Probing the Quark Gluon Liquid using Transverse Momentum Fluctuations

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onset of thermal equilibration – common centrality dependence of $\langle p_t \rangle$, p_t and charge dynamic fluctuations   Quark Matter 04

I. **Fluctuations measure two-body correlations**

- multiplicity and p_t fluctuation observables

II. **Multiple-Scattering Contributions**

- thermalization   *PRL* 92 (2004) 162301
- radial flow   Abdel-Aziz & SG, DPF 04

III. **Hydrodynamic Correlations**

- viscosity info from p_t fluctuations?

with M. Abdel-Aziz

in progress

Dynamic Fluctuations

variance minus thermal contribution

Pruneau, Voloshin & S.G.

multiplicity N

$$R = \frac{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle}{\langle N \rangle^2}$$

mean p_t

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N_{\text{pairs}} \rangle} \left\langle \sum_{\text{pairs } i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

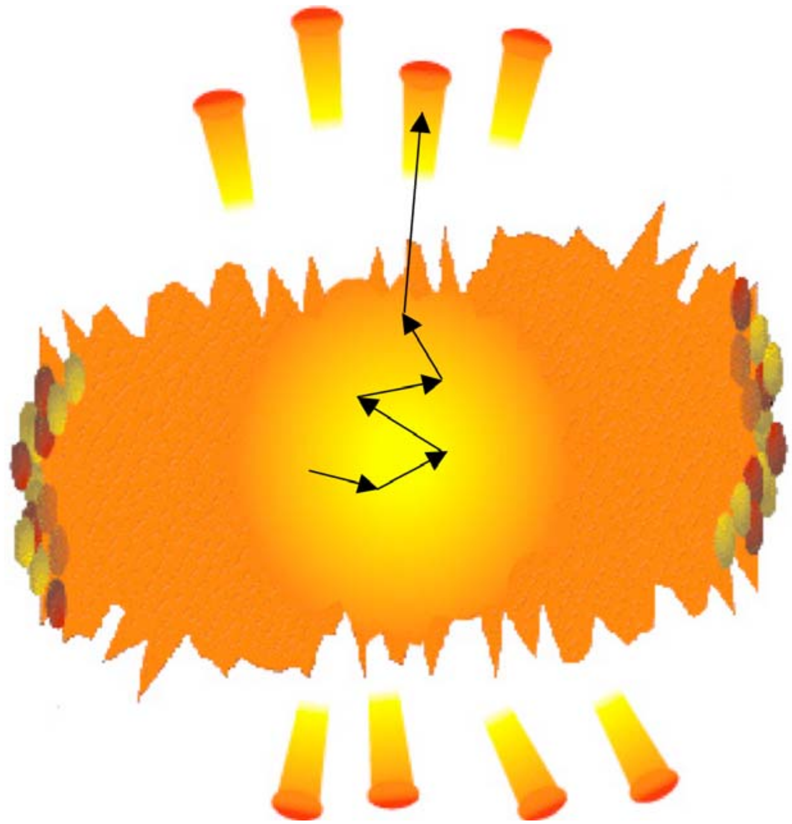
$$\delta p_t \equiv p_t - \langle p_t \rangle$$

correction function: $r(p_1, p_2) = \text{pairs} - (\text{singles})^2$

$$R \propto \iint r(p_1, p_2)\, dp_1 dp_2$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \iint \delta p_{t1} \delta p_{t2}\, r(p_1, p_2)\, dp_1 dp_2$$

Thermalization



scattering drives particles toward local thermal equilibrium

fluctuations enhanced

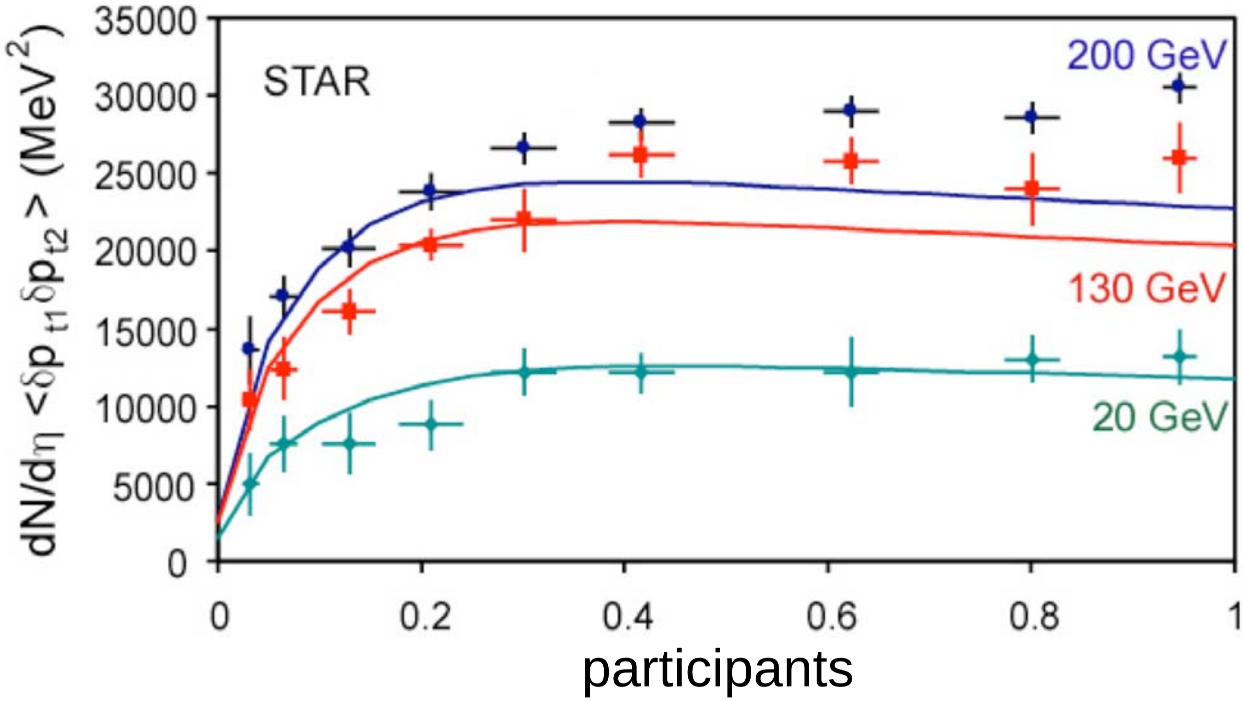
$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \iint \delta p_{t1} \delta p_{t2} \, r(p_1, p_2)$$

