Violation of $k_\perp$ factorization in quark production from the CGC

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Outline

- Introduction
- Quark production in p–A collision
- Violation of $k_\perp$ factorization
- Open charm spectrum of MV model
- Quark pair and quarkonium

- HF, Gelis, Venugopalan, hep-ph/0504047 and in progress

Introduction

- Process with single large scale $\sqrt{s} \sim \boldsymbol{p}_\perp \gg \Lambda_{\mathrm{QCD}}$

  $\Rightarrow$ Collinear factorization

$$d\sigma = \int \hat{\sigma} \;\; x_1 G(x_1) \;\; x_2 G(x_2)$$

- Process with two large scales $\sqrt{s} \gg \boldsymbol{p}_\perp \gg \Lambda_{\mathrm{QCD}}$

  $\Rightarrow$ $\boldsymbol{k}_\perp$ factorization Collins Ellis 1991, Catani Ciafaloni Hautmann 1991

$$d\sigma = \int \frac{|\mathcal{M}|^2}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2} \;\; \phi_1(x_1, \boldsymbol{k}_{1\perp}) \;\; \phi_2(x_2, \boldsymbol{k}_{2\perp}) \;\; \delta(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} - \boldsymbol{q}_\perp)$$

  - Useful devise to include the intrinsic $k_\perp$
  - Extension to include rescattering corrections in pA ?

