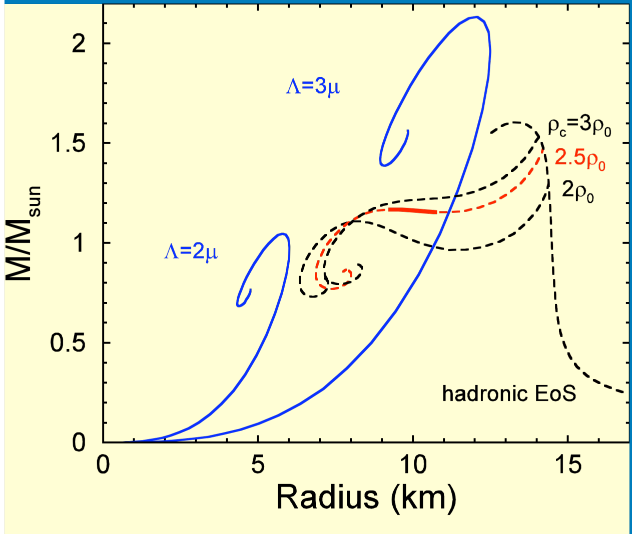
[For ~20 years… since Witten (1984), Alcock, Farhi & Olinto (1986), etc]

Results to be discussed: a physical mass for the strange quark could bring up to 25% corrections to the EoS !!!
[ESF & Romatschke, 2005]

Important influence on the physics of the chiral transition: position of the critical value of μ, strength of the 1^{st} order transition, etc -> observables in compact stars!

Drawing the QCD phase diagram close to T=0…

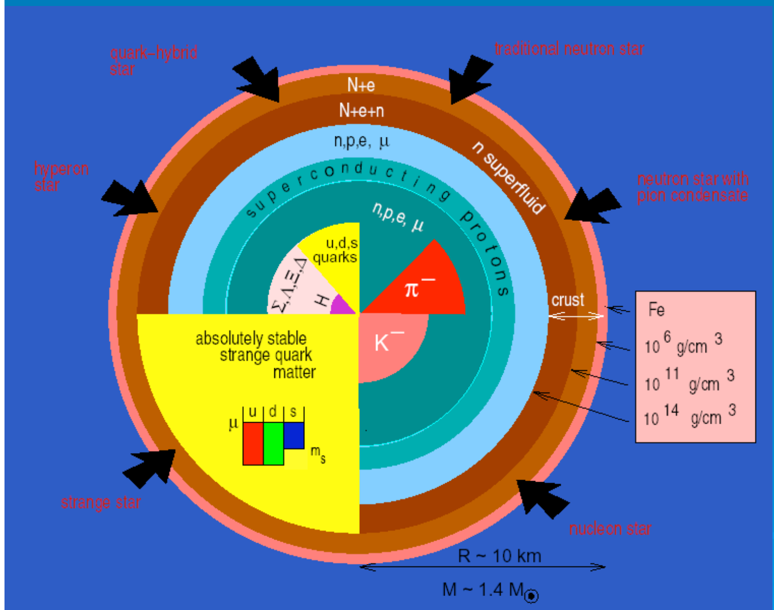




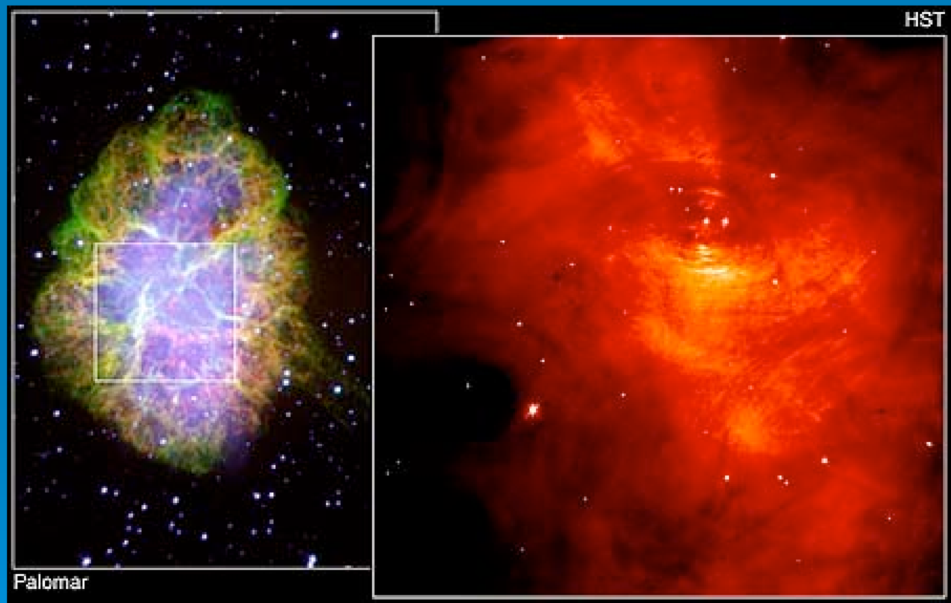
(ESF, Pisarski, Schaffner-Bielich, 2002)