$$\propto \iint \delta T(x_1) \delta T(x_2) \, r(x_1, x_2) \, dx_1 dx_2$$

- $p_t \propto$ local temperature $T(x)$, difference $\delta T = T(x) - \langle T \rangle$
- correlation function $r(x_1,x_2)$ – more likely to find particles near “hot spots”
- approach to equilibrium → Boltzmann eq’n with Langevin noise

Onset of Thermalization?

Data → Pruneau's talk Monday, 4c



thermalization ⇒ centrality dependence of mean pt and fluctuations

- describes peripheral collisions

PRL 92 (2004) 162301; Quark Matter 04

- deviation in central collisions – **radial flow?**

Collective Flow

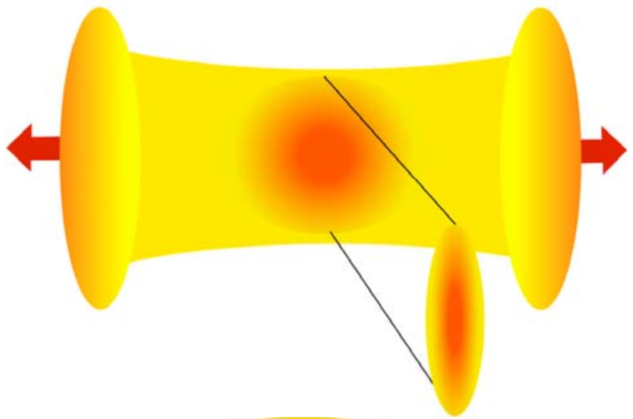
near equilibrium: pressure ⇒ flow
Voloshin

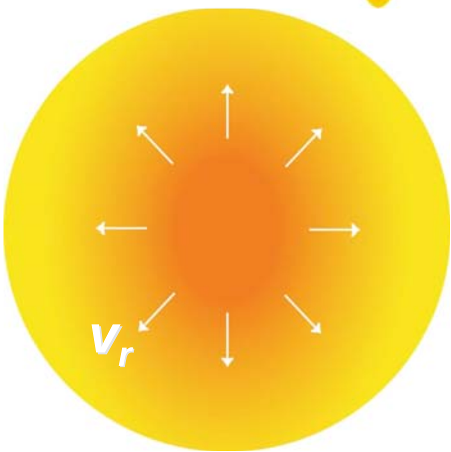
blue shifted temperature $T_{eff} \approx T\left(\frac{1+v_r}{1-v_r}\right)^{1/2}$

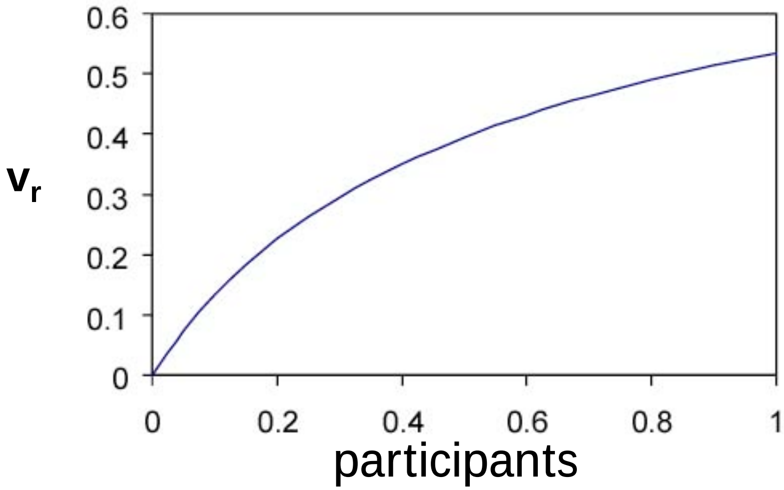
radial velocity v_r
Schnedermann, et al.

flow increases with centrality

STAR data: $\langle v_r \rangle = 0.59 \pm 0.05$
for central collisions
Barannikova, nucl-ex/0403014