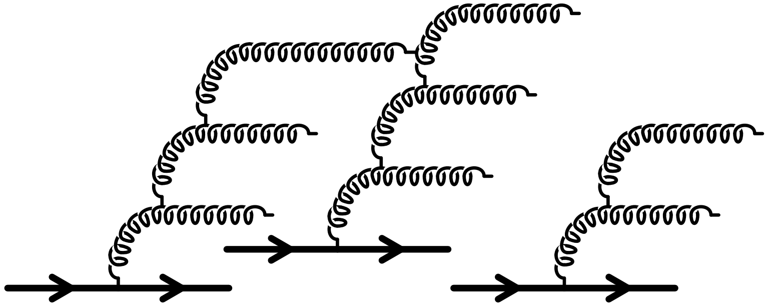
Introduction

Particle procution from the CGC

- Nucleus at high energy

Introduction

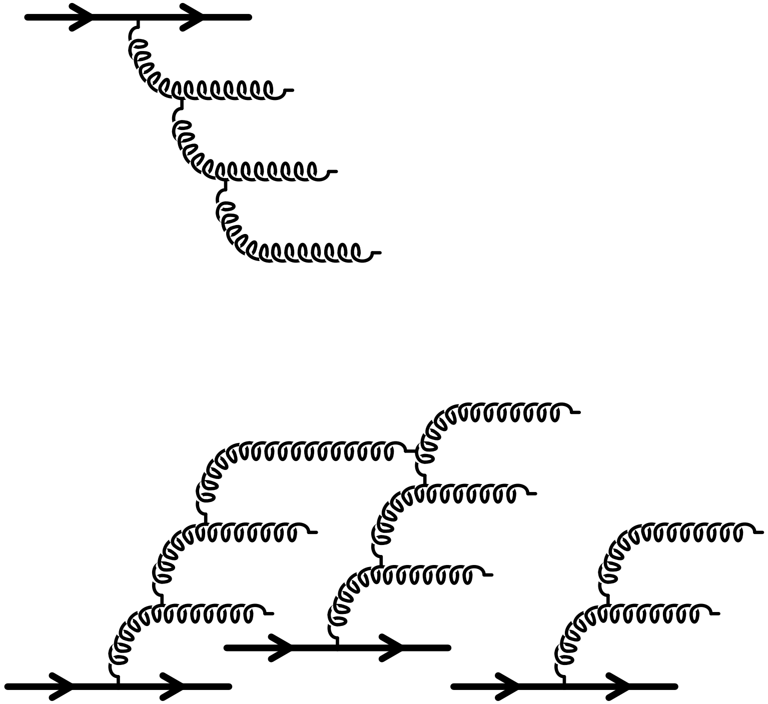
Particle procution from the CGC



- Nucleus at high energy = dense small-x gluons

Introduction

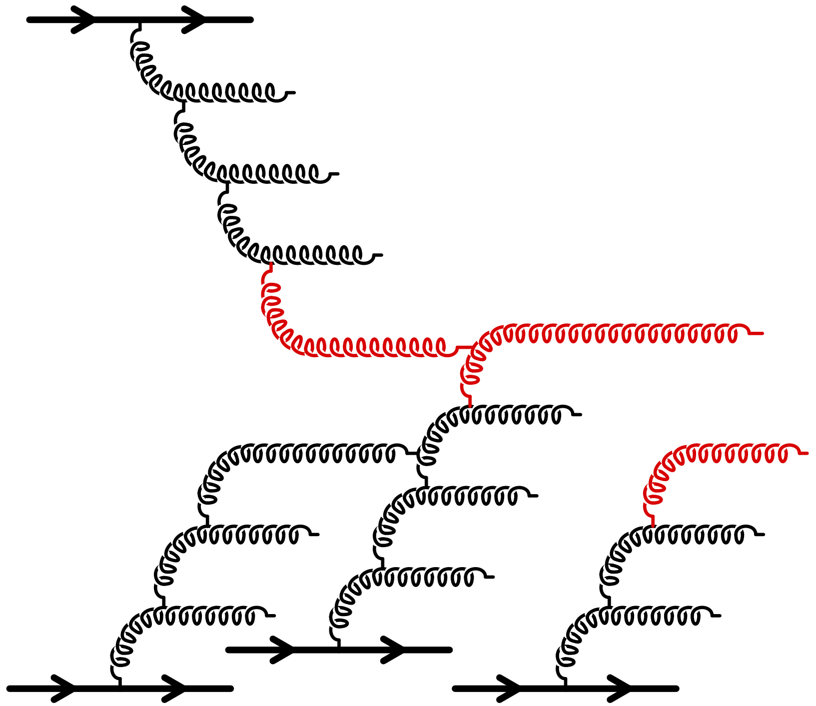
Particle procution from the CGC



- Nucleus at high energy = dense small-x gluons
- Proton probes Nucleus

Introduction

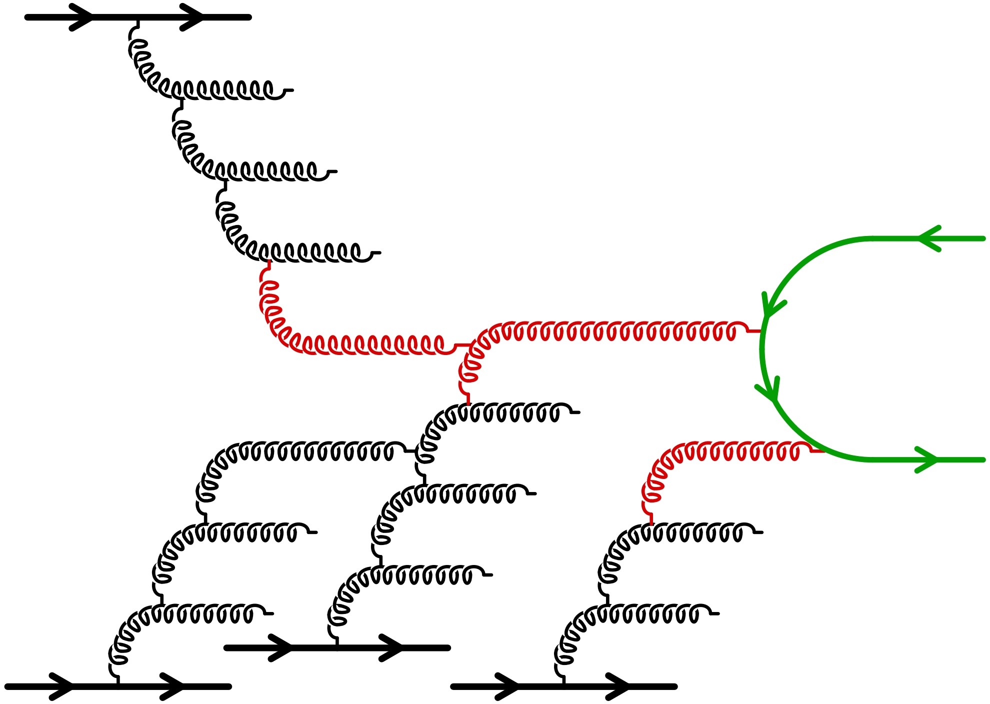
Particle procution from the CGC



- Nucleus at high energy = dense small-x gluons
- Proton probes Nucleus
  - gluon production

Introduction

Particle procution from the CGC



- Nucleus at high energy = dense small-x gluons
- Proton probes Nucleus
  - gluon production
  - quark production

Introduction

- Interests in quark production from the CGC in pA
  - Probing the nuclear wave function
  - Quantitative test of $k_\perp$-factorization
  - Phenomenology in pA collisions
  - Baseline for AA collisions
  - $\cdots$

Classical field and gluon production

- Small x-gluons are described as classical field generated by source charge $\rho_{p,A}$ on the light-front McLerran Venugopalan 1994
- Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad , \qquad [D_\nu, J^\nu] = 0 \quad , \qquad \partial_\mu A^\mu = 0$$

$$J^\nu|_{\mathrm{LO}} = \delta^{\nu+}\delta(x^-)\rho_p(\boldsymbol{x}_\perp) + \delta^{\nu-}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$

- Gluon and Quark production amplitudes are known to first order in ρ_p and to all orders in ρ_A Kovchegov Muller 1998
  Blaizot Gelis Venugopalan 2004
- Gluon production:

$$\frac{d\sigma}{d^2\boldsymbol{q}dy_q} \sim \frac{\alpha_s N}{\pi^4 d_A \boldsymbol{q}_\perp^2} \int_{\boldsymbol{k}_\perp} \varphi_p^{g,g}(\boldsymbol{k}_\perp)\phi_A^{g,g}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)$$

  - $\boldsymbol{k}_\perp$-factorization is valid with $\phi_A^{g,g} = FT\langle U(\boldsymbol{x}_\perp)U^\dagger(0)\rangle$