Compact stars:

Mass effects may be crucial:
for a strongly 1st order χ transition,
new class of stars!

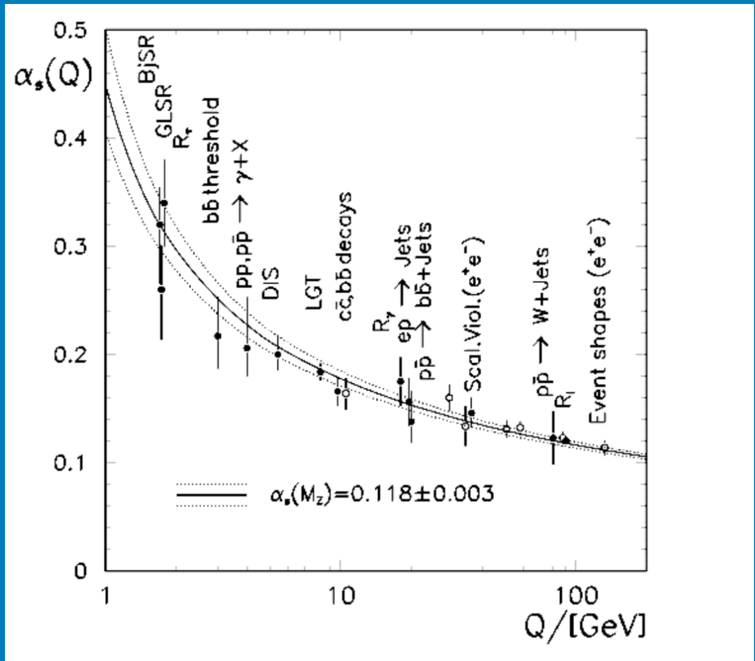


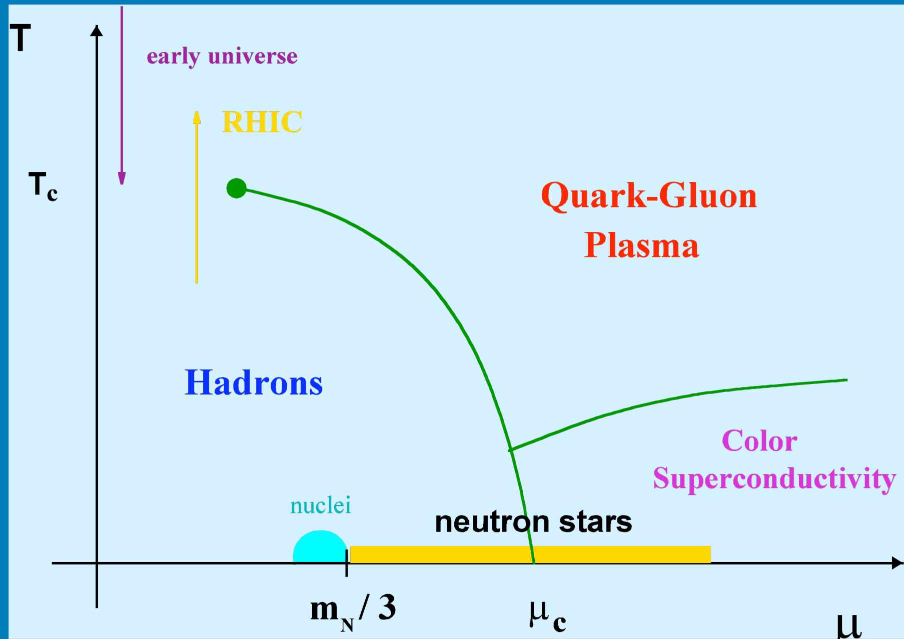
(F. Weber, 2000)