Thermalization + Flow

measured $\langle p_t \rangle$ at 200 GeV fixes T_{eff} vs centrality

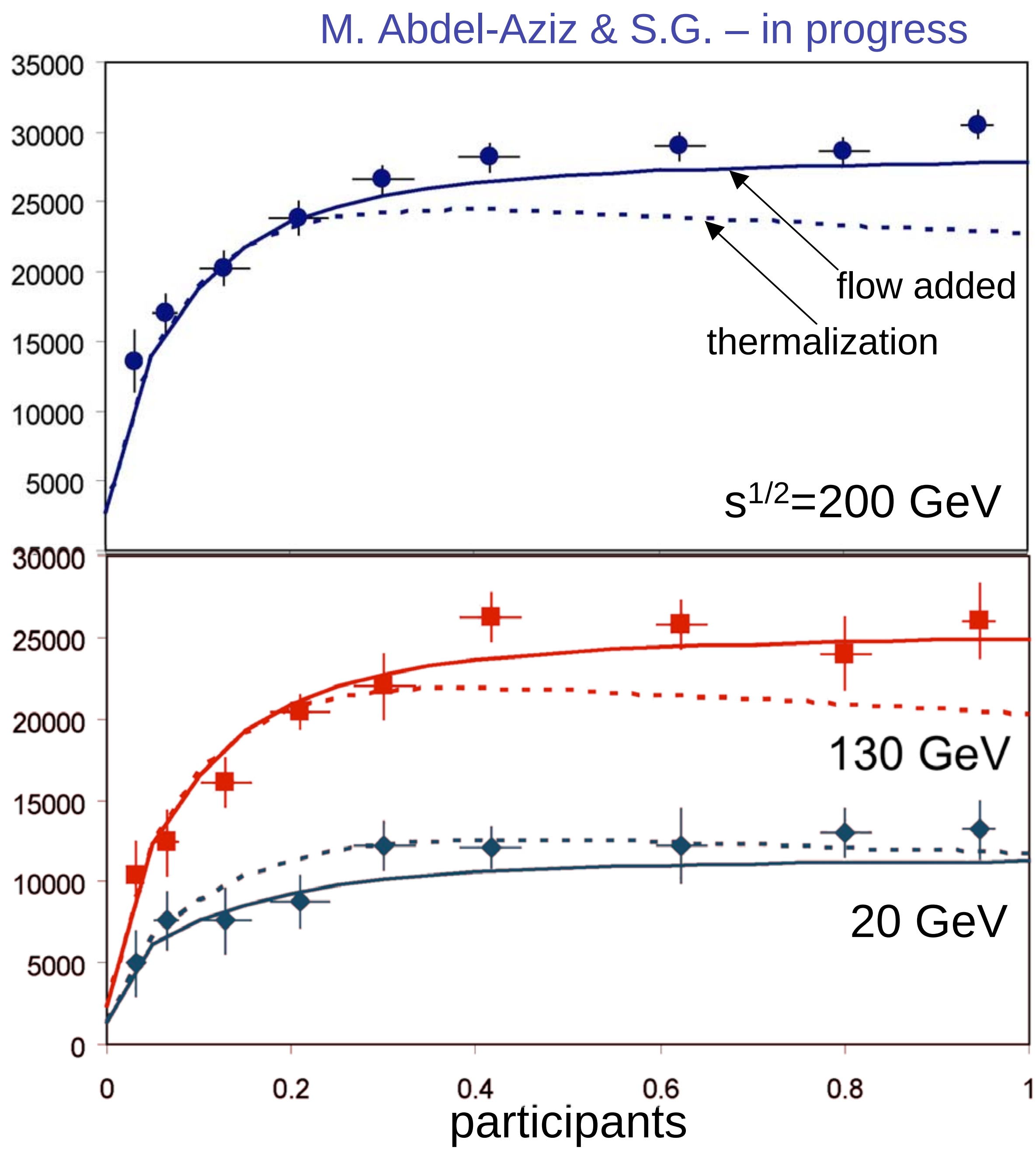
blue-shift:

- enhances equilibrium contribution

$$\langle \delta p_{t1} \delta p_{t2} \rangle \propto \int \delta T_1 \delta T_2 \, r_{12}$$

$$\propto T_{eff}^2 \propto \frac{1+v_r}{1-v_r}$$

- compensates effect of longitudinal cooling

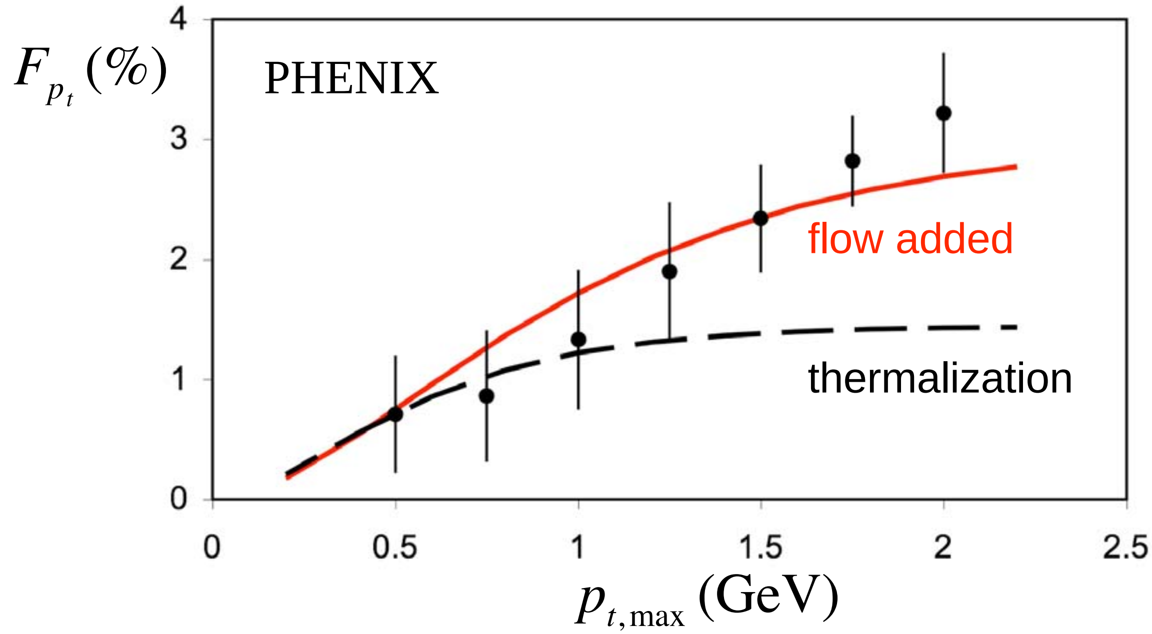


Flow Contribution

dependence on maximum $p_{t,\,max}$

$$F_{p_t} \approx \langle N \rangle \langle \delta p_{t1} \delta p_{t2} \rangle / 2\sigma^2$$

$$\sigma^2 = \langle p_t^2 \rangle - \langle p_t \rangle^2$$



explained by blueshift?

$$F_{p_t}^{obs} \propto \frac{\langle N \rangle_{obs}}{\langle N \rangle} = 1 - e^{-p_{t,\max}/T_{eff}} \left(1 + p_{t,\max}/T_{eff}\right)$$

$$T_{eff} \approx T \left(\frac{1 + v_r}{1 - v_r} \right)^{1/2}$$

needed: **more realistic hydro + flow fluctuations**

Hydrodynamic Density Correlations

hydro: stress-energy tensor $T^{\mu\nu}$ and current j^{μ}
number density $n = j^0$ and momentum density $g^i = T^{0i}$

phase space density $f(x,p)$ fluctuates:

$$r(p_1, p_2) = \int dx_1 dx_2 \left(\langle f_1 f_2 \rangle - \langle f_1 \rangle \langle f_2 \rangle - \delta_{12} \langle f_1 \rangle \right)$$

multiplicity fluctuations probe **density** $n(x) = \int dp\, f(x, p)$

$$R = \int dp_1 dp_2 \frac{r(p_1, p_2)}{\langle N \rangle^2}$$

Measured: NA49; PHENIX; PHOBOS

density correlation function

$$r_n = \langle n(x_1) n(x_2) \rangle - \langle n(x_1) \rangle \langle n(x_2) \rangle$$

$$\Delta r = r - r_{eq}$$

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multiplicity fluctuations probe **density** $n(x) = \int dp \; f(x,p)$

$$R = \int dx_1 dx_2 \frac{\Delta r_n(x_1, x_2)}{\langle N \rangle^2}$$

density correlation function

$$r_n = \langle n(x_1) n(x_2) \rangle - \langle n(x_1) \rangle \langle n(x_2) \rangle \qquad \Delta r = r - r_{eq}$$

Hydrodynamic Momentum Correlations

p_t fluctuations probe **transverse momentum density** $g_t(x) = \int dp\ p_t f\left(x,p\right)$

$$\left\langle N_{pair}\right\rangle \left\langle \delta p_{t1}\delta p_{t2}\right\rangle = \int dp_1 dp_2 \left(p_{t1} - \left\langle p_t\right\rangle\right)\left(p_{t2} - \left\langle p_t\right\rangle\right) r\left(p_1,p_2\right)$$

momentum density correlation function

$$r_g = \left\langle g_t(x_1) g_t(x_2)\right\rangle - \left\langle g_t(x_1)\right\rangle \left\langle g_t(x_2)\right\rangle \qquad \Delta r = r - r_{eq}$$

observable: $$S = \int dx_1 dx_2 \frac{\Delta r_g\left(x_1,x_2\right)}{\left\langle N_{pair}\right\rangle} = \left\langle \delta p_{t1}\delta p_{t2}\right\rangle + \left\langle p_t\right\rangle^2 \frac{R}{1+R}$$

measured $\langle \delta p_{t1}\delta p_{t2}\rangle$ plus HIJING $R \Rightarrow r_g$ and r_n are **comparable**

Hydrodynamic Momentum Correlations

p_t fluctuations probe **transverse momentum density** $g_t(x) = \int dp\ p_t f\left(x,p\right)$

$$\left\langle N_{pair}\right\rangle\left\langle \delta p_{t1}\delta p_{t2}\right\rangle = \int dx_1 dx_2 \left[\Delta r_g(x_1,x_2) - \left\langle p_t\right\rangle^2 \Delta r_n(x_1,x_2)\right]$$

momentum density correlation function

$$r_g = \left\langle g_t(x_1) g_t(x_2)\right\rangle - \left\langle g_t(x_1)\right\rangle\left\langle g_t(x_2)\right\rangle \qquad \Delta r = r - r_{eq}$$

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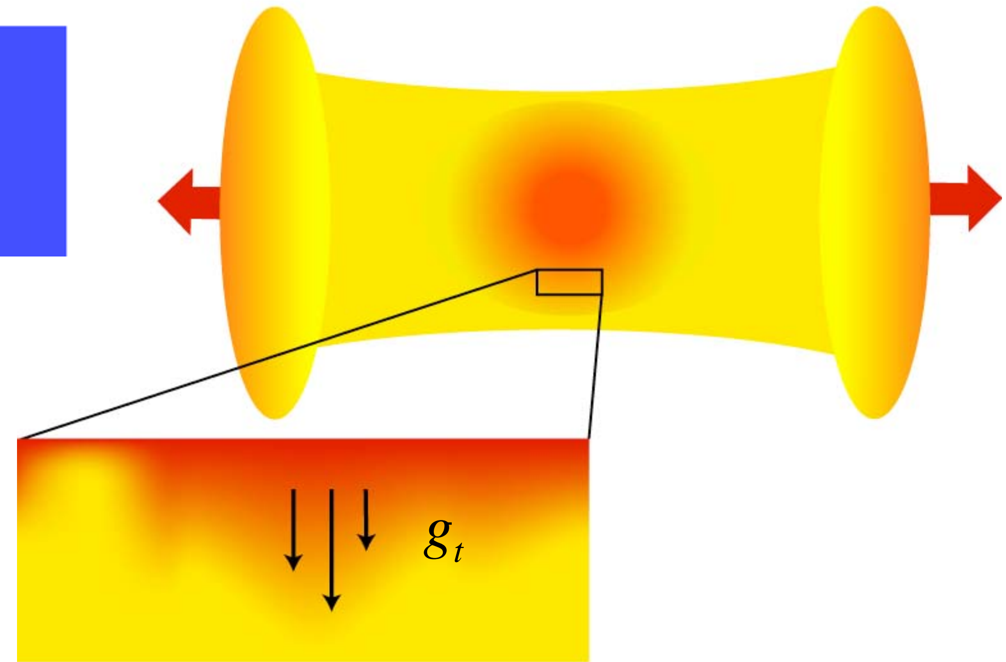
Evolution of Momentum Fluctuations



assume average boost invariant radial flow

$$\langle g_t \rangle = (e+P)\gamma^2 v_r$$

shear viscosity damps small fluctuations $g_t - \langle g_t \rangle$

viscous hydro: Muronga et al.; Teaney et al.; Moore et al.; Hirano and Gyulassy

flow fluctuations: Mrowczynski and Shuryak

diffusion of small fluctuations

$$\left(\frac{\partial}{\partial t} - \Gamma_s \nabla^2 \right) g_t = 0$$

momentum diffusion length

$$\Gamma_s = \frac{\eta}{e+P}$$

shear viscosity η

energy density e, pressure P

Viscosity Broadens Rapidity Correlations

analogous effect in charge diffusion

Stephanov & Shuryak; Abdel-Aziz & Gavin; Koide; Sasaki et al.; Wolchin; Teaney & Moore; Bass, Pratt & Danielowicz

viscous diffusion + Bjorken flow

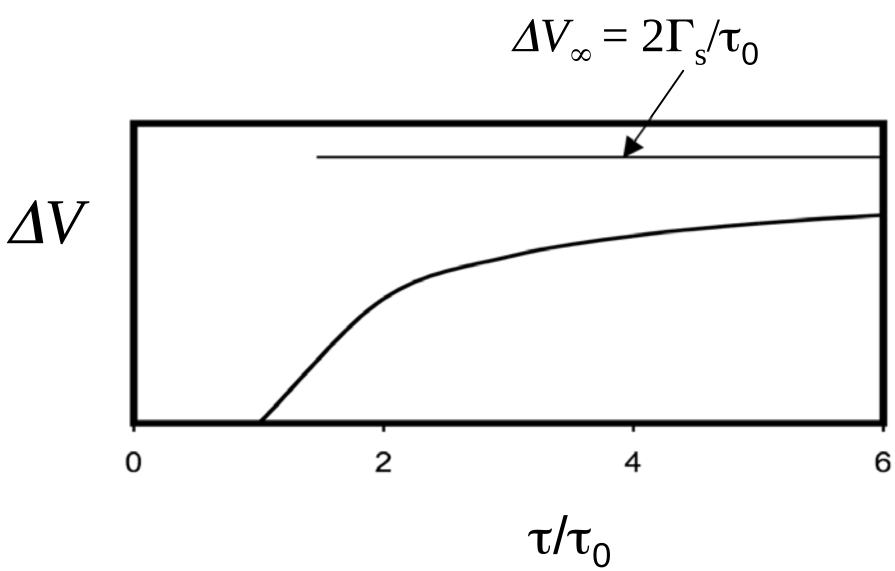
$$\frac{\partial g_t}{\partial \tau} = \frac{\nu}{\tau^2}\frac{\partial^2 g_t}{\partial y^2} \longrightarrow \frac{d}{d\tau}\Delta V = \frac{2\nu}{\tau^2}, \qquad \Delta V = \int y^2 g_t\, dy$$

random walk in rapidity y vs. proper time τ

$$\Delta V = 2\Gamma_s\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$

momentum diffusion length $\Gamma_s = \eta/(e+P)$

formation at τ_0



How Much Viscosity?

sQGP core + hadronic corona

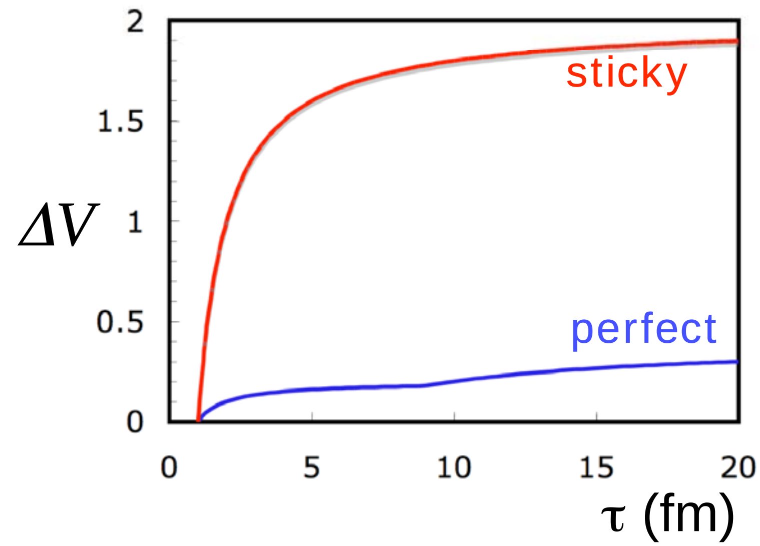
see Hirano & Gyulassy

two extreme scenarios:

sticky liquid $\Gamma_{\text{pQGP}} \sim \Gamma_{\text{HRG}} \sim 2$ fm

perfect liquid $\Gamma_{\text{sQGP}} \sim (4\pi T_c)^{-1}$, $\Gamma_{\text{HRG}} \sim 2$ fm

Abdel-Aziz, S.G - in progress

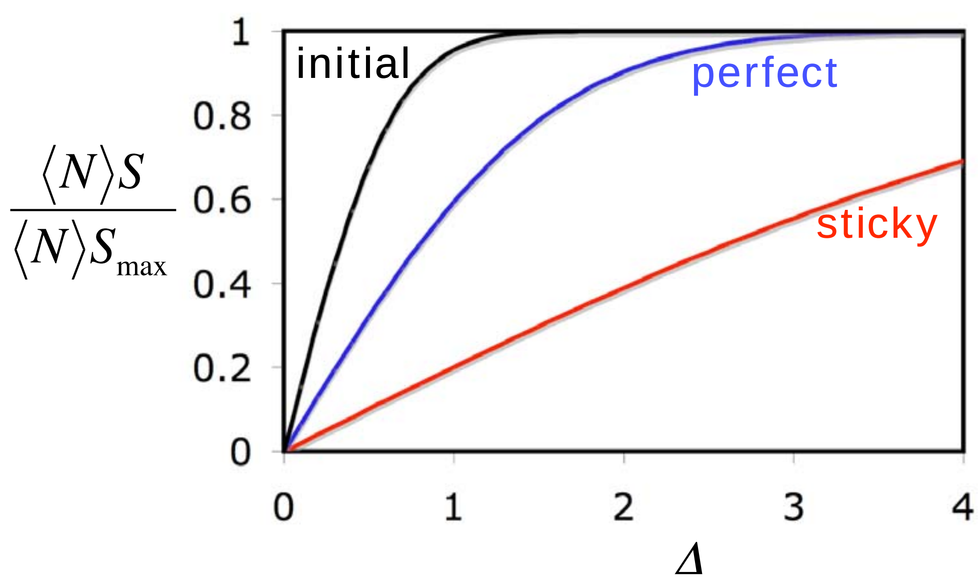


fluctuations, rapidity window Δ:

- solve diffusion eq'n
- Gaussian r_g, width σ
- initial $\sigma_0 \sim 0.25 \sim$ balance function

$$\langle N \rangle S \propto \iint_{\Delta} r_g \, dy_1 dy_2$$

$$\sigma^2 = \sigma_0^{\,2} + 2\Delta V(\tau_f)$$



Summary

p_t fluctuations from 20 to 200 GeV

- peripheral collisions ⇒ onset of thermalization
- central ⇒ flow

multiple scattering phenomena

- random walk in momentum space
- **broadens width of rapidity distribution**

testing the perfect liquid → viscosity info

- diffusion coefficient ∝ shear viscosity
- **compare rapidity width of momentum fluctuations for different projectile sizes and energies**
- cross-check: combine with other indirect viscosity measures, e.g., mach cone Casalderry-Solana, Shuryak & Teaney

Nonequilibrium $\langle p_t \rangle$

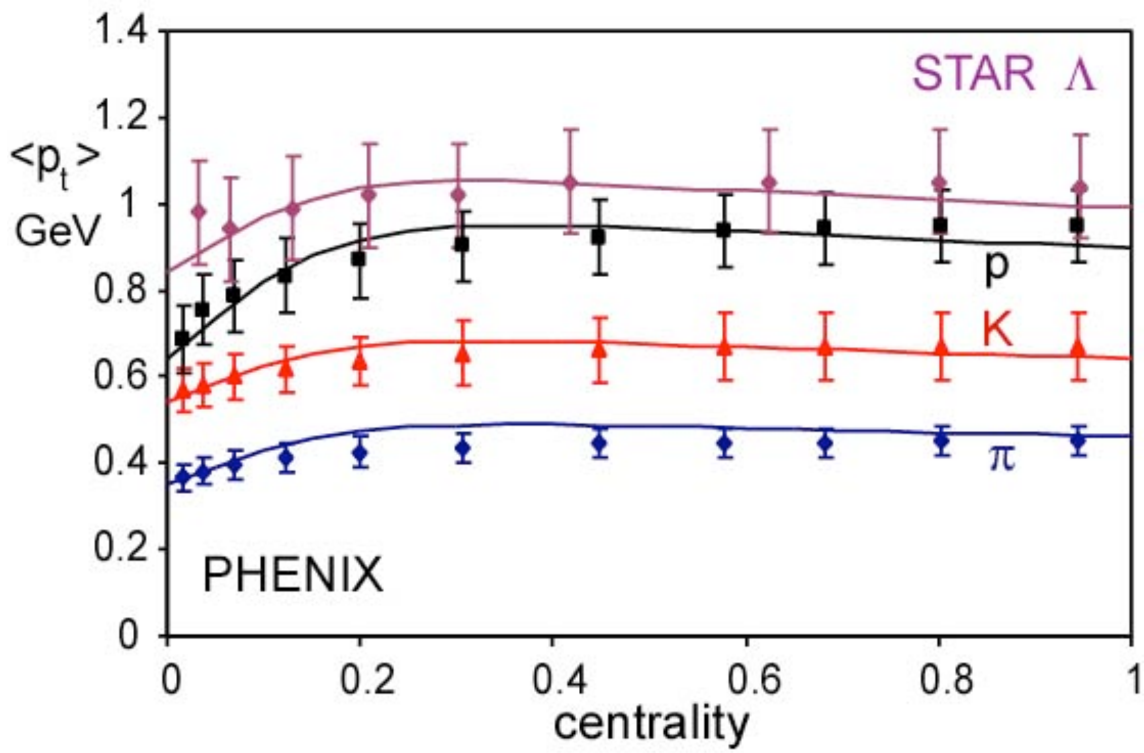
more participants $N \Rightarrow$ longer lifetime $\Rightarrow$ smaller survival probability S

$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1 - S)$$

Boltzmann equation + longitudinal expansion

- survival probability
- $S = (\tau_0 / \tau_F)^{\alpha}$
- equilibrium cooling $\langle p_t \rangle_e \propto (\tau_0 / \tau_F)^{\gamma}$
- centrality dependence $\tau_F \propto N^x, \alpha \propto N^y$

$\tau_F(0) = 6$ fm, $\tau_0 = 1$ fm, $T_0 = 450$ MeV
$\alpha(0) = 4$, $\gamma = 0.15$, $x = 1$, $y = 1/2$



Nonequilibrium Dynamic Fluctuations

Boltzmann equation with Langevin noise ⇒ phase-space correlations ⇒ dynamic fluctuations

simple limit: initial correlations equilibrium-like

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \langle \delta p_{t1} \delta p_{t2} \rangle_0 S^2 + \langle \delta p_{t1} \delta p_{t2} \rangle_{le} \left(1 - S^2\right)$$

$\langle \delta p_{t1} \delta p_{t2} \rangle_0$ **initial fluctuations** – production mechanism

$\langle \delta p_{t1} \delta p_{t2} \rangle_{le}$ **near equilibrium fluctuations –** range of temperatures in the volume

S **survival probability** vs. centrality from $\langle p_t \rangle$ data

compute initial and local equilibrium fluctuations – is magnitude reasonable?

Effect of Flow on Mean p_t

radial flow → blue shift → increases local equilibrium mean p_t

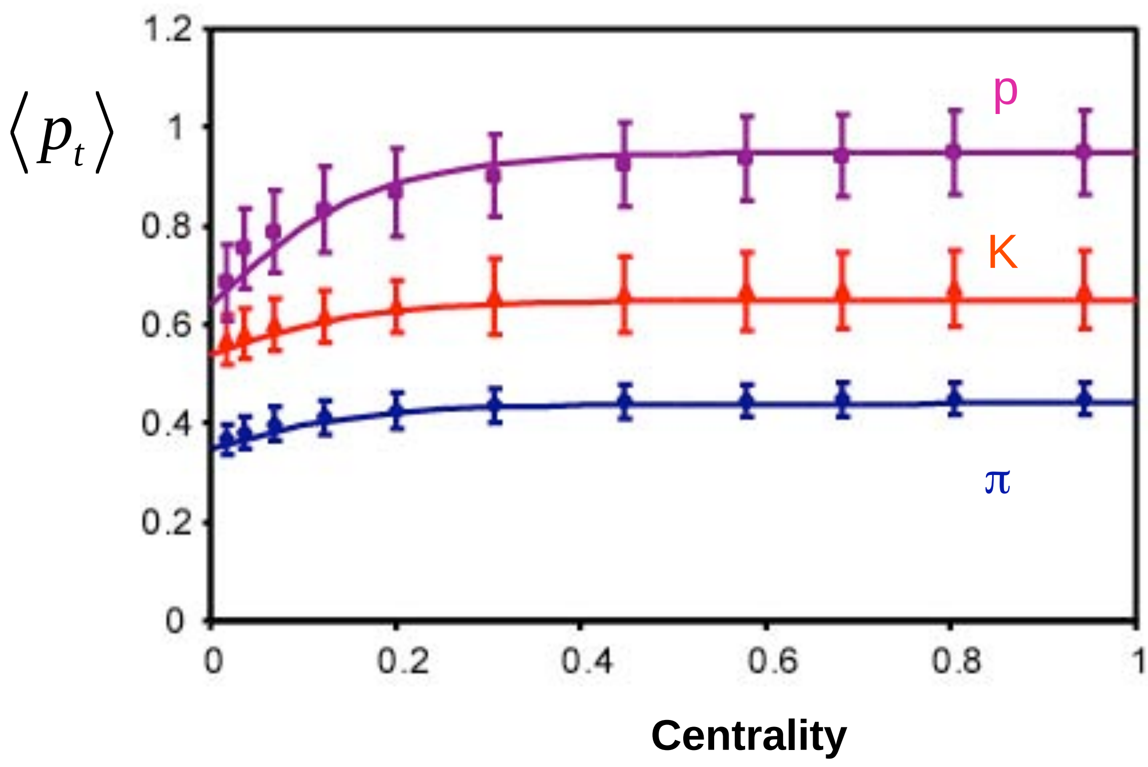
$$\langle p_t \rangle = \langle p_t \rangle_0 S + \langle p_t \rangle_e (1 - S) \quad \text{with} \quad \langle p_t \rangle_e \approx \langle p_t \rangle_{th} \left(\frac{1 + v_r}{1 - v_r} \right)^{1/2}$$

velocity vs. centrality

fit: coincides with

$$T_{eff} \approx T \left(\frac{1 + v_r}{1 - v_r} \right)^{1/2} \approx const.$$

fixed freeze out temp in comoving frame

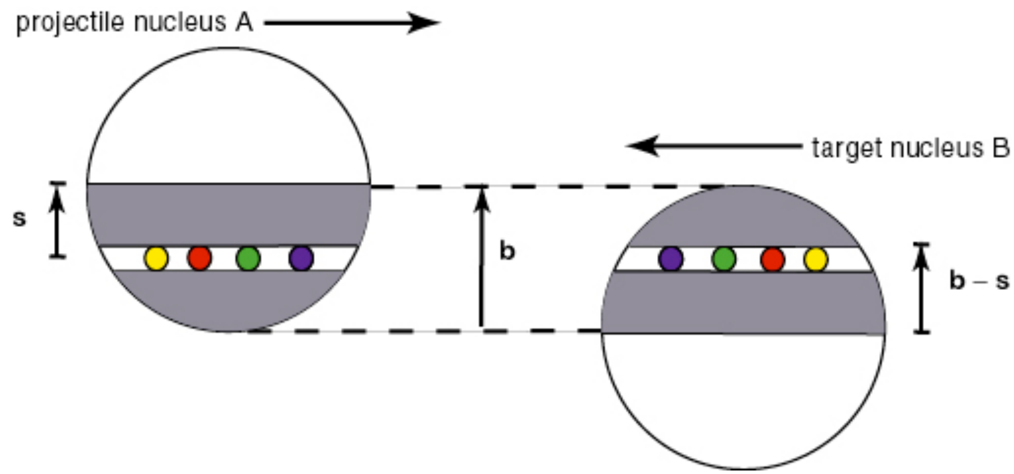


Initial Fluctuations

AA collision:

M participant nucleons

→ independent strings



multiplicity ∝ *M*

variance $R_{AA} \propto M^{-1}$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \equiv \frac{1}{\langle N(N-1) \rangle} \left\langle \sum_{i \neq j} \delta p_{ti} \delta p_{tj} \right\rangle$$

$$\langle N(N-1) \rangle = \langle N \rangle^2 (1 + R_{AA})$$

$$\langle \delta p_{t1} \delta p_{t2} \rangle \approx \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{M} \left(\frac{1 + R_{pp}}{1 + R_{AA}} \right)$$

ISR data → $\langle \delta p_{t1} \delta p_{t2} \rangle_{pp}$; HIJING R_{pp}, R_{AA}

Fluctuations Near Equilibrium

correlations from non-uniformity – more likely to find particles near “hot spots” → spatial correlation function

$$\langle \delta p_{t1} \delta p_{t2} \rangle_e \propto \iint \delta T(x_1) \delta T(x_2)\, r(x_1, x_2)\, dx_1 dx_2$$

assume:

- longitudinal Bjorken expansion up to freeze out time
- independent longitudinal and transverse d.o.f.
- $\delta p_t \propto \delta T(x)$ independent of rapidity
- gaussian densities and correlation function

obtain:

$$\langle \delta p_{t1} \delta p_{t2} \rangle_e \approx \frac{\langle p_t \rangle^2 R_{AA}}{1 + R_{AA}} F(\xi / R_t)$$

transverse size R_t, correlation length ξ