Quark cross section in pA

Blaizot Gelis Venugopalan 2004

- Quark production amplitude:

$$\mathcal{M}_F = g^2 \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{p,a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

$$\times\overline{u}(\vec{\boldsymbol{q}})\Big\{[\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) + [t^b U_{ba}(\vec{\boldsymbol{x}}_\perp)]\not{L}\Big\}v(\vec{\boldsymbol{p}})$$

with

$$T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) \equiv \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2] + 2q^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

and $\not{L}$ Lipatov's vertex

- Interpretation of solution:

Quark cross section in pA

Blaizot Gelis Venugopalan 2004

- Pair production cross section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times \Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)$$

$$+ \int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

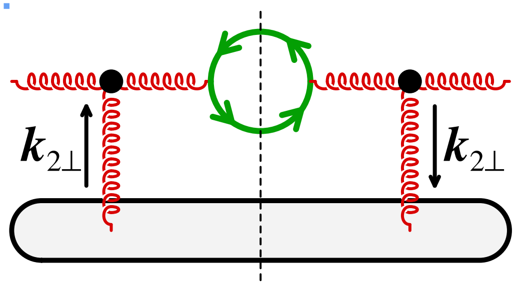
$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_p(\vec{\boldsymbol{k}}_{1\perp})$$

▷ $k_\perp$-factorization violated on the nucleus side:
we need three different “distributions”

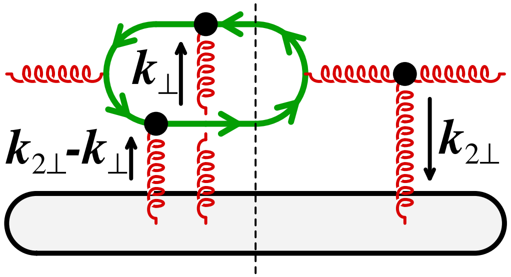
Quark cross section in pA

- Interpretation of nuclear distributions:

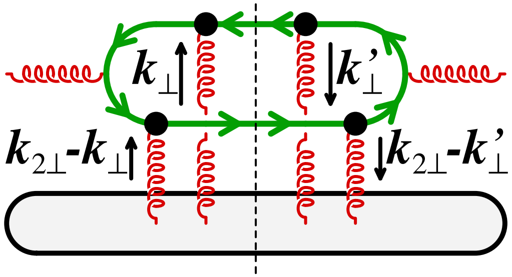
$$\phi_A^{g,g} \sim \mathrm{FT}\ \mathrm{tr}\langle U\ U^\dagger\rangle \quad \propto$$



$$\phi_A^{q\bar q,g} \sim \mathrm{FT}\ \mathrm{tr}\langle \widetilde U\ t^a\ \widetilde U^\dagger\ t^b\ U_{ba}\rangle \quad \propto$$



$$\phi_A^{q\bar q,q\bar q} \sim \mathrm{FT}\ \mathrm{tr}\langle \widetilde U\ t^a\ \widetilde U^\dagger \widetilde U\ t^a \widetilde U\rangle \quad \propto$$



Quark cross section in pA

- Properties of ϕ_A's:
  - Sum rule

$$\int_{\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp} \phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp) = \int_{\vec{\boldsymbol{k}}_\perp} \phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = \phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$$

▷ $k_\perp$-factorization is recovered if one can neglect $\boldsymbol{k}_\perp$ in $T_{q\bar{q}}(\boldsymbol{k}_\perp)$

  - In large N limit, $\phi_A^{q\bar{q},q\bar{q}}$ becomes a product of 2-pnt fns

$$\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp) \xrightarrow{\text{large } N} \# C(\boldsymbol{k}_\perp) C(\boldsymbol{k}_{2\perp} - \boldsymbol{k}_\perp)(2\pi)^2 \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}'_\perp)$$

▷ multiple scattering of a single quark in the nucleus

$$C(\boldsymbol{k}_\perp) = FT\langle \tilde{U}(\boldsymbol{x}_\perp)\tilde{U}^\dagger(0)\rangle$$

- Large N approximation is rather good:
  we present the large N results (mainly)

Quark cross section in pA

- Single quark cross section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{q}}_\perp dy_q} = \int d^2\vec{\boldsymbol{p}}_\perp dy_p \frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q}$$

  - Only 2- and 3-point functions appear thanks to sum rule for ϕ_A's
  - $k_\perp$-factorization is still violated for the nucleus

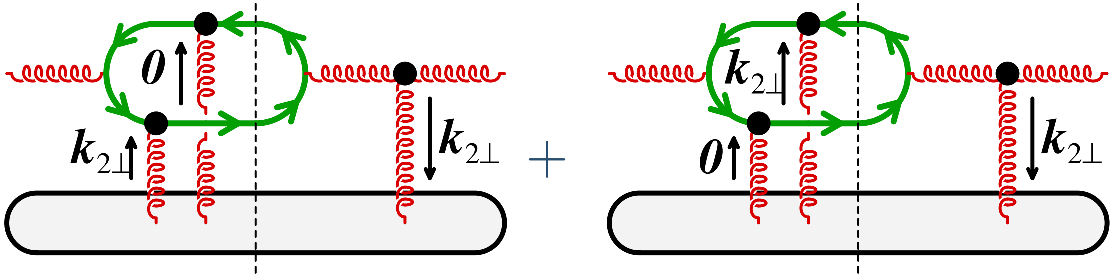
$k_\perp$ factorization

HF, Gelis, Venugopalan 2005

- $\boldsymbol{k}_\perp$-factorization for single quark is recovered if one can replace