(NASA)

pQCD: QCD is asymptotically free – matter becomes "simpler" at high T and μ





$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 u}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln(u)}{u} + \frac{4\beta_1^2}{\beta_0^4 u^2}\left(\left(\ln(u) - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]$$

$$u = \ln(\bar{\Lambda}^2/\Lambda^2_{\overline{\text{MS}}})$$

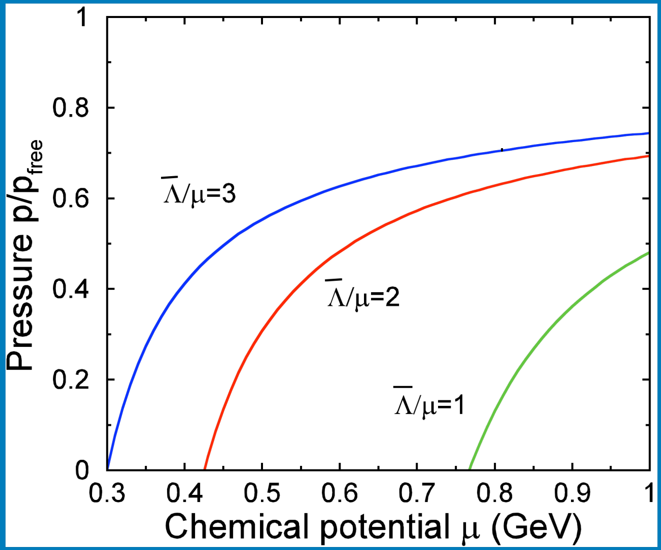
QCD at “high” density – a simple model:

(ESF, Pisarski & Schaffner-Bielich, 2001/2002)

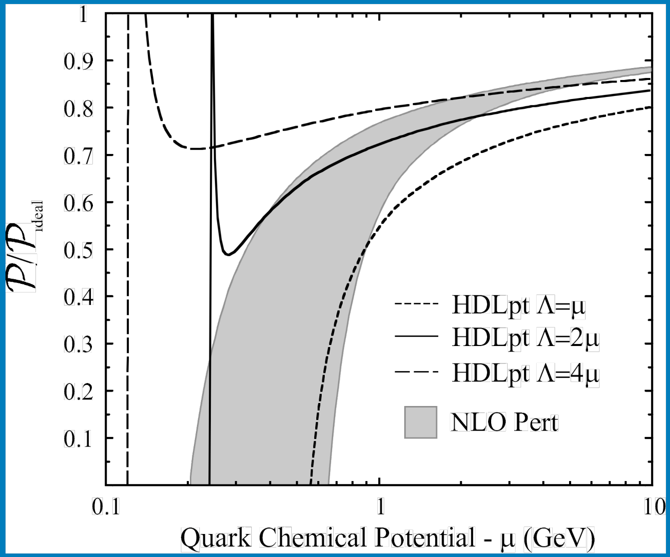
- gas of massless u, d, s quarks
- interaction taken into account perturbatively up to α_s^2
- α_s^2 runs according to the renormalization group eqn.
- no bag constant
- charge neutrality and β-equilibrium: $\mu_s=\mu_d=\mu_u$

Thermodynamic potential in $\overline{MS}$ scheme +
physics constraints on renormalization scale freedom

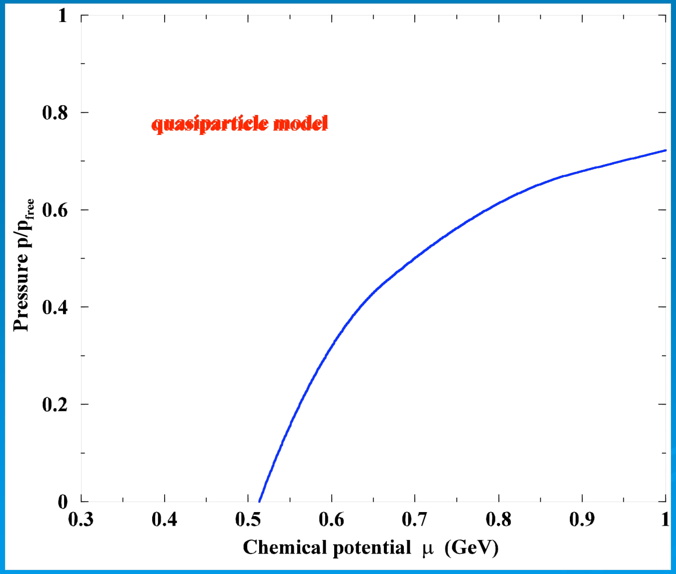
Results for the pressure:



(ESF, Pisarski & Schaffner-Bielich, 2001)



(Andersen & Strickland, 2002)

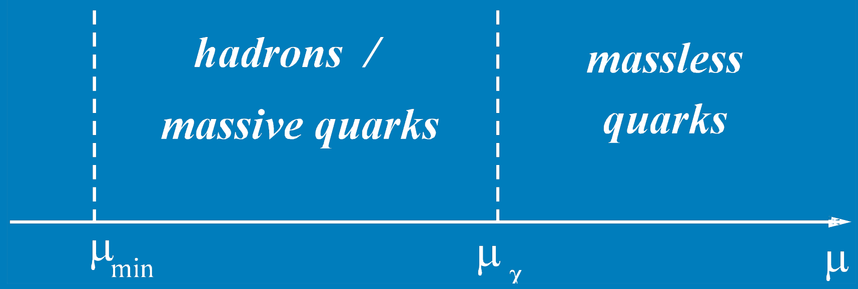


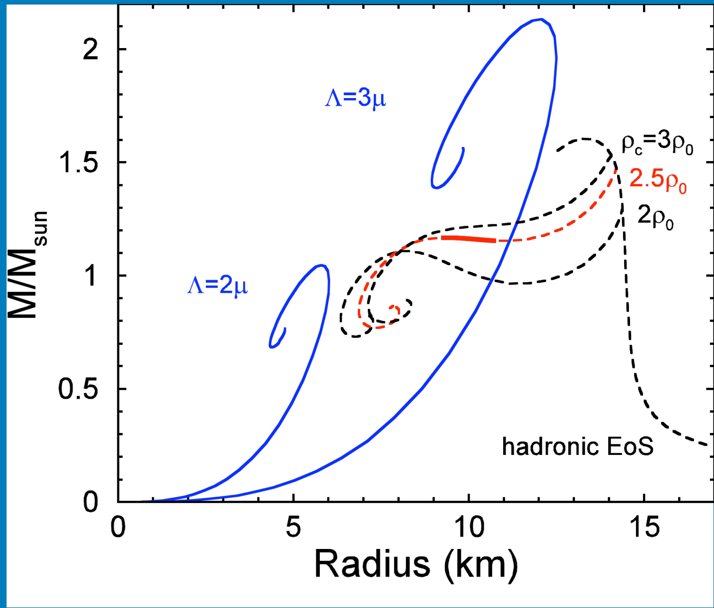
(Peshier, Kämpfer & Soff, 2002)

+ similar results from

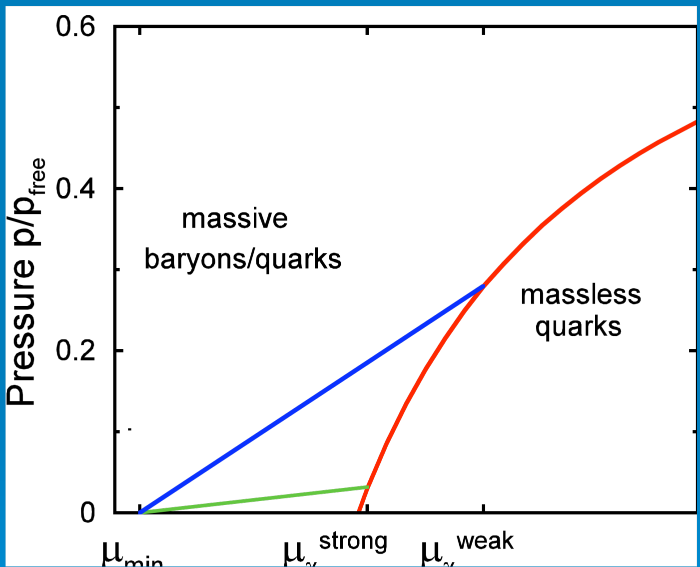
Blaizot, Iancu & Rebhan (2001)
Rebhan & Romatschke (2003)

pQCD, HDL, quasiparticle models, ...





Matching to "low" density - Two scenarios:



(ESF, Pisarski & Schaffner-Bielich, 2002)

- For strongly 1st order chiral transition: new class of compact stars!

- Pure neutron matter up to ~$2n_0$:
  (Akmal, Pandharipande & Ravelhall, 1998)
  $p/p_{free} \approx 0.04\,(n/n_0)^2$
- Dilute nuclear matter in χpert. theory at finite density and T=0:
  (ESF, Hatta, Pisarski & Schaffner-Bielich, 2003/2004)
  hadronic phase with small pressure viable!

$m_q > 0$ up to $O(\alpha_s)$:

ESF & Romatschke, 2005
$O(\alpha_s^2)$: ESF & Romatschke, in prep.

Does a nonzero mass for the strange quark matter?

- The original approach to dense matter made use of the bag model with corrections $\sim \alpha_s$ from pQCD to compute the thermodynamic potential.

- In the massless case, first-order corrections cancel out in the EoS, so that one ends up with a free gas of quarks modified only by a bag constant.

- Finite quark mass effects were then estimated to modify the EoS by less than 5% and were essentially ignored for ~20 years.

- Is the effect of m_s~100 MeV really that small for quark stars?

• Even the most recent QCD approaches generally neglected quark masses and the presence of a color SUC gap as compared to the typical scale for the chemical potential in the interior of compact stars, ~400 MeV and higher.

• However, it was recently argued that both effects should matter in the lower-density sector of the EoS [Alford et al., 2004].

• Although quarks are essentially massless in the core of quark stars, m_s runs up, and becomes comparable to the typical scale for the chemical potential, as one approaches the surface of the star !

Then: do exploratory analysis of the effects of a finite mass for the strange quark on the EoS for pQCD at high density including renormalization group running of α_s and m_s !

Thermodynamic potential (1 flavor)

Leading-order piece:

$$\Omega^{(0)} = -\frac{N_c}{12\pi^2}\left[\mu u(\mu^2 - \frac{5}{2}m^2) + \frac{3}{2}m^4 \ln\left(\frac{\mu+u}{m}\right)\right]$$

$$u \equiv \sqrt{\mu^2 - m^2}$$

The exchange term:

$$\tilde{\Omega}^{(2)} = -2\pi\alpha_s \frac{N_c^2-1}{2} \sum\!\!\!\!\!\!\!\int_{P,Q,K} \delta(P-Q-K) \times \frac{\mathrm{tr}\left[\gamma_\mu(\not{P}+m_f)\gamma^\mu(\not{Q}+m_f)\right]}{(P^2-m_f^2)(Q^2-m_f^2)K^2}$$

$$\sum\!\!\!\!\!\!\!\int_P \equiv T\sum_{p_0}\int \frac{d^3p}{(2\pi)^3} \quad , \quad \int_{\mathbf{p}} \equiv \int \frac{d^3p}{(2\pi)^3}$$

$$\delta(P-Q) = \beta\delta_{p_0,q_0}(2\pi)^3\delta^3(\vec{p}-\vec{q})$$

Results:

Using standard QFT methods, one obtains the complete renormalized exchange energy for a massive quark in the $\overline{MS}$ scheme:

$$\Omega^{(2)} = \frac{\alpha_s(N_c^2-1)}{16\pi^3}\left[3\left(m^2\ln\frac{\mu+u}{m}-\mu u\right)^2 - 2u^4 + m^2\left(6\ln\frac{\bar{\Lambda}}{m}+4\right)\left(\mu u - m^2\ln\frac{\mu+u}{m}\right)\right]$$

- Ω depends on the quark chemical potential μ and on the renormalization subtraction point Λ both explicitly and implicitly through the scale dependence of the strong coupling constant $\alpha_s(\Lambda)$ and the mass $m(\Lambda)$.

- The scale dependencies of both α_s and m are known up to 4-loop order in the $\overline{MS}$ scheme [Vermaseren, 1997]. Since we have only determined the free energy to first order in α_s, we choose

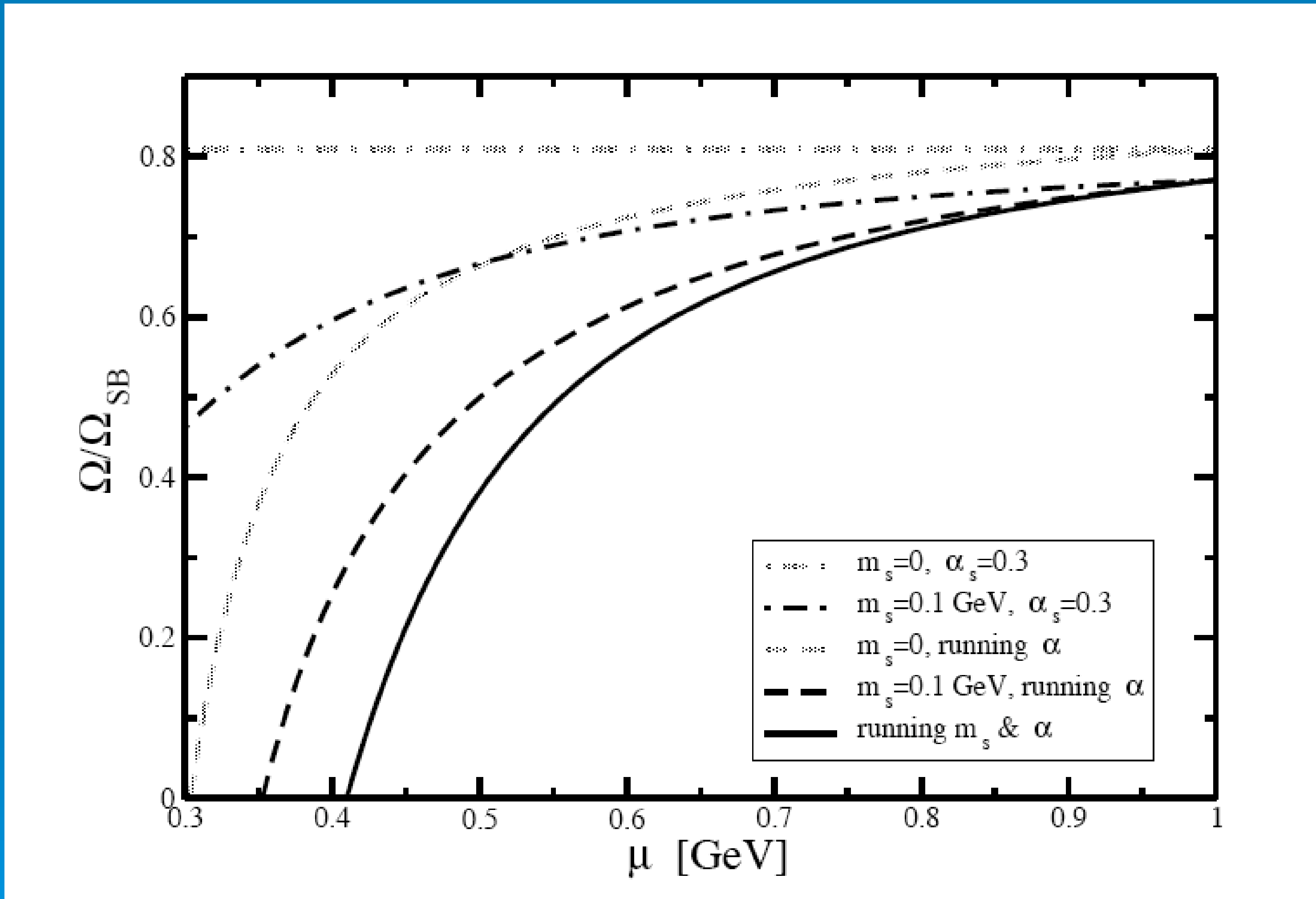
$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 L}\left[1 - 2\frac{\beta_1}{\beta_0^2}\frac{\ln L}{L}\right]$$

$$m_s(\bar{\Lambda}) = \hat{m}_s\left(\frac{\alpha_s}{\pi}\right)^{4/9}\left[1+0.895062\frac{\alpha_s}{\pi}\right]$$

$$L = 2\ln\left(\bar{\Lambda}/\Lambda_{\overline{\mathrm{MS}}}\right)$$

+ rest fixed by PDG data

Thermodynamic potential for one massive flavor



Pressure and energy density for quark matter + electrons (neutral and β equilibrated)

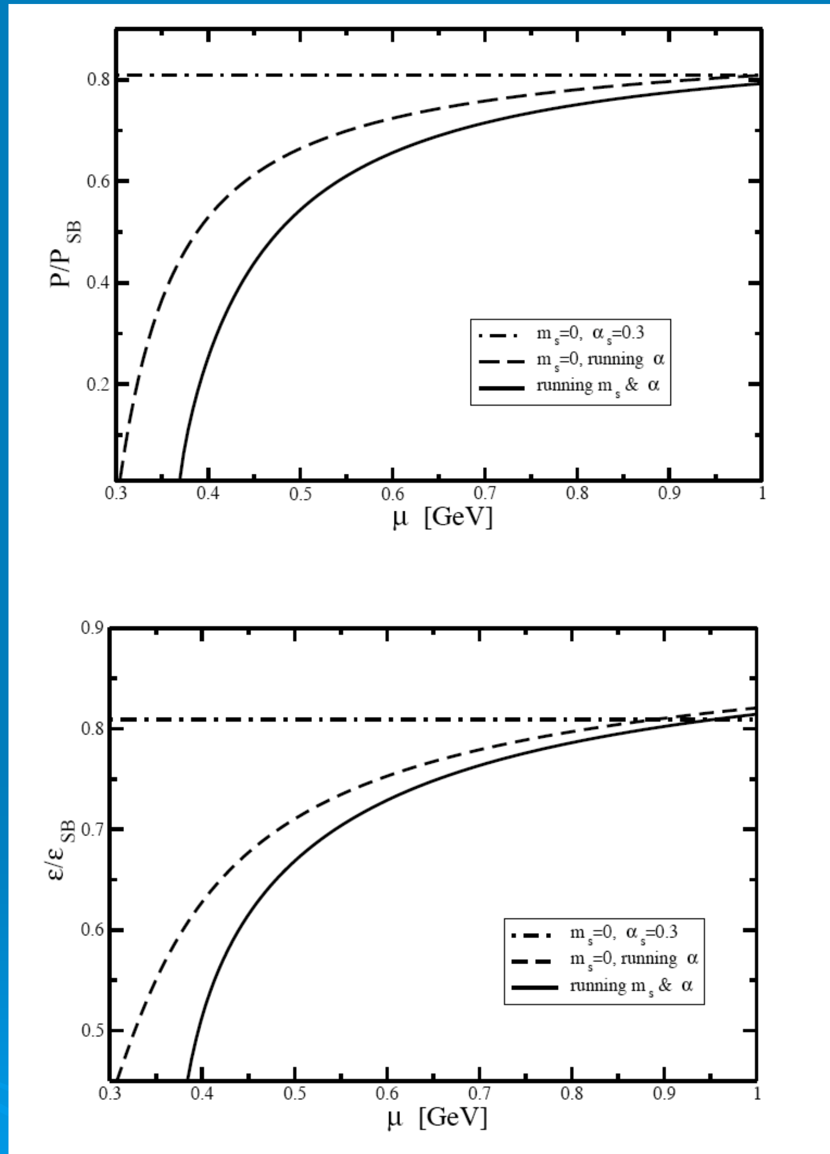
$$d \to u + e + \bar{\nu}_e, \quad u + e \to d + \nu_e,$$
$$s \to u + e + \bar{\nu}_e, \quad u + e \to s + \nu_e,$$
$$s + u \leftrightarrow d + u$$

$$\mu_d = \mu_s = \mu, \quad \mu_u + \mu_e = \mu$$

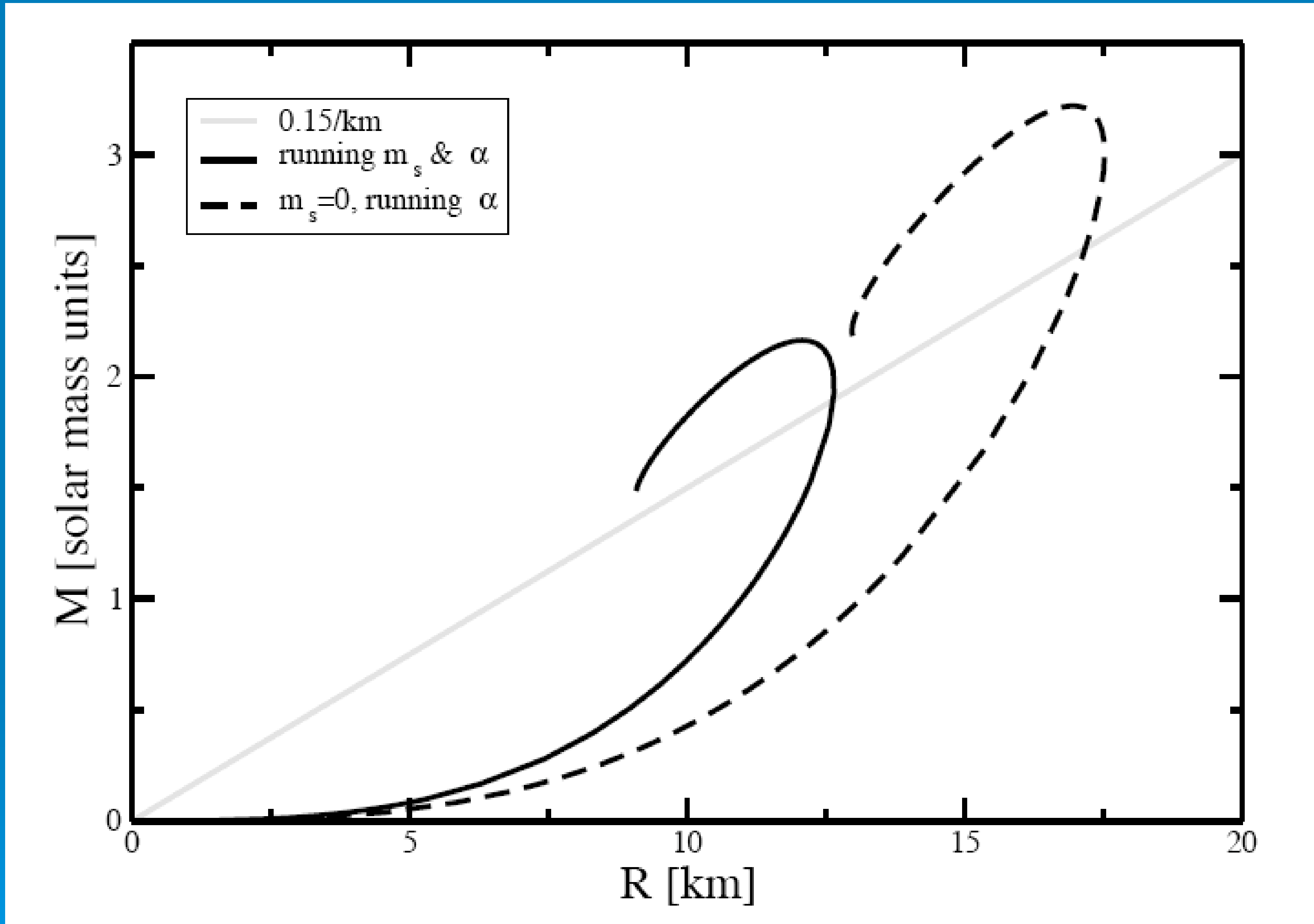
$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$$

$$\epsilon = \Omega + \sum_i \mu_i n_i$$

$$P = n_B \frac{\partial \epsilon}{\partial n_B} - \epsilon$$



Mass-radius relation for quark stars



Final Comments

❖ Finite quark mass effects can dramatically modify the EoS for cold and dense QCD -> definitely one should take them into account for quark stars !

❖ The numbers shown in this talk are meant to be illustrative of the strength of the effect. $O(\alpha_s^2)$ corrections will modify them significantly [ESF & Romatschke, work in progress]

❖ The computation of $O(\alpha_s^3)$ for the thermodynamic potential is crucial for a better control and understanding of the stability of the perturbative series at finite μ.

❖ Related work in progress: non-Fermi liquid behavior in quark matter, cooling of neutron stars, etc.