Two particle correlations in jets and triggered distributions in dense matter

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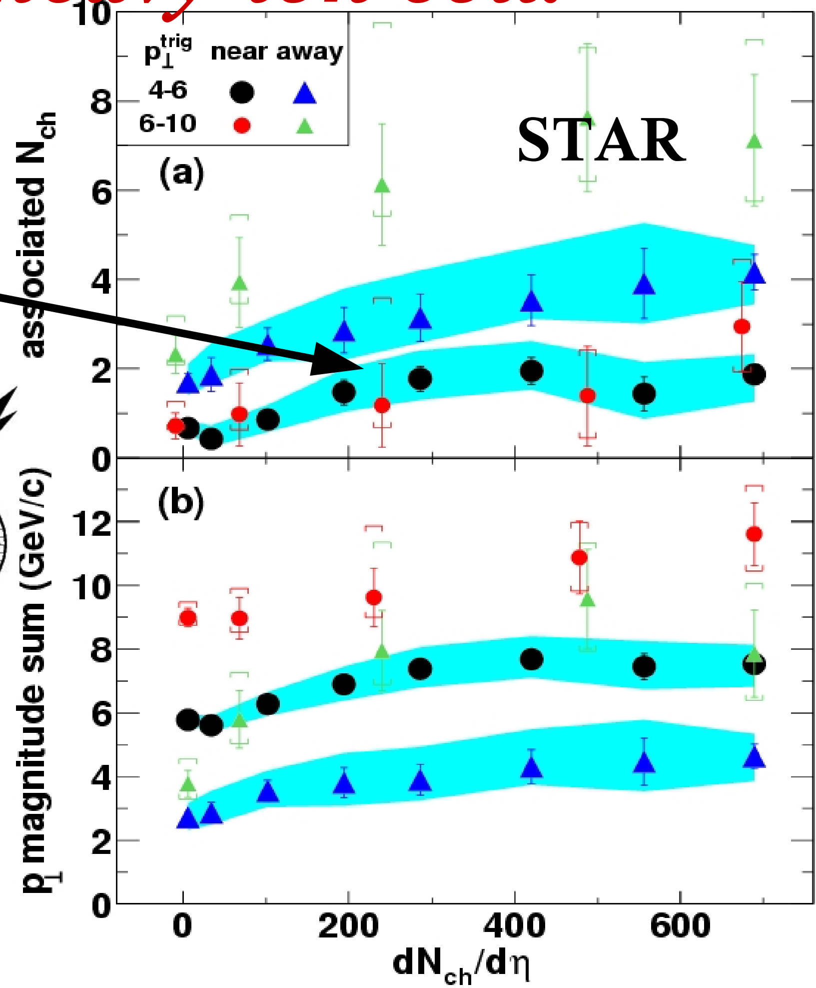
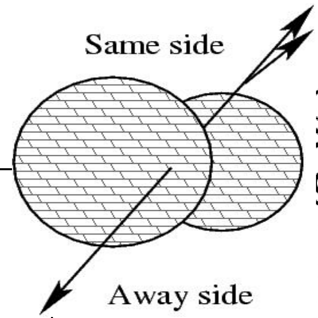
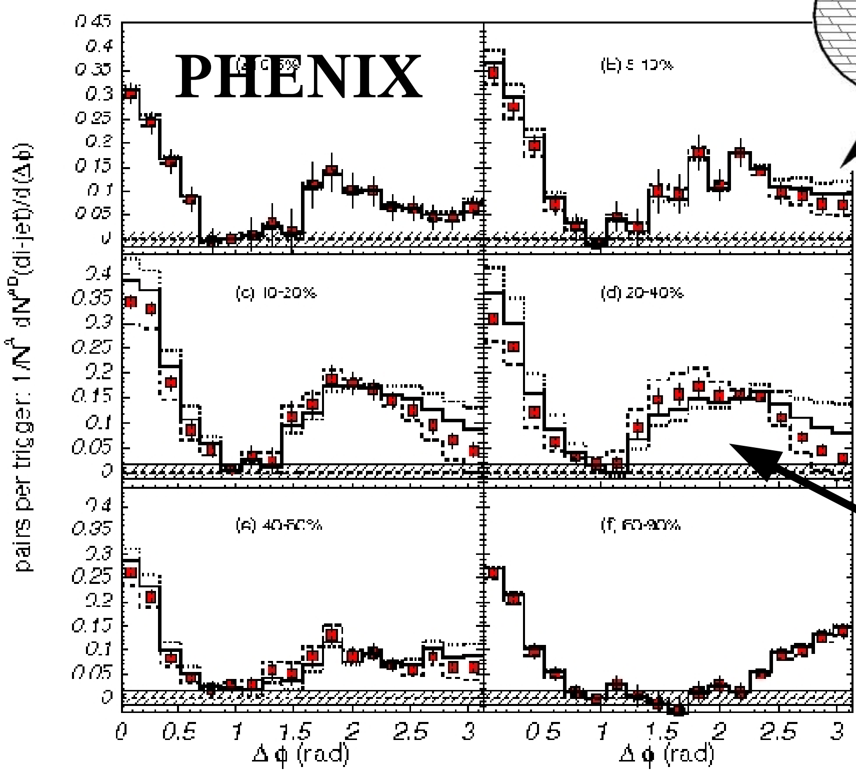
Hadron correlations in heavy-ion coll.

Enhancement in assoc. yield on near side

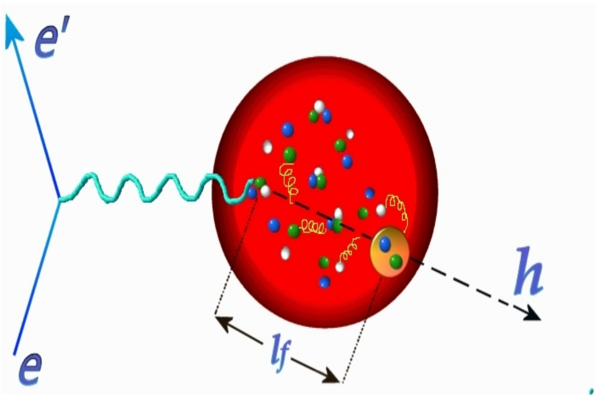


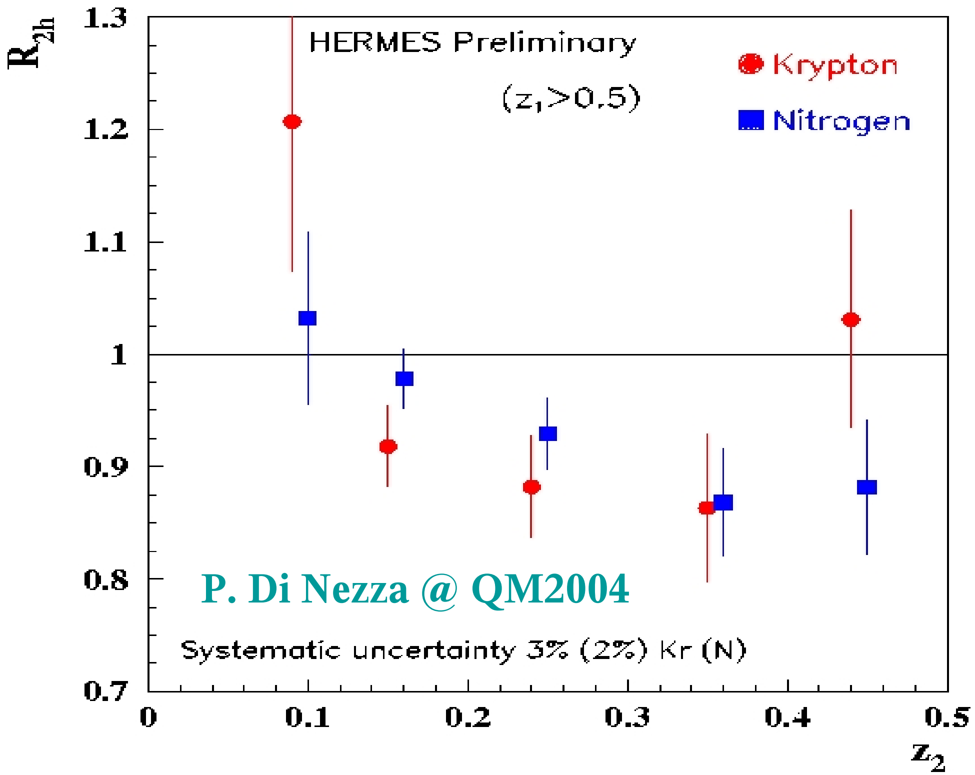




Bump like structure on the away side

Analogue of near side in cold matter: DIS on nuclei



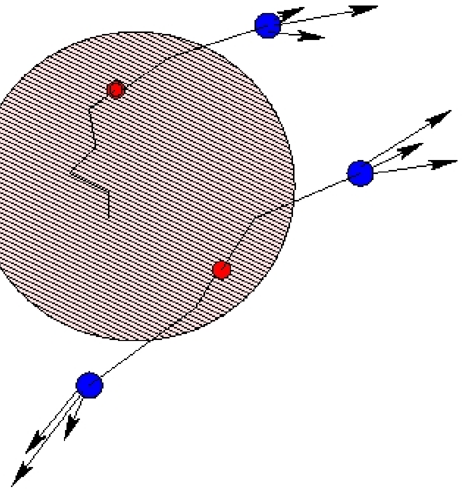


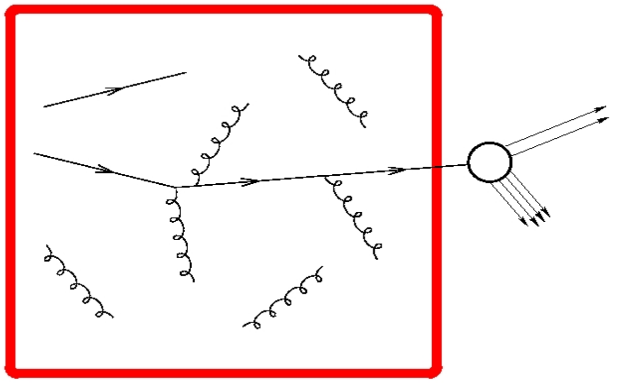
- ***R_{2h} is like I_{AA} :***
- ***replace p with D***
- ***replace p_T with $z\nu$***

$$R_{2h} = \frac{\dfrac{No.\,of\;events\;with\;at\;least\;2\;hadrons\;with\;z_1 > 0.5}{No.\,of\;events\;with\;at\;least\;one\;hadron\;with\;z > 0.5}}{same\;ratio\;on\;deuterium}$$

THE PICTURE OF PARTON ENERGY LOSS :

- ***High energy partons are created over the entire collision zone***

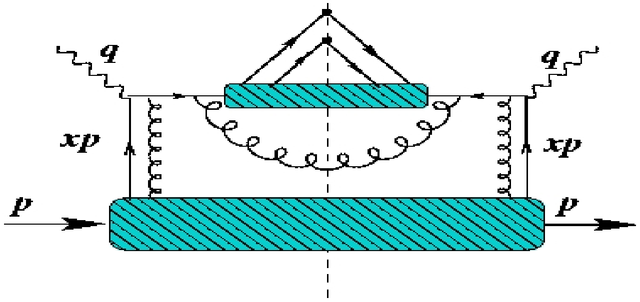


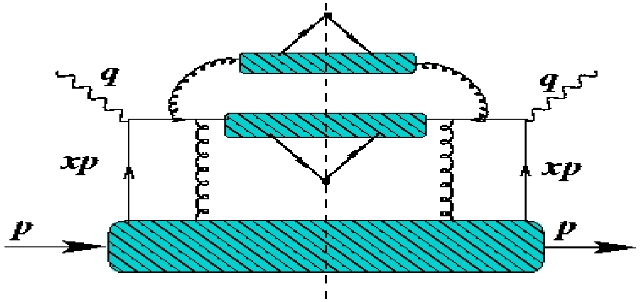


- ***Lose energy by partonic interaction, medium may be hadronic or partonic***
- ***Emerge as partons and then fragment***
- ***Require knowledge of single and double fragmentation functions***

$$D^{h}_{q,g}(z) \qquad\qquad D^{h_1 h_2}_{q,g}(z_1, z_2)$$

The kinds of diagrams that need to be evaluated !

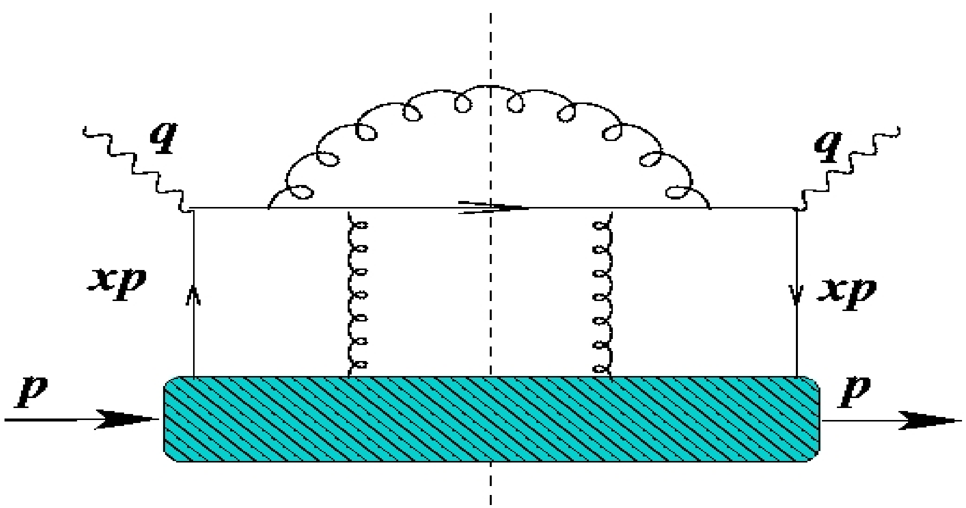




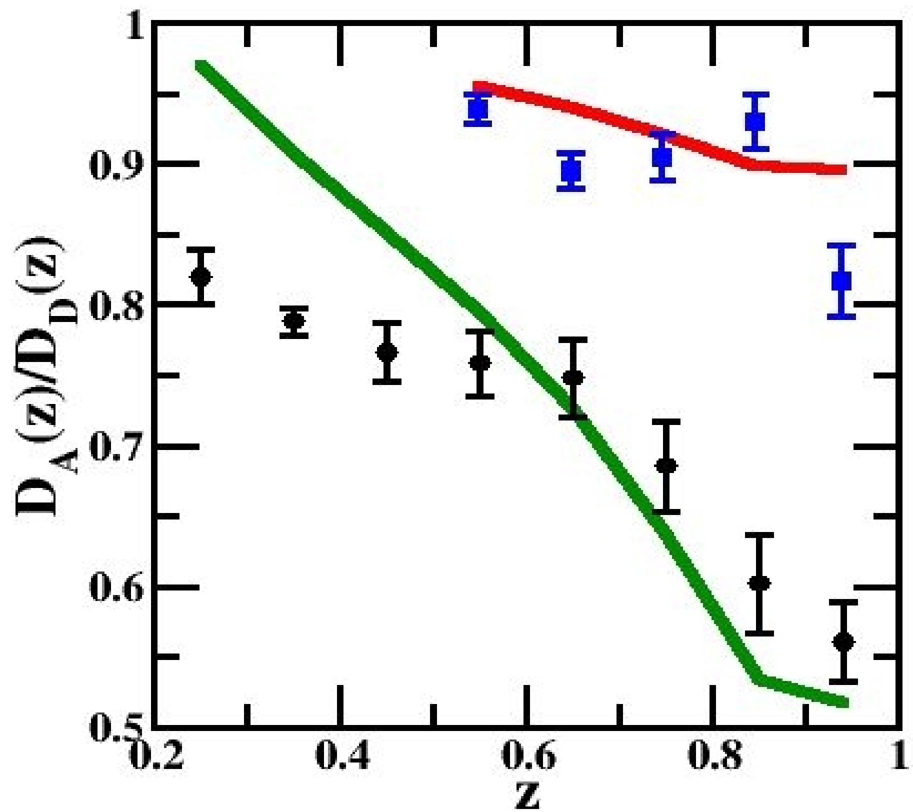
The second diagram has two independent fragmentations can be factorized in the collinear limit as...

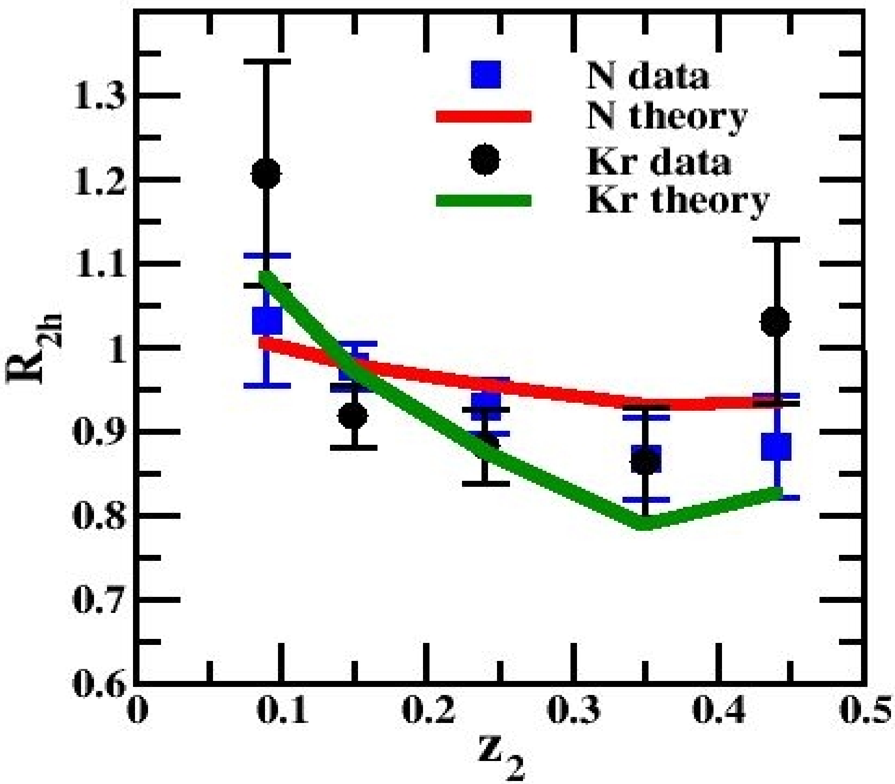
$$\frac{dW^{\mu\nu}}{dz_1 dz_2} = \int \frac{dz}{z^2} D_q(z_1/z, z_2/z) H^{\mu\nu} + \int \frac{dz}{z(1-z)} D_q(z_1/z) D_g(z_2/(1-z)) H^{\mu\nu}$$

$H^{\mu\nu} =$



- *Note the drop with N_part*
- *ν is measured, initial parton energy known (No triggering!)*





Dividing by the single inclusive takes out a lot of the suppression

Medium modification in a deconfined medium

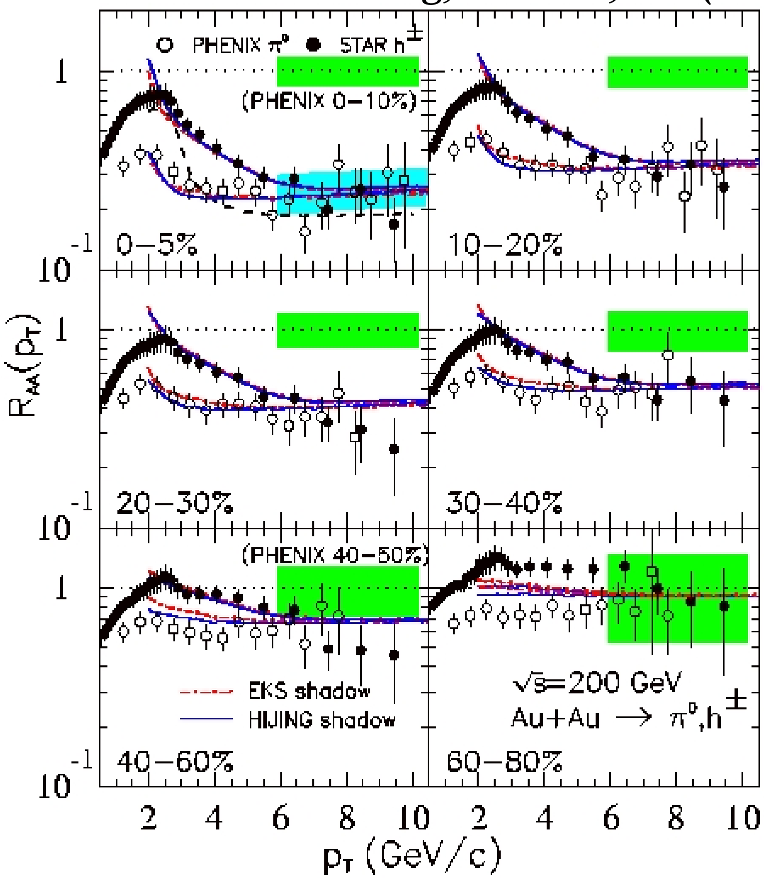
Calculate medium modification in a 1-D expanding medium,

Dial up gluon density to get suppression.

Gluon density proportional to number of participants

dE/dx ~ 14 GeV/fm

X.N.Wang, PLB 595, 165 (2004)



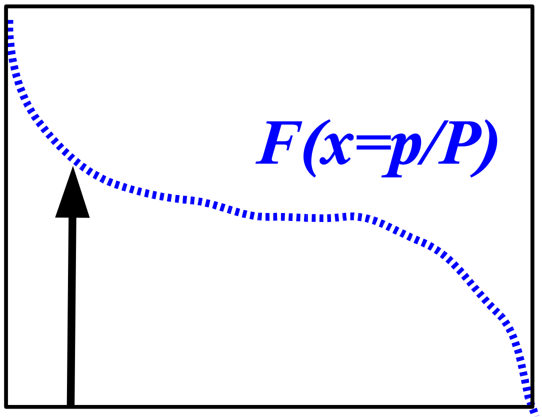
Simplified picture of triggering effect

$$\frac{d\sigma}{dp_h} = \int d\,q\,d\,p\, f_q^A(p/P)\, f_q^A(q/P)\, d\sigma_{hard}(p+q)\, \tilde{D}^h(p_h/p)$$

$$\tilde{D}(p_h/p) = \frac{p}{p-\Delta p} D\left(\frac{p_h}{(p-\Delta p)}\right)$$

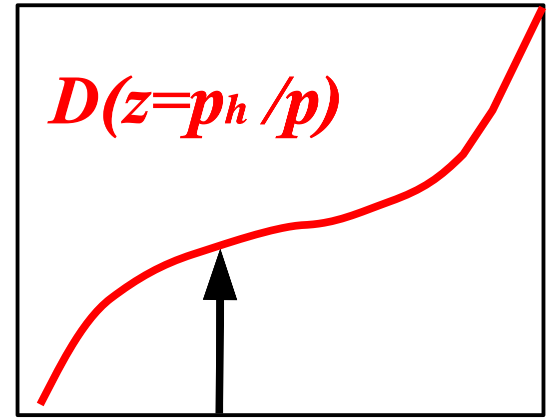
$$z_{effective} = \frac{p_h}{p-\Delta p} > z_{pp} = \frac{p_h}{p}$$

For a given p_h, there is a distribution of p



p/P

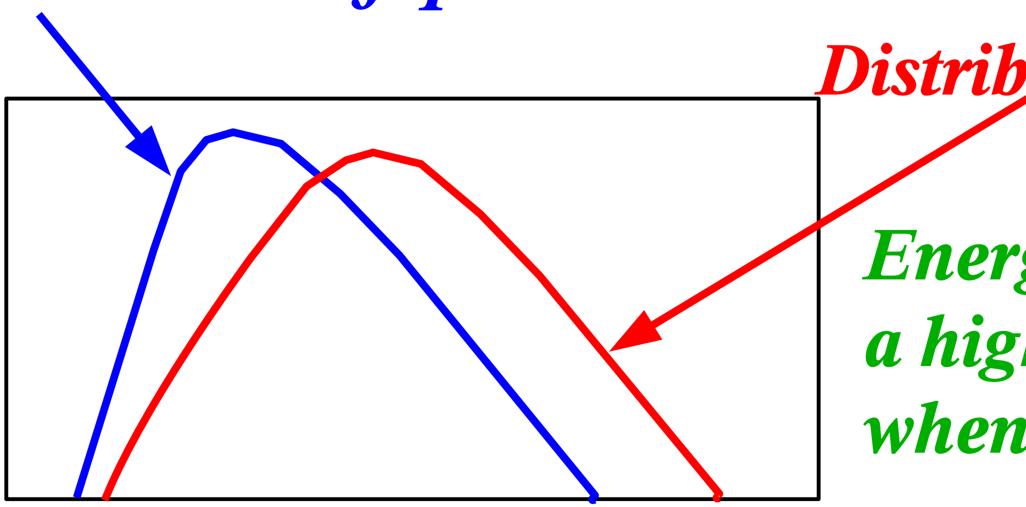
$* \quad \sigma(p+q) \quad *$



p/p_h

Distribution of p without E-Loss

Distribution of p with E-Loss



p

Energy loss selects events with a higher initial parton energy when you trigger on hadron p_T

Assoc. hadron fragments of higher energy parton than in p-p

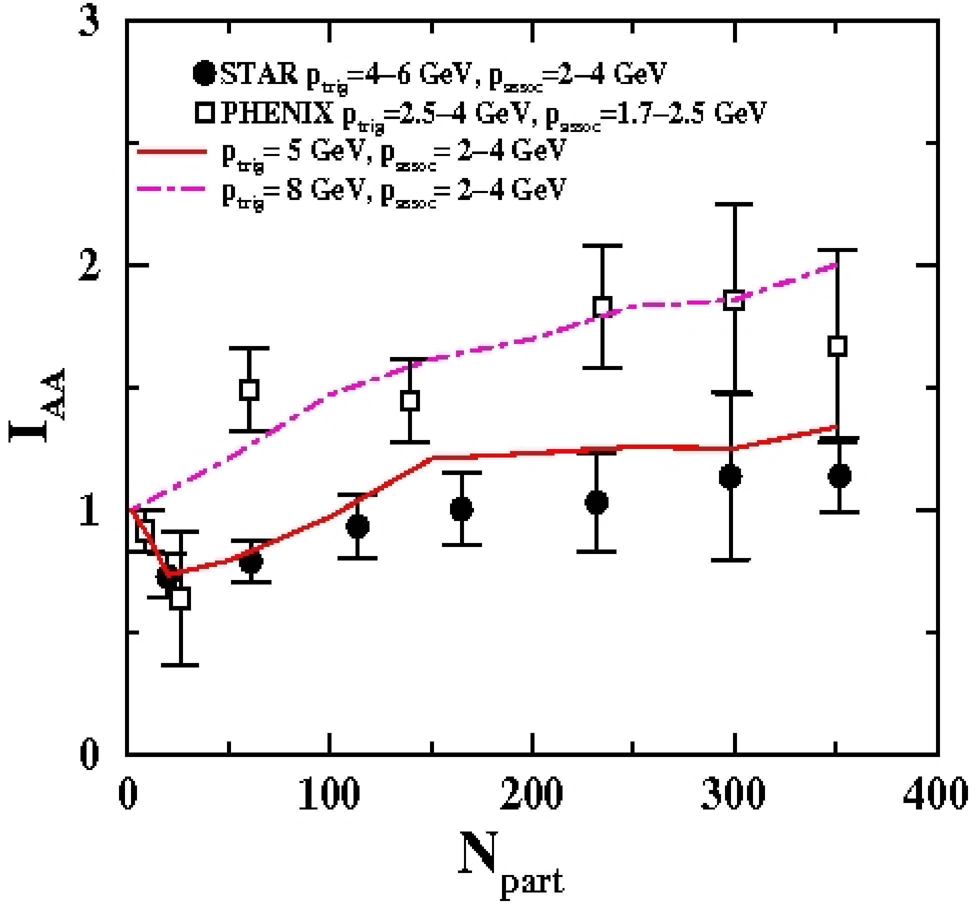
Dihadron results for hot medium

Without the trigger effect, I_{AA} would drop as in DIS off nuclei.

Drop is always less for the soft particles, so these get enhanced due to trigger bias.

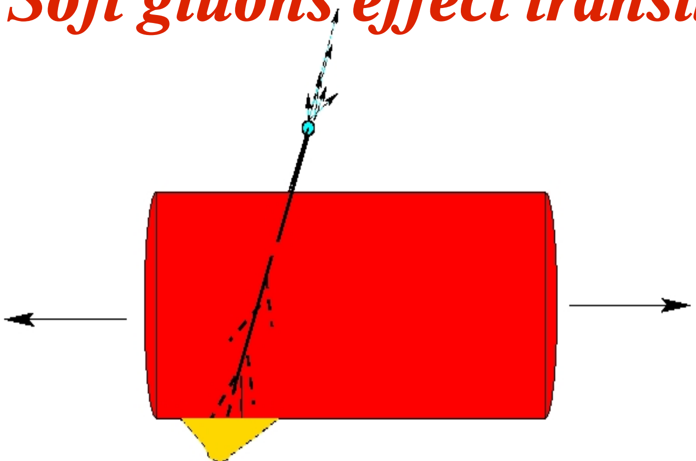
Cronin effect boosts the hard scattering towards the trigger, opposite bias to E-Loss

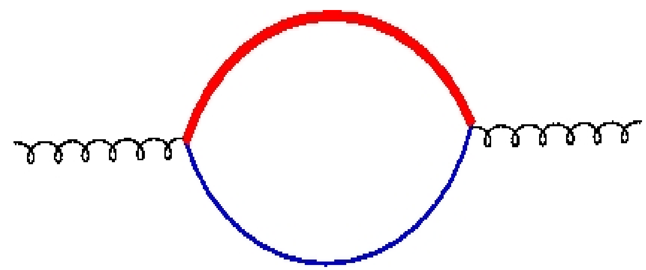
Recombination also enhances soft assoc. mult. see R. Hwa , E. Wang (earlier today).



Cherenkov radiation from fast partons

- *Away side partons travel through a **lot more** matter*
- *If the s-QGP has **heavy colored (bound)** states*
- *Soft gluons effect transitions between these states*

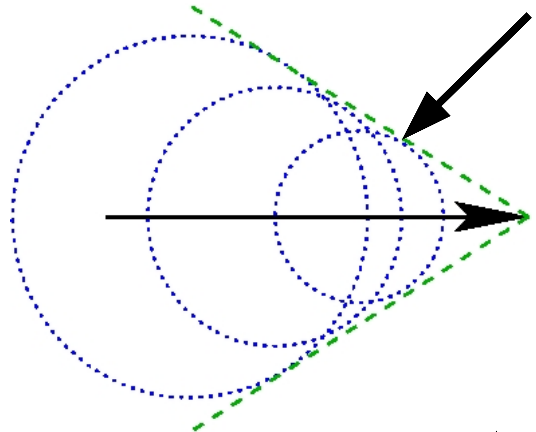




- ***Gluon dispersion relation will be modified***
- ***Large mass diff. → space-like dispersion***
- ***Space-like dispersion → large ε → Cherenkov radiation***

To get Cherenkov radiation:

$$A(x,t) \sim \exp\left(ip\left(x - \frac{p^0(p)}{p}t\right)\right)$$

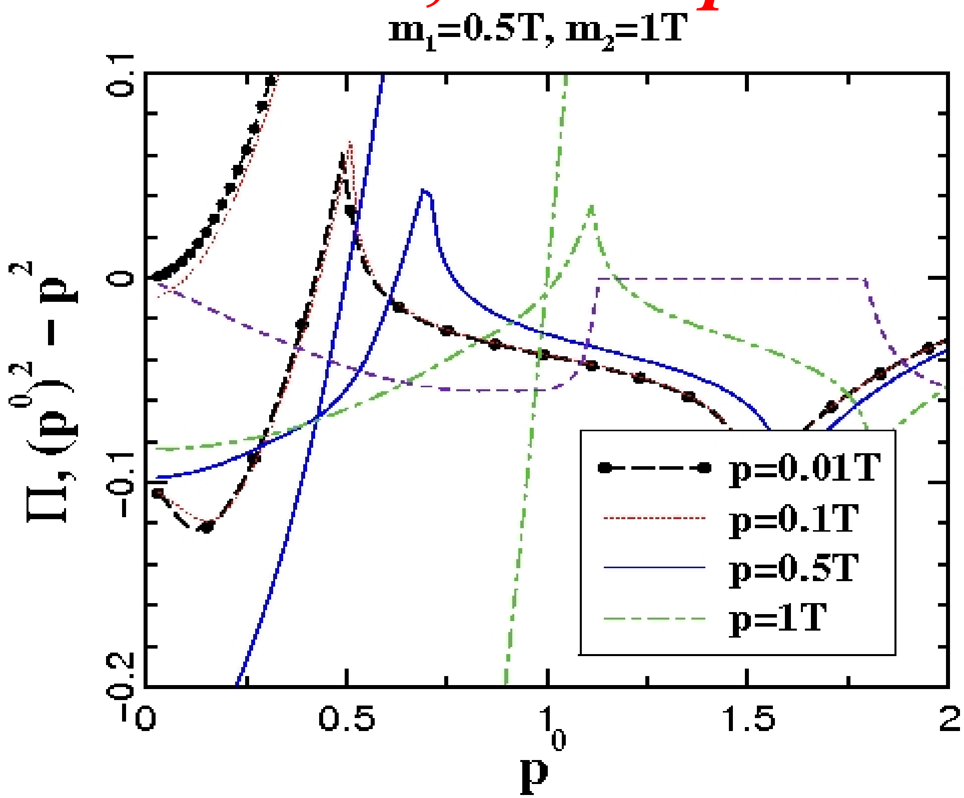


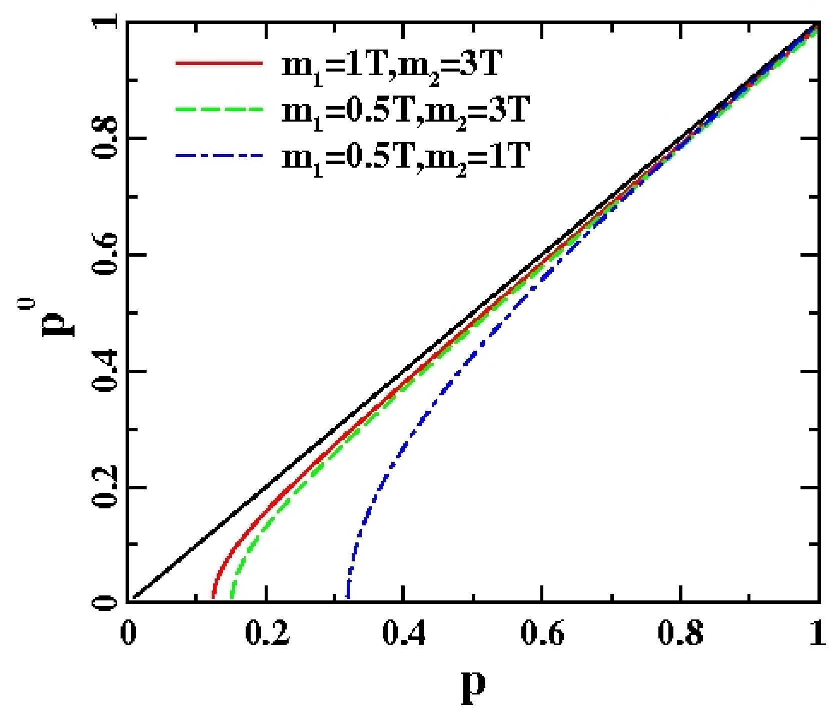
$$v = \frac{1}{\sqrt{\epsilon}} = \frac{p^0}{p} < 1$$

Scaler 3 Φ theory !

In the medium, solve dispersion relation

$$(p^0)^2 - p^2 = \Re[\Pi(p^0, p, T)]$$



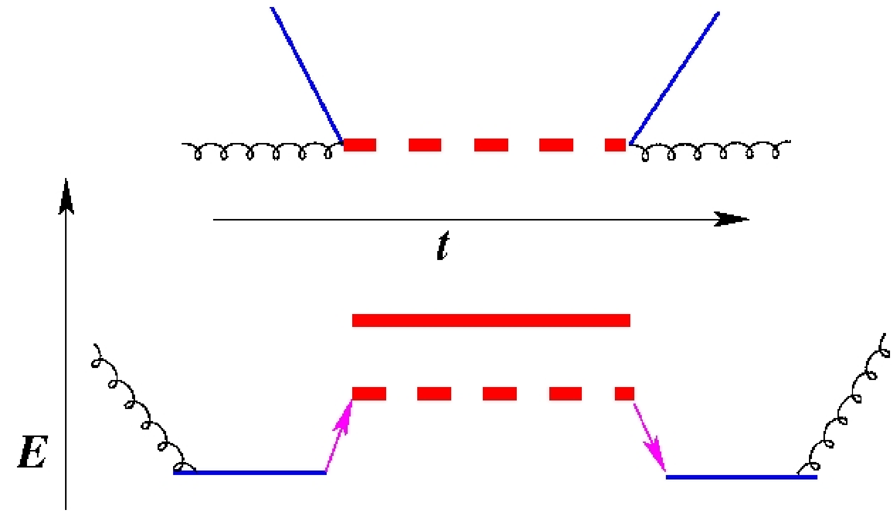


What causes the large negative self energy?

Open the thermal loop!

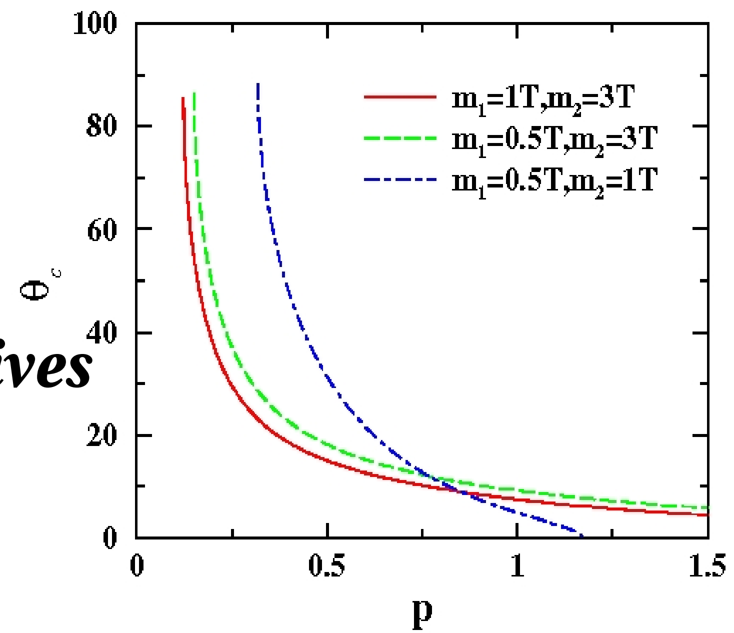
- Space like virtual intermediate state

- Due to large mass gap, cannot excite the intermediate state to its on-shell energy



Regular Cherenkov, gives small E-loss

Scattering induced Cherenkov Brem. gives large E-loss, see Xin-Nian Wang's talk tomorrow !



Summary & Conclusions!

- *Jet correlations: a diagnostic of structure of dense matter*

- *On near-side, hard brem. resurrects picture of partonic Int.*
- *Independent measure of partonic density*
- *Enhancement of mult. = trigger bias due to E-loss*

- *On far-side, soft brem. sensitive to emergent d. o. f.*
- *Use Cherenkov like brem. :measure masses of colored res.*
- *Variation of Ch. angle distinguish it from Mach cone.*

Back Up ..

- *The calculation of the double is now in lock-step with the calculation of the single... hard part is the same*
- *Multiple higher twist diagrams need to be evaluated*

Fourier transform from momentum to position space

Simple expressions for a Gaussian density distribution for nucleons in a medium sized nucleus

$$\rho(r)=\rho_0 e^{\frac{-r^2}{2R_A^2}}$$

$$Mod \sim C A^{1/3}(F(x_B)\, x G^N(x))(1-e^{-x_L^2/x_A^2})$$

$$\frac{x_L^2}{x_A^2}=\frac{R_A^2}{\gamma^2\tau_f^2}$$

τ_f = *Formation time*

R_A = *Nuclear size*

γ = *boost*

Final results are quite simple...

$$x = \frac{-Q^2}{2p^+ q^-}$$

$$\tilde{D}(z_1, z_2, \mu^2) = D(z_1, z_2, \mu^2) + \frac{\alpha_s}{2\pi}\int_0^{\mu^2} \frac{dl_\perp^2}{l_\perp^2}\int \frac{dy}{y^2}\left(\frac{1+y^2}{1-y} T_{qg}(x, y, Q^2, l_\perp) + V.C.\right) D(z_1/y, z_2/y, \mu^2)$$

$$+ \frac{\alpha_s}{2\pi}\int_0^{\mu^2} \frac{dl_\perp^2}{l_\perp^2}\int \frac{dy}{y(1-y)}\left(\frac{1+y^2}{1-y} T_{qg}(x, y, Q^2, l_\perp)\right) D(z_1/y, \mu^2) D(z_2/(1-y), \mu^2)$$

$$T_{qg} = \int dy\, dy_1\, dy_2 \langle A|\bar{\Psi}(y) F(y_1) F(y_2) \psi(0)|A\rangle e^{i(phase\ factors)}$$

The nuclear state can be expressed in terms of nucleons

$$|A\rangle = \int \Pi_i \frac{d^3 p_i}{(2\pi)^3 2 p_i} \Theta(p_i) \Phi([p_1, \ldots, p_i, \ldots]) |p_1, \ldots p_i, \ldots\rangle (2\pi)^3 \frac{2P_A}{A} \delta^3\left(\sum_i p_i - P_A\right)$$

Medium modification

- Apply to DIS of Nuclei (HERMES expt. at DESY)
- A parton in a nucleon is struck by EM probe
- Parton traverses cold medium and then fragments
- Fragmentation function is medium modified.

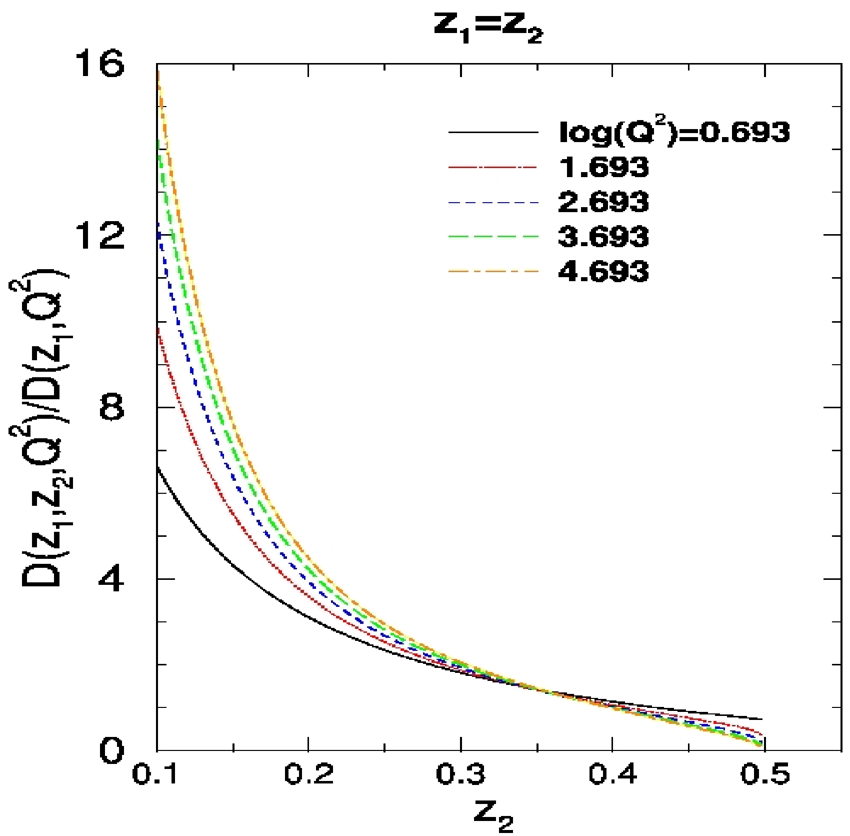
The medium modification equation looks very similar to the vacuum evolution equation...

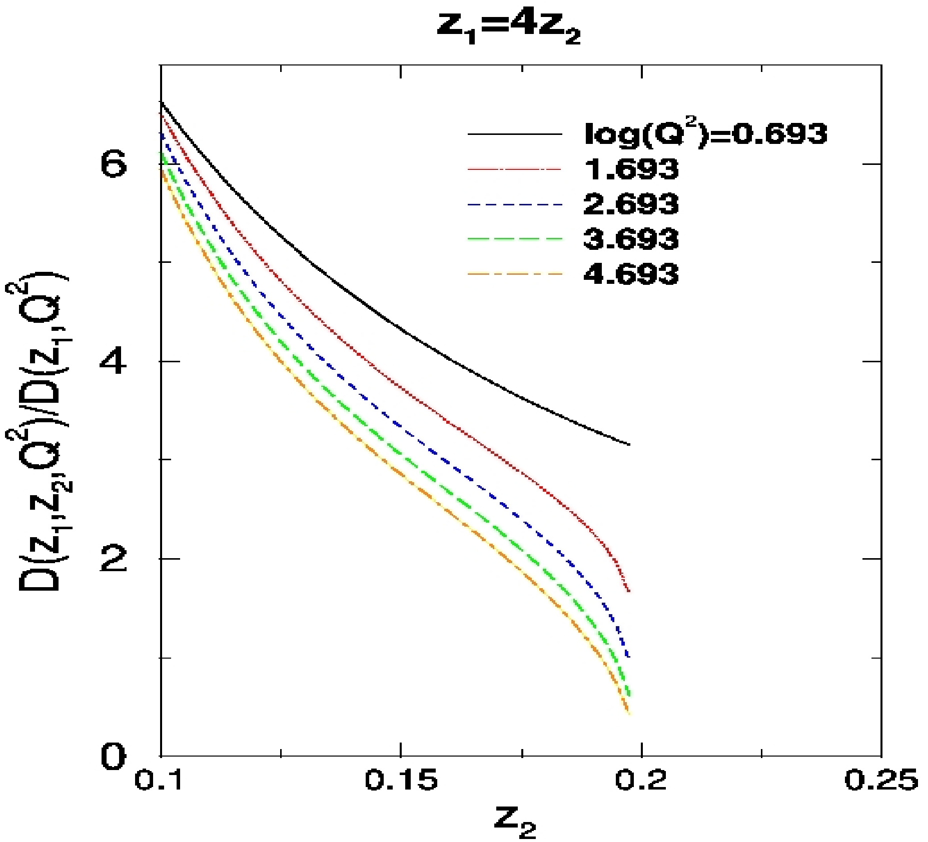
$$\tilde{D}_q(z_1, z_2, \mu^2) = D_q(z_1, z_2, \mu^2) + \int dl_T \int_{z_1+z_2}^{1} \frac{dy}{y^2} \tilde{P}_{q \to qg}(y) D(z_1/y, z_2/y, \mu^2)$$

$$+ \int dl_T \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \hat{\tilde{P}}_{q \to qg}(y) D(z_1/y, \mu^2) D(z_2/(1-y), \mu^2)$$

$$\tilde{D} = medium\ modified\ fragmentation\ function$$

$$\tilde{P} = medium\ modified\ splitting\ function$$



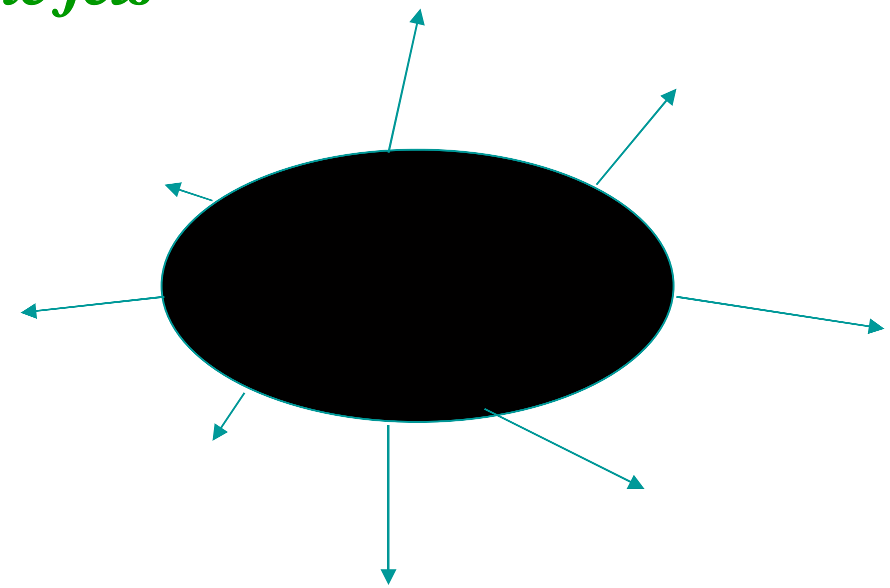


Measure the function at the scale μ, from JETSET

JETSET is a Monte Carlo event algorithm that generates jet like events with a parton shower followed by a string fragmentation routine to get hadrons. It has many parameters tuned to fit almost all experimental data.

SURFACE EMISSION PICTURE

- Suppose the matter produced is very opaque
- Hence only hard collisions on the surface will produce observable jets



- Inconsistent with an R_{AA} near participant scaling
- Inconsistent with all energy loss models which require bulk emission and fit single inclusive data!

How does partonic interaction effect dihadrons?

- *Important to A+A, d+A, DIS and* $e^+ e^-$ *experiments.*
- *To date observations in A+A d+Au and DIS*

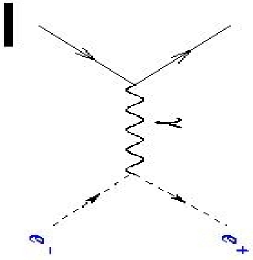
Wish List!

- *Definition and factorization of fragmentation functions*
- *Calculate the effect of medium modification*
- *Requires the evaluation of twist 4 diagrams,*
- *But medium modification similar to vacuum evolution*
- *Calculate and check vacuum evolution first (simpler!)...*

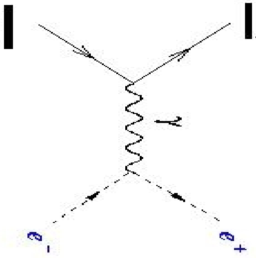
Basic Methodology at L. O. in α_s :

Replace partonic basis with hadronic basis

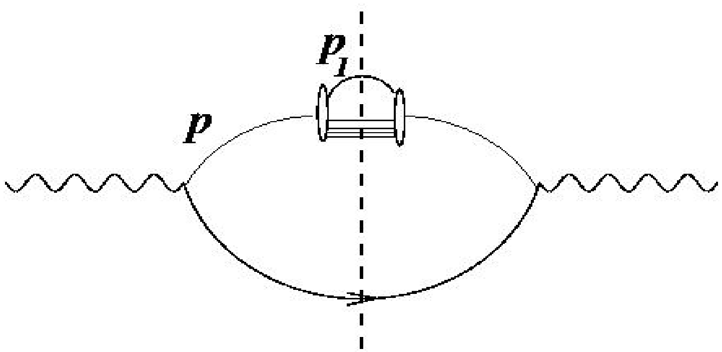
$$D_q^h(z) \longrightarrow \sum_{S-1} \left| \langle k| \quad \cdots \quad |P_h, S-1\rangle \right|^2$$

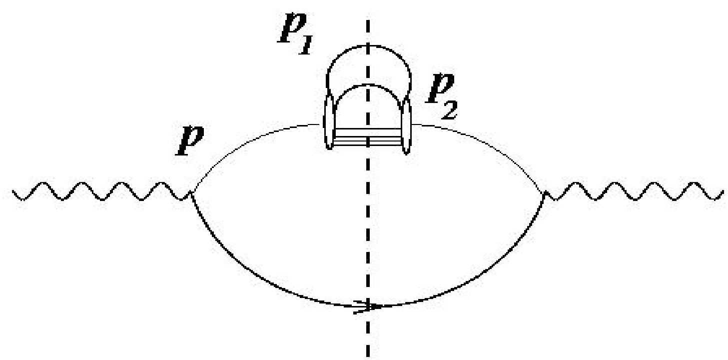


$$D_q^{h_1 h_2}(z_1, z_2) \longrightarrow \sum_{S-2} \left| \langle k| \quad \cdots \quad |P_1 P_2, S-2\rangle \right|^2$$

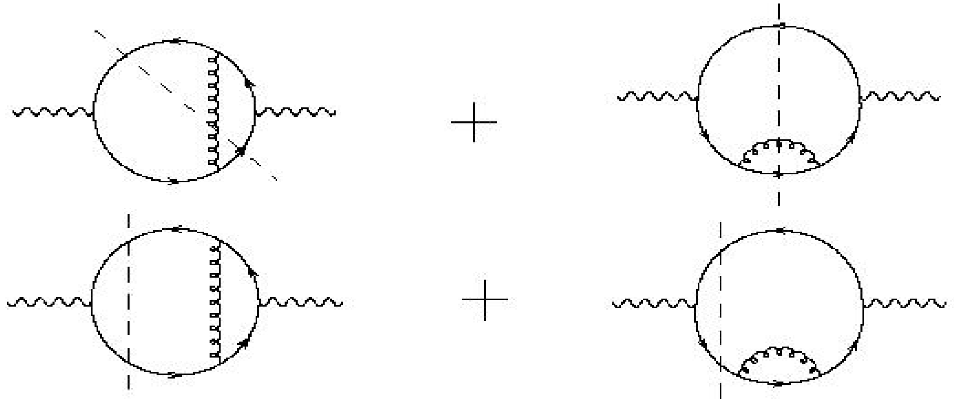


In self-energy diagrams

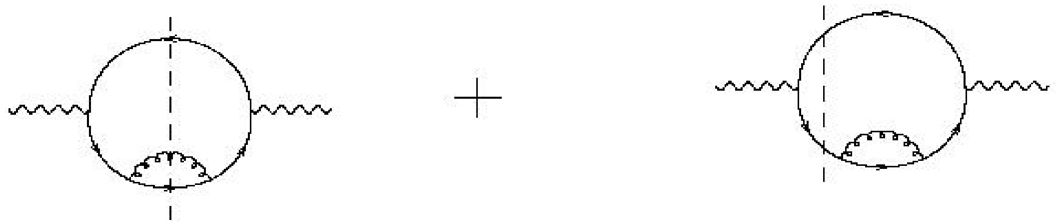




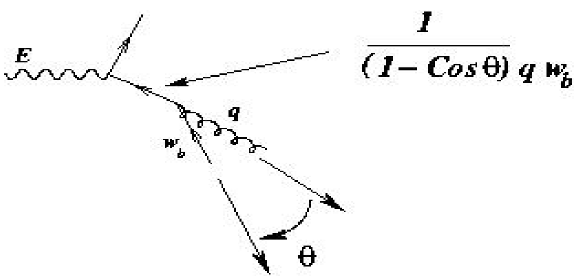
- *Isolation of matrix element is the first step of factorization*
- *Calculate higher order corrections and isolate leading log and power contributions. Isolate the hard part.*
- *Leading log contributions have divergences, absorb into fragmentation functions*



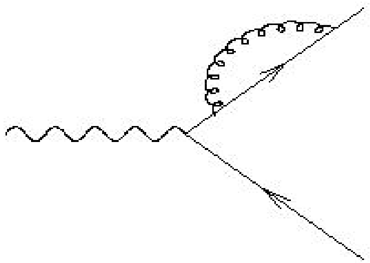
In Light cone gauge, leading log contributions only come form



Infrared and collinear divergences from soft and collinear gluons



Infrared divergences cancelled by self energy diagrams



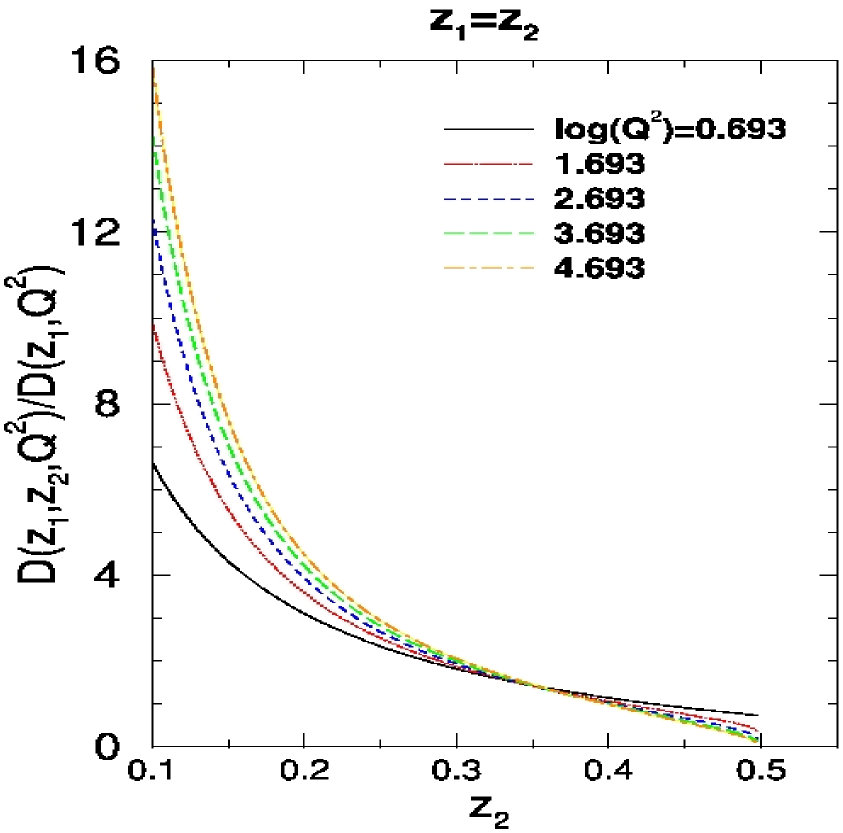
Use factorized distribution for Non-singlet fragmentation function

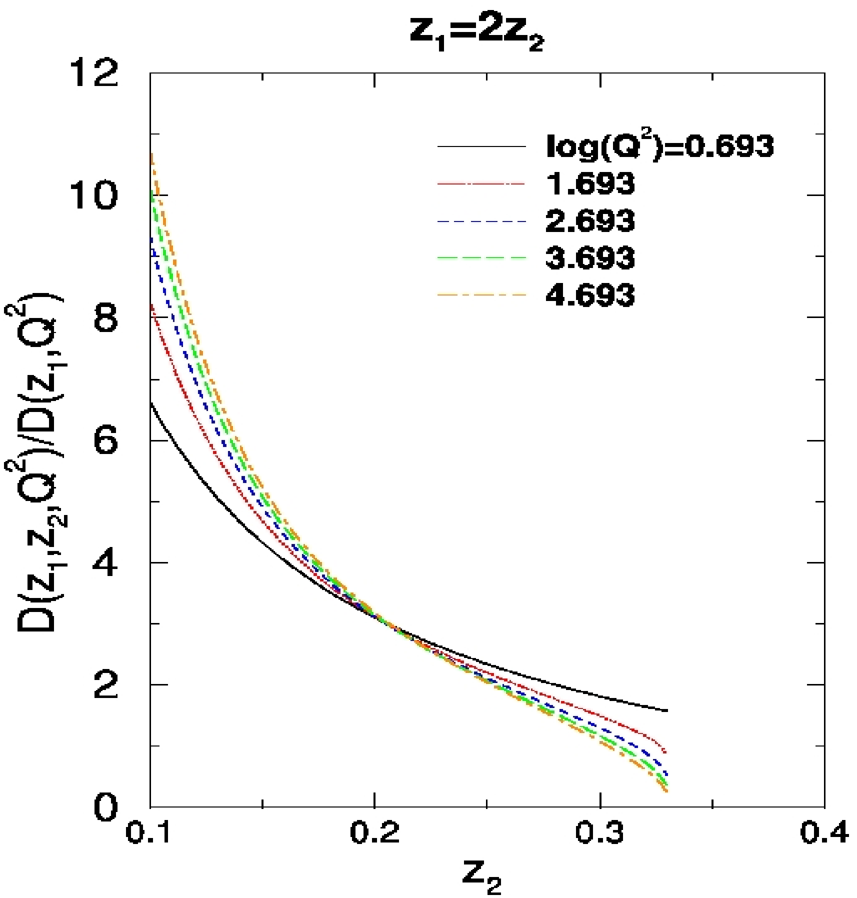
$$Factorized\ Distribution: D(z_1, z_2, \mu) = D(z_1, \mu)\, D(z_2, \mu)$$

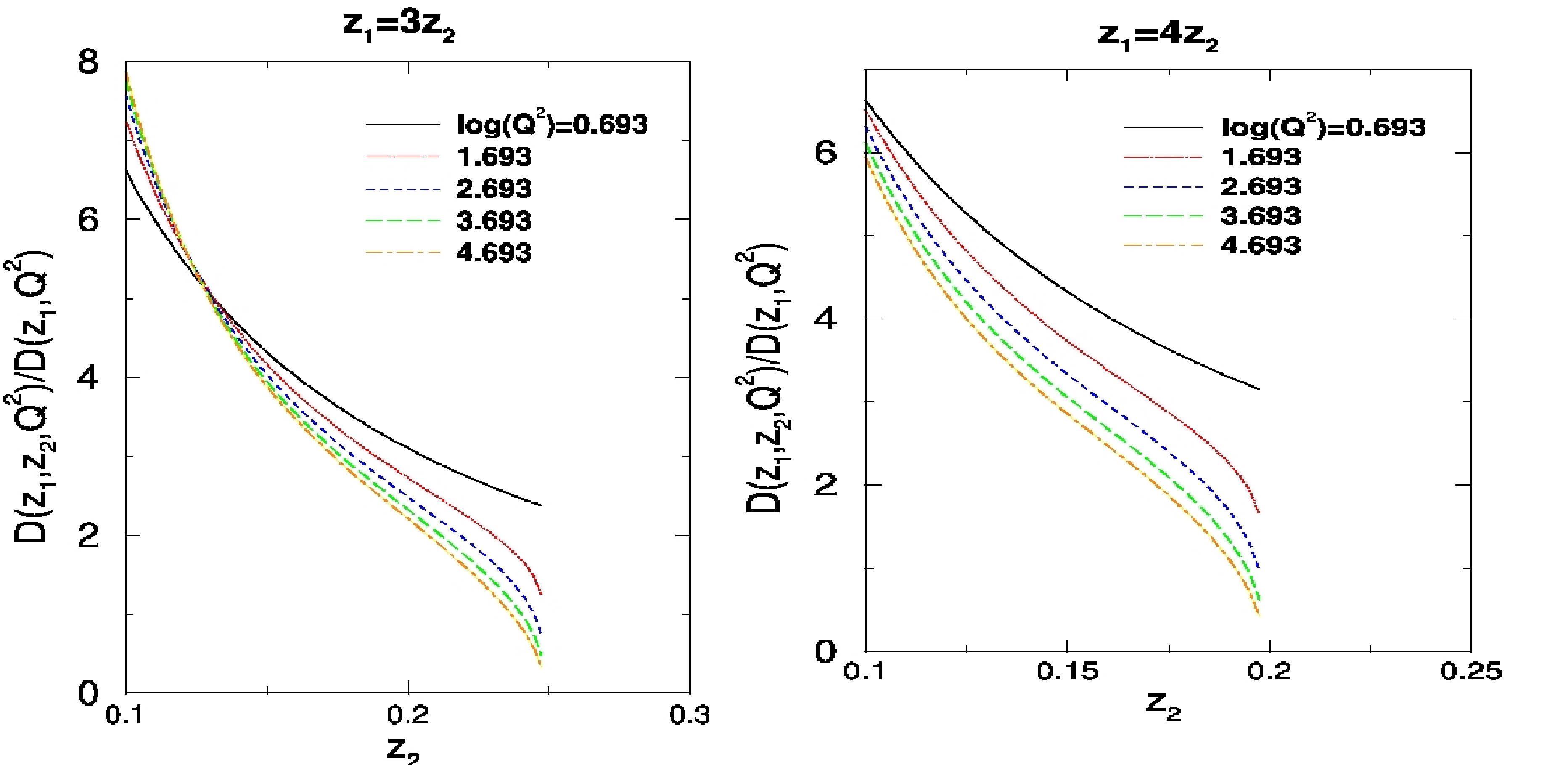
Single fragmentation functions taken from KKB

Plots for z_1/z_2 = 1,2,3,4

$$\log(Q^2 = 2 \ldots 110\, GeV^2) = 0.693 \ldots 4.693$$







Note: ratio double/single shows little change at intermediate z. Why ?

Ratio is the number of associated particles for given trigger !

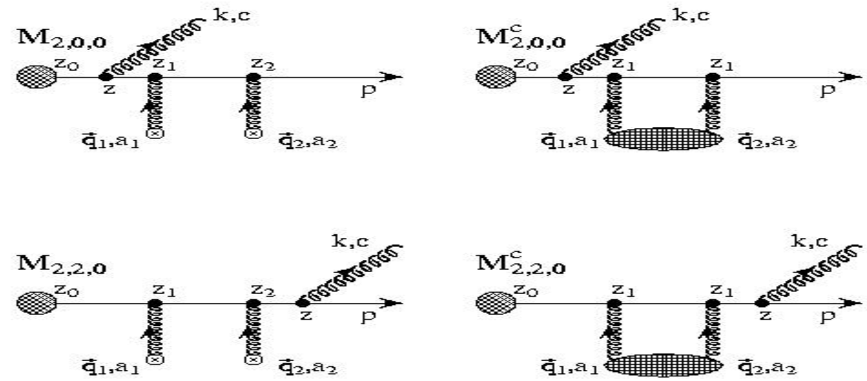
Regular evolution softens the spectrum: as for single hadrons

Single gluon fragmentation increases multiplicity!

CALCULATION OF THE MODIFICATION

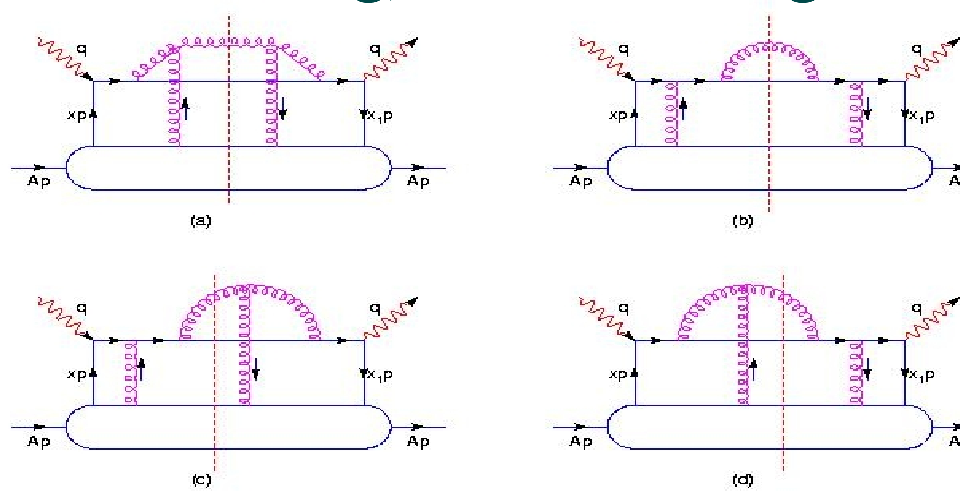
1) Potential scattering model : Gyulassy Wang; Gyulassy Levai Vitev; Weidemann Salgado..

Scattering of static color sources, e-loss by gluon radiation, followed by radiation reinteraction..



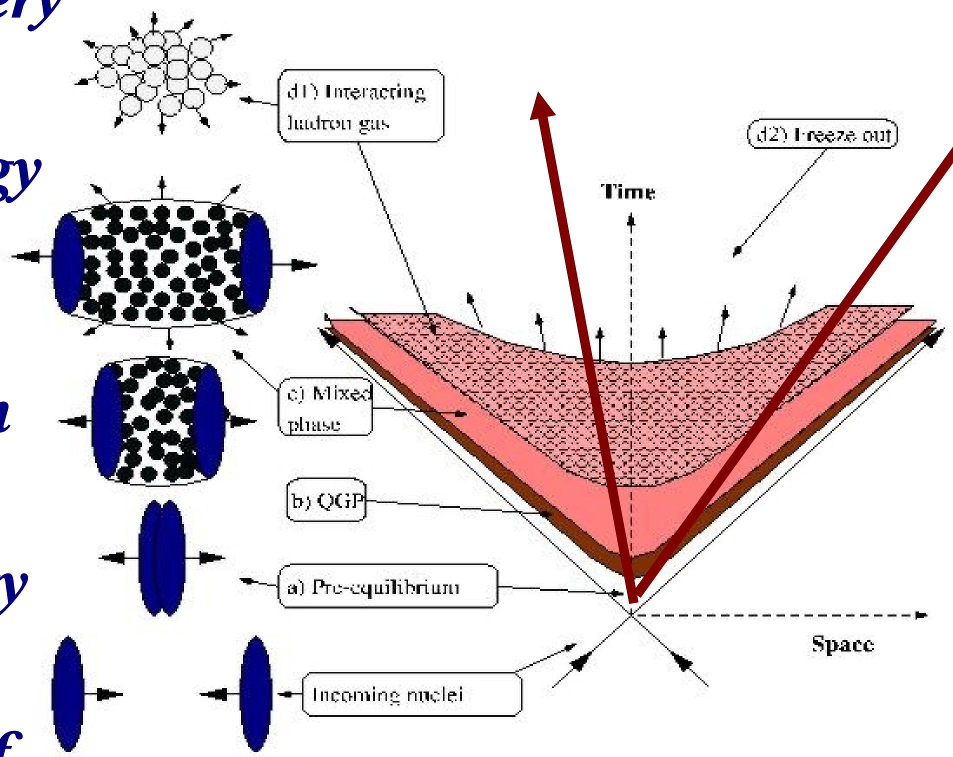
2) Modification of fragmentation function by higher power corrections from the medium: Guo Wang; Osborne Wang..

Based directly on DIS formalism,
Power corrections from structure functions enhanced by size!



HEAVY-ION COLLISIONS AND JETS

- ***Collide 2 heavy ions at very high energy.***
- ***Given high enough energy density a QGP may be created.***
- ***But QGP turns to hadron gas and freezes out.***
- ***Occasionally, high energy jets produced.***
- ***Jets sample the history of the collision.***
- ***Study of jet properties may produce insight into matter produced***

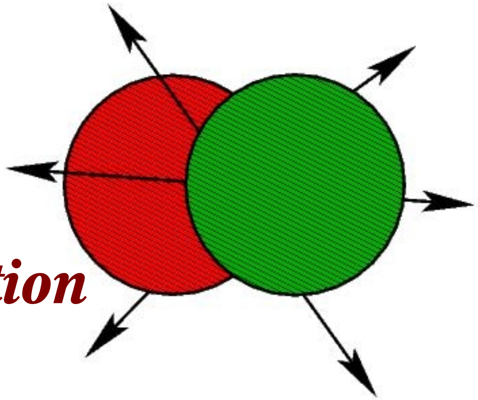


Compare with jet properties in p-p or electron positron annihilation

And with modification in cold nuclear matter

SINGLE PARTICLE MEASUREMENTS in heavy-ion coll.

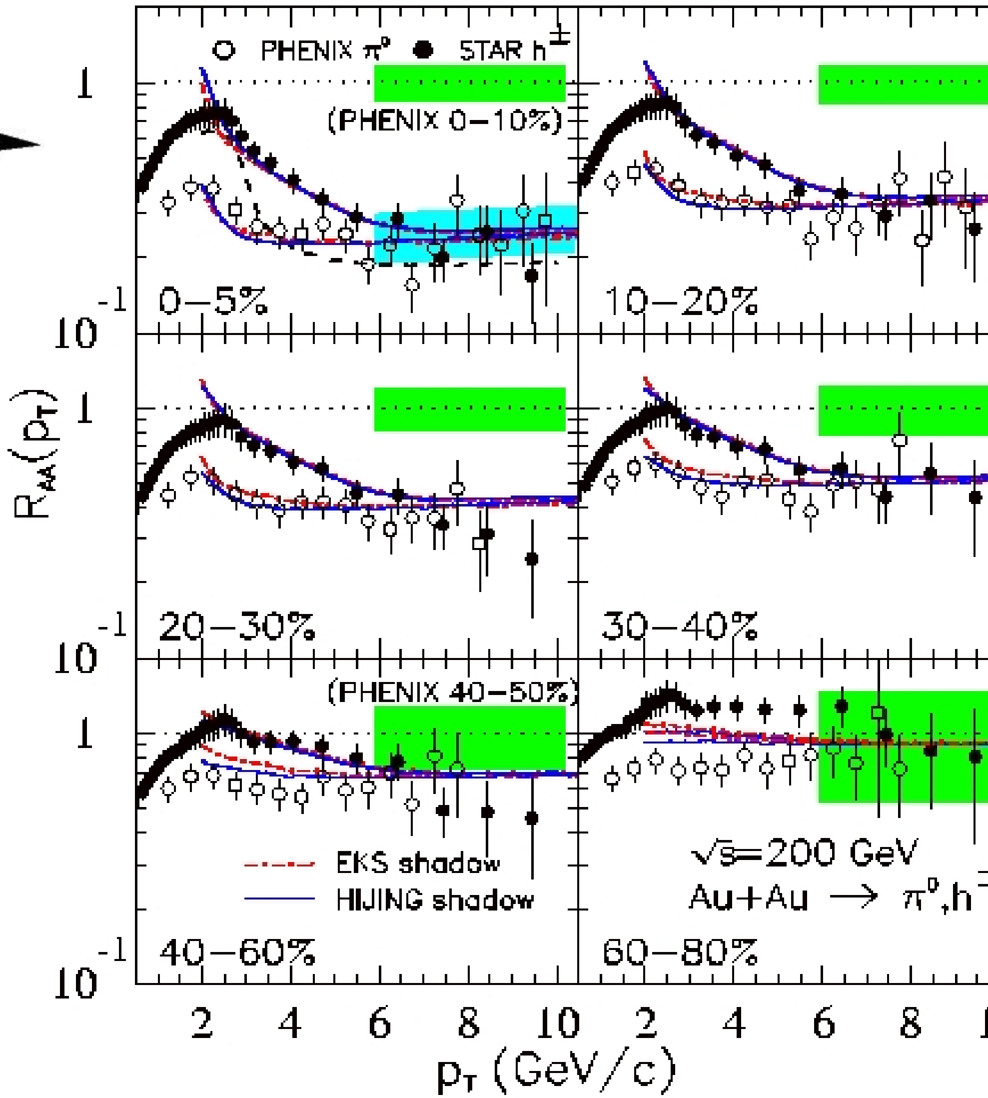
High P_T particle production at midrapidity
Nuclear modification factor R_{AA}



$$R_{AA} = \frac{d^2 N_{AA} / d\eta \, dp_\perp}{(N_{coll} / \sigma_{pp}^{inelastic}) d^2\sigma / d\eta \, d p_\perp}$$

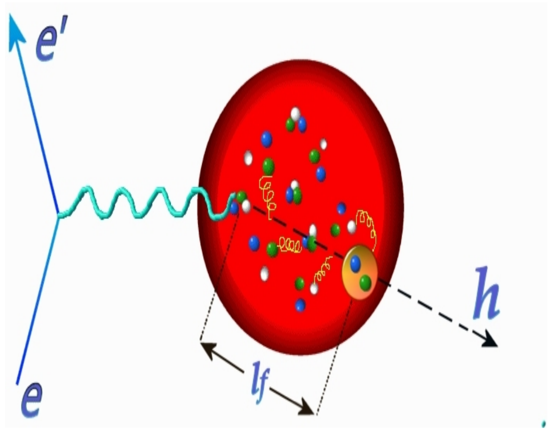
1 = particle production scales with number of expected p-p collisions

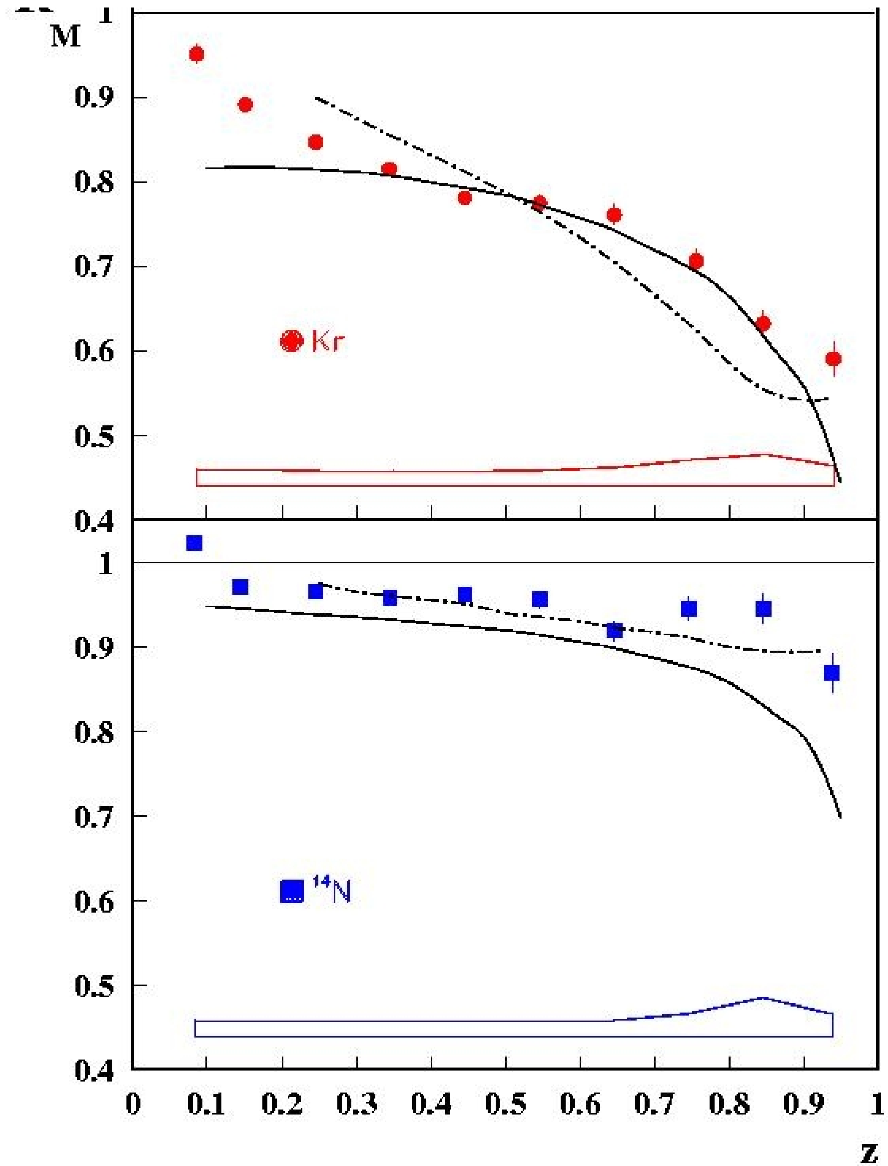
includes initial state modification of structure functions



$\eta = 0$

Single hadron attenuation in Deep-Inelastic Scattering





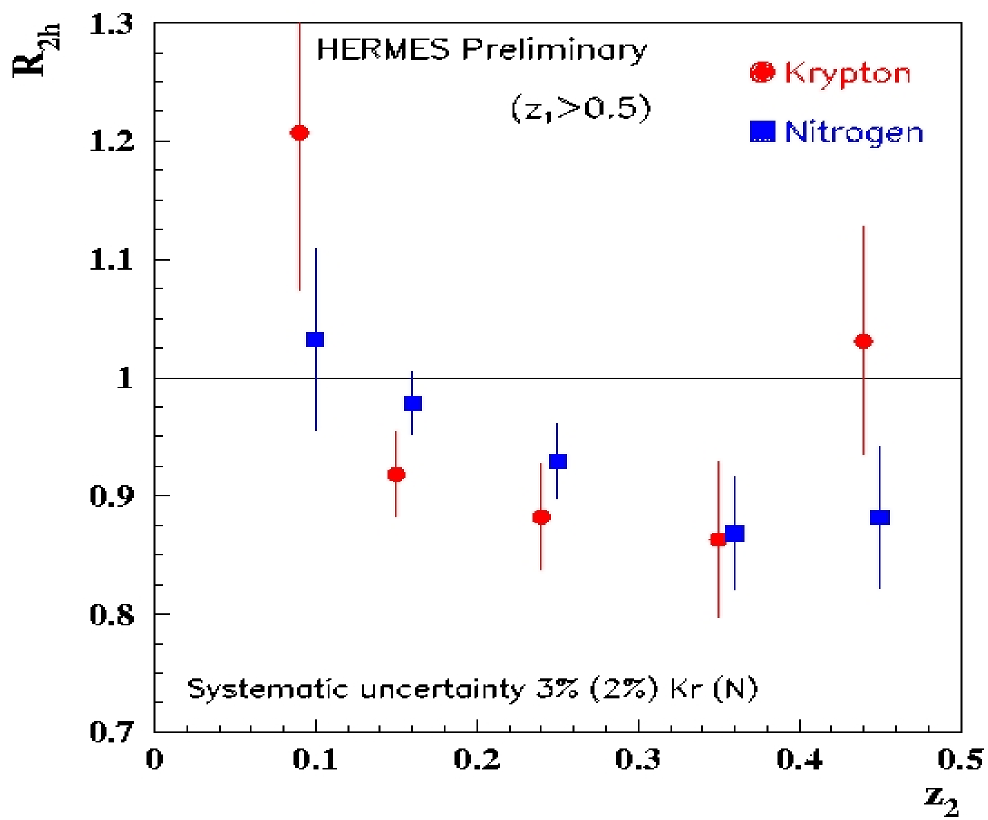
Perform DIS of nuclei

Look at particle production in the forward region

compare with DIS off light nuclei: Deuterium

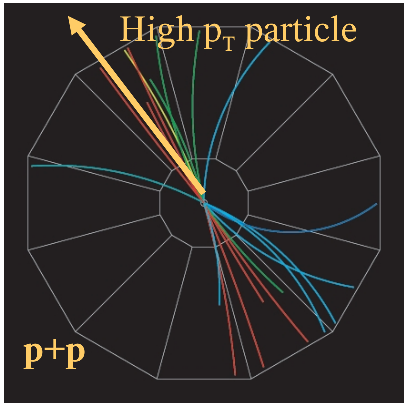
Double hadron measurements in DIS

- ***Always measure a ratio of double to single production***
- ***Divide by same ratio in deuterium to remove detector systematics***
- ***As in Heavy-ion collisions very little change of double/single ratio.***

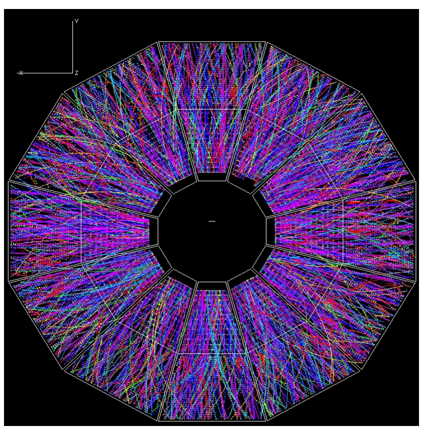


$$R_{2h} = \frac{\dfrac{No.\ of\ events\ with\ at\ least\ 2\ hadrons\ with\ z_1 > 0.5}{No.\ of\ events\ with\ at\ least\ one\ hadron\ with\ z > 0.5}}{same\ ratio\ on\ deuterium}$$

SINGLE HADRONS AND DI-HADRONS