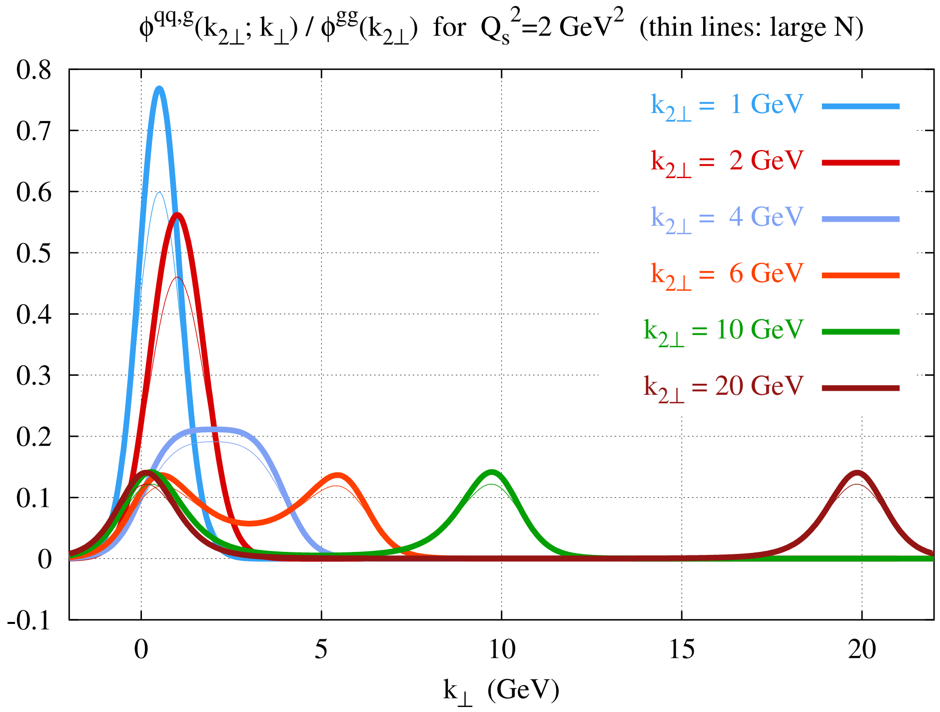
$$\phi_A^{q\bar{q},g}(\boldsymbol{k}_{2\perp}|\boldsymbol{k}) \quad \Rightarrow \quad \frac{1}{2}(2\pi)^2[\delta(\boldsymbol{k}_\perp)+\delta(\boldsymbol{k}_\perp-\boldsymbol{k}_{2\perp})]\ \phi_A^{g,g}(\boldsymbol{k}_{2\perp})$$

- This means, we use the approximation that either of Q and $\bar{Q}$ exchanges all the momentum



$k_\perp$ factorization

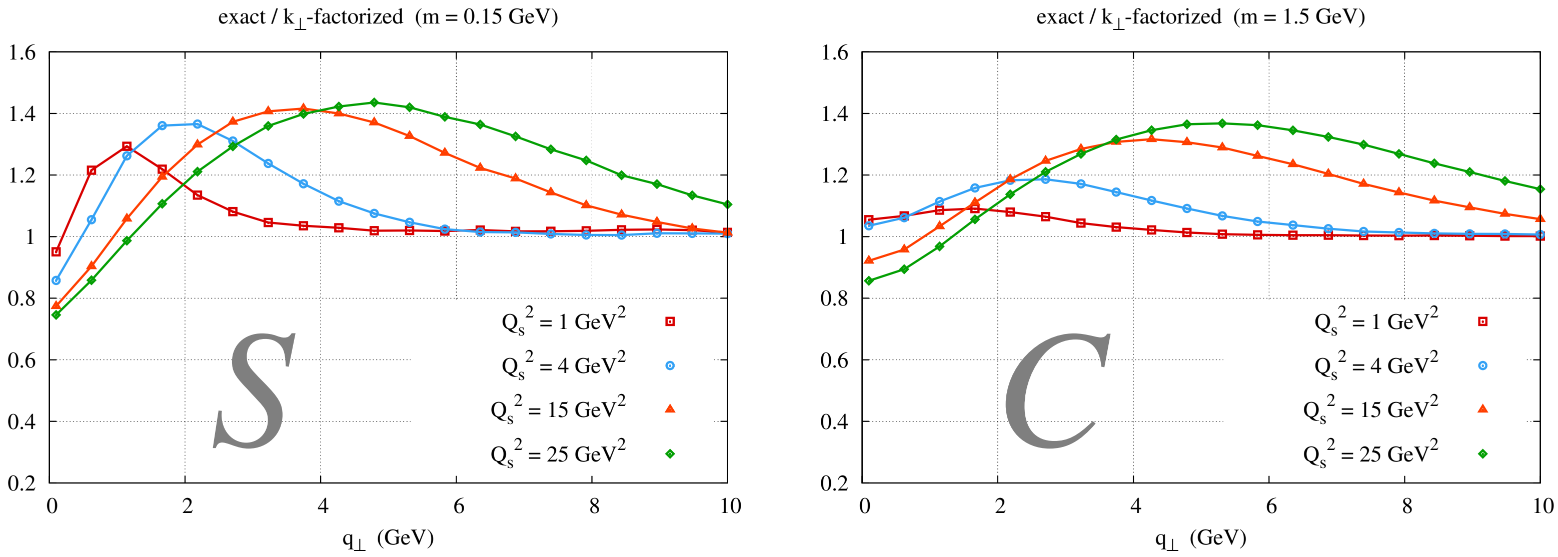
Exact 3-point function in the MV model:



- For $\boldsymbol{k}_{2\perp} \gg Q_s$ there are two peaks with width of order Q_s
- When $\boldsymbol{k}_{2\perp} \lesssim Q_s$, two peaks merge into one
- $\boldsymbol{k}_\perp$-factorization holds if $\sqrt{s} \gg (p_\perp \text{or } m) \gg Q_s$
  - typical $\boldsymbol{k}_{2\perp}$ is large since $\boldsymbol{p}_\perp + \boldsymbol{q}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$
  - $T_{q\bar{q}}$ is nealy constant if the width $\ll m$

$k_\perp$ factorization

Ratio of “full” / “$k_\perp$-factorized” for single quark production



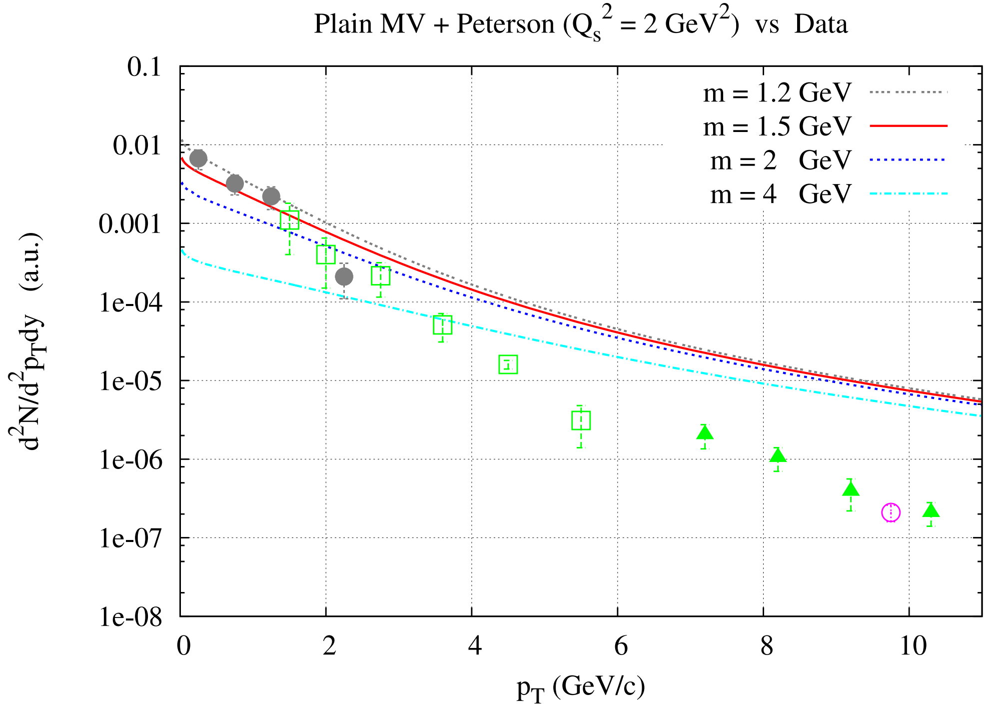
- Breaking is larger for smaller m and/or larger Q_s
- maximum around $q_\perp \sim Q_s$
- At RHIC ($Q_s^2 \sim 1$ GeV^2) the breaking is mild for heavy quark
- At LHC and at forward rapidities ($Q_s^2 \sim 15$ GeV^2) the correction can be significant even for the heavy

- Please don’t confuse this with the Cronin peak

Open charm spectrum in pA

Quark spectrum of the MV model is convoluted with a frag fn

Cf. Kharzeev-Tuchin 2003



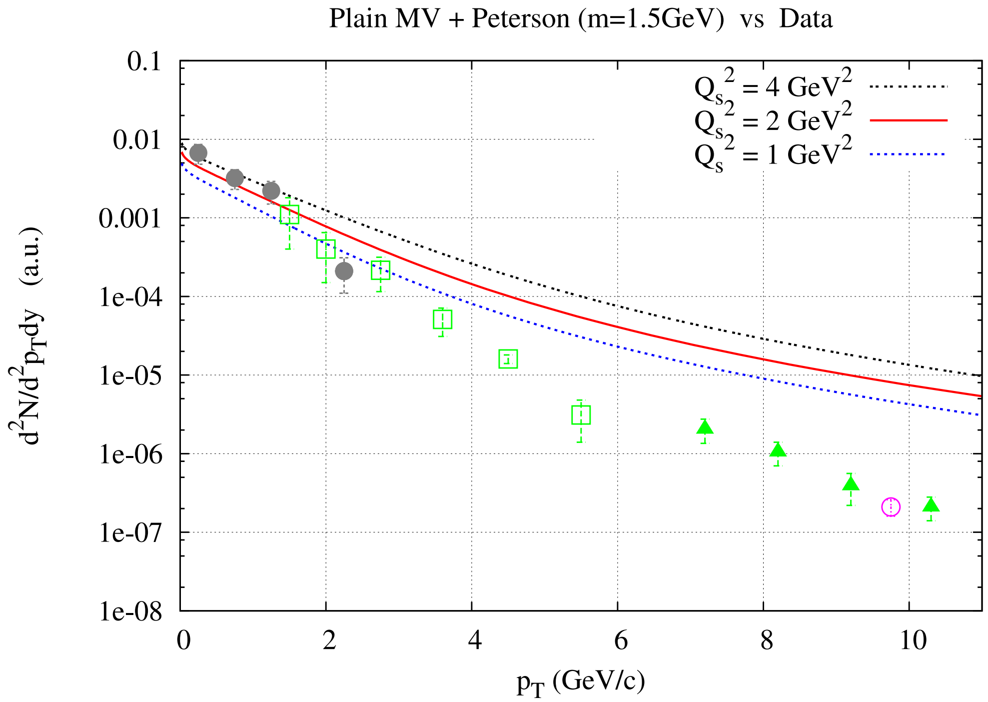
- Harder spectrum than STAR dAu data
  - At large $p_\perp$, slope $\to \ln^2(p_\perp)/p_\perp^4$
  - m changes slope at $p_\perp \lesssim m$

Note: relative normalization between the curve and data is arbitrary

Open charm spectrum in pA

Quark spectrum of the MV model is convoluted with a frag fn

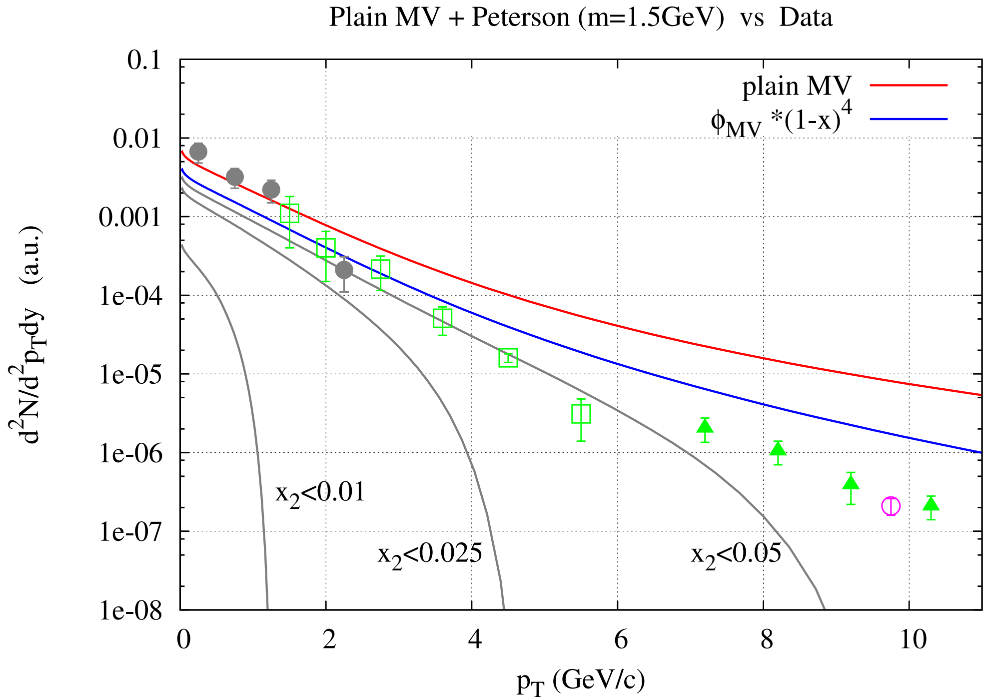
Cf. Kharzeev-Tuchin 2003



- Effect of Q_s:
  Q_s changes slope for $\boldsymbol{p}_\perp \lesssim Q_s$

Open charm spectrum in pA

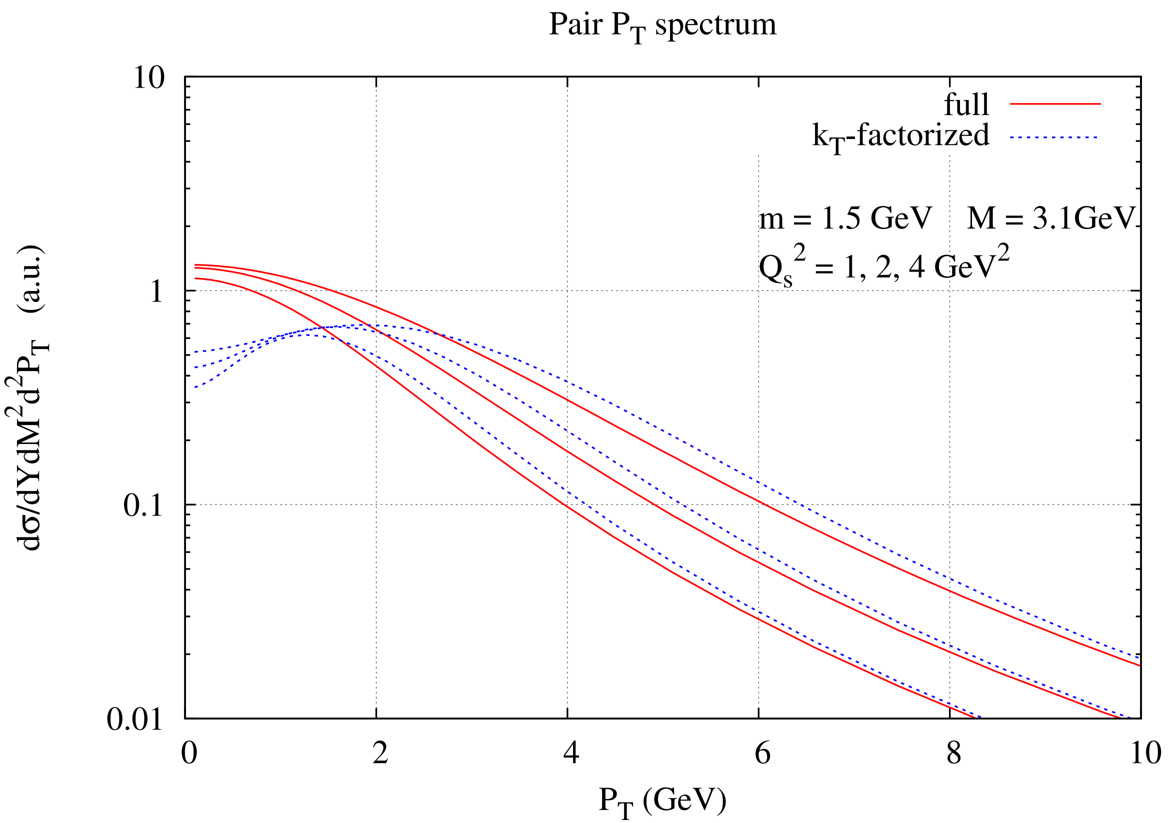
Introduce x-dependence into ϕ_A's of MV model by hand, $(1-x)^4$ with $\sqrt{s}$=200GeV *Cf.* Kharzeev-Tuchin 2003



- it helps
  - ▷ At larger $\boldsymbol{p}_\perp$, finite-x behavior of ϕ_A's plays a role
- but that's not enough
  - ▷ one needs systematic x-dependence, e.g., $Q_s^2(x)$, anom.dim.:

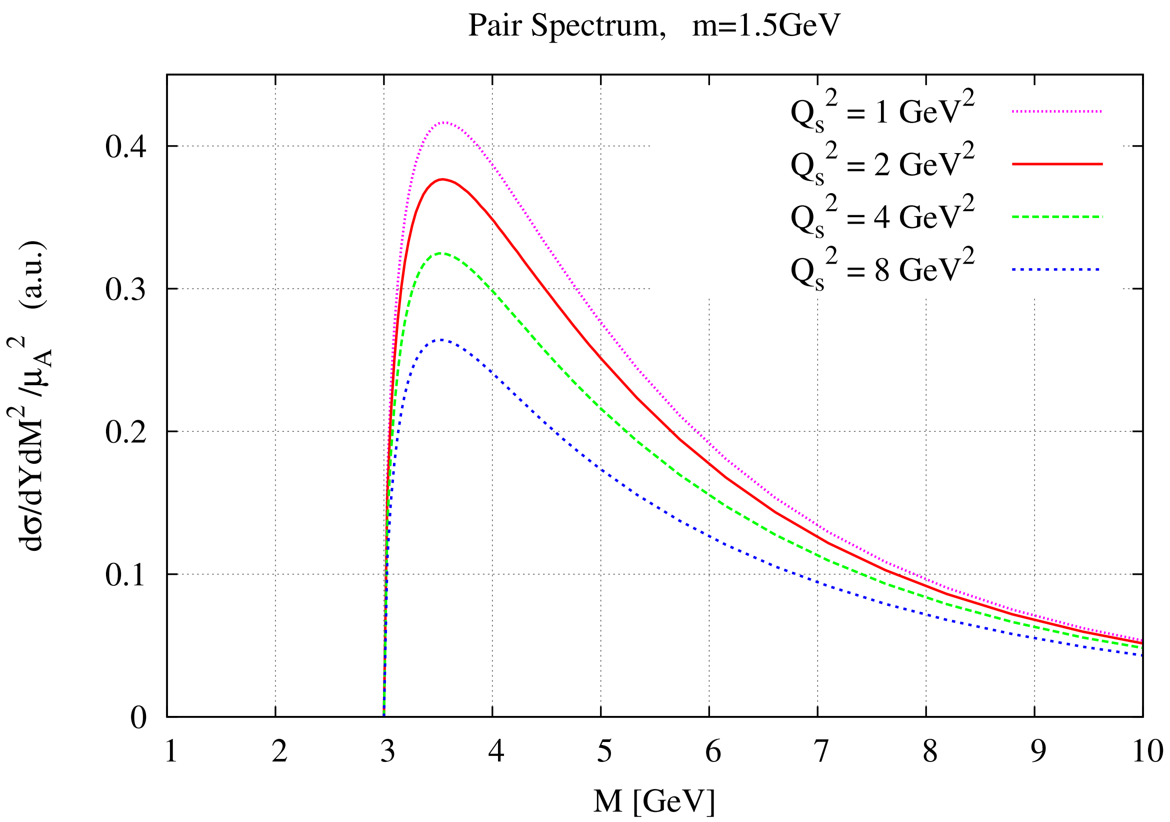
study on x-evolution for 2-, 3-pnt fns in large N is in progress

Pair production



- In $k_\perp$ factorization approach a bump shows up around Q_s $\Leftarrow$ shape of $\phi_A^{g,g}(\boldsymbol{k}_{2\perp})$
- Multiple scatterings of Q and $\bar{Q}$ in nucleus, smear it out
- At large $P_\perp$, $\sim \ln(P_\perp)/P_\perp^4$

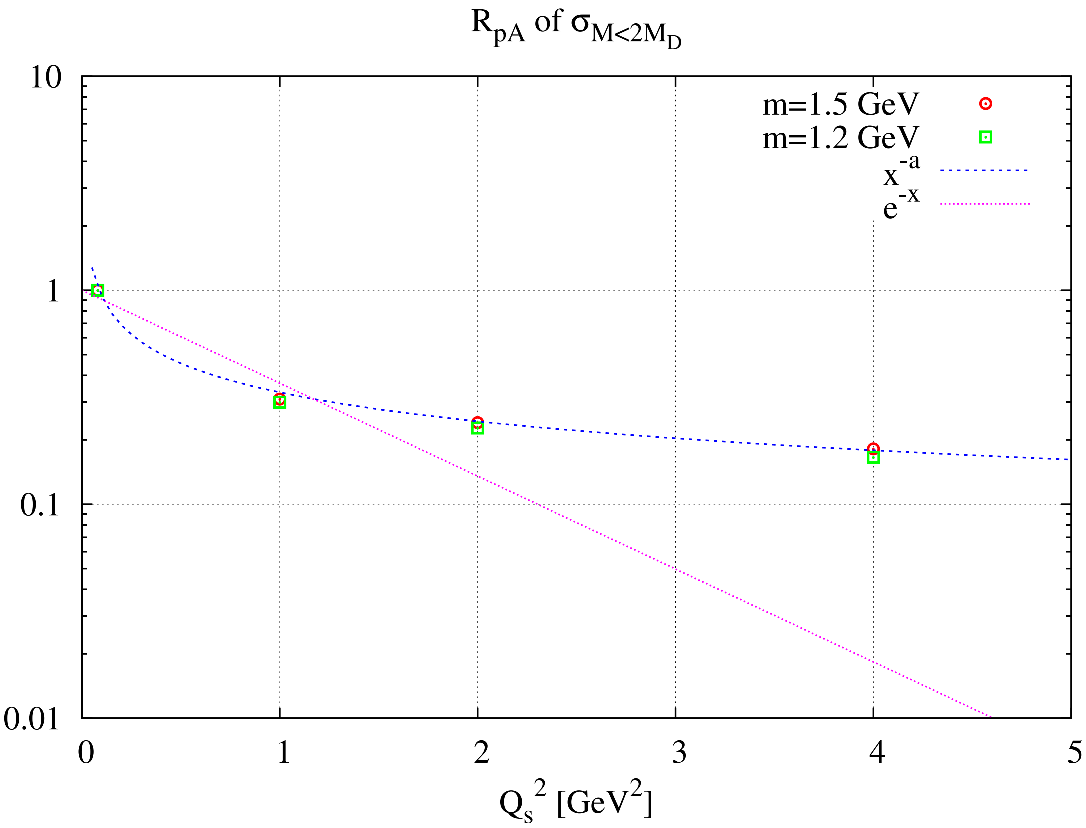
Pair production



- Invariant mass spectrum of the pair after integration over $P_\perp$
- $\mu_A^2 \sim Q_s^2$ up to log
- Binary scaling is recovered at larger M
- Non-trivial Q_s-dependence at smaller M

Quarkonium production in pA

Introduction
Classical Field and Gluon
Quark Cross Section in pA
kT factorization
Open charm spectrum in pA
Pair and Quarkonium in pA
Conclusions



- Color Evaporation Model: quarkonium rate $\propto$ rate of pair below $2m_D$ $\Rightarrow R_{pA}$
- Suppression is fit by Q_s^{-a} with $a \sim 0.9$ (very preliminary)

Conclusions

- Violation of $\boldsymbol{k}_\perp$-factorization in pA
  - first quantitative estimate is shown
  - can be significant when $Q_s \gtrsim m,\ \boldsymbol{p}_\perp$
    - is mild for charm quark at RHIC
    - will be significant at LHC and/or at very forward rapidities
- Single quark production in pA
  - open charm spectrum in plain MV model is shown
  - nonsmall-x pdf plays some role at larger $\boldsymbol{p}_\perp$ at RHIC
  - study of 2-, 3-pnt fns w/ “MV model + x-evolution” in progress
- Pair production pA
  - is useful for studying nuclear distribution
  - quarkonium suppression is weaker than the exponential within the MV model (preliminary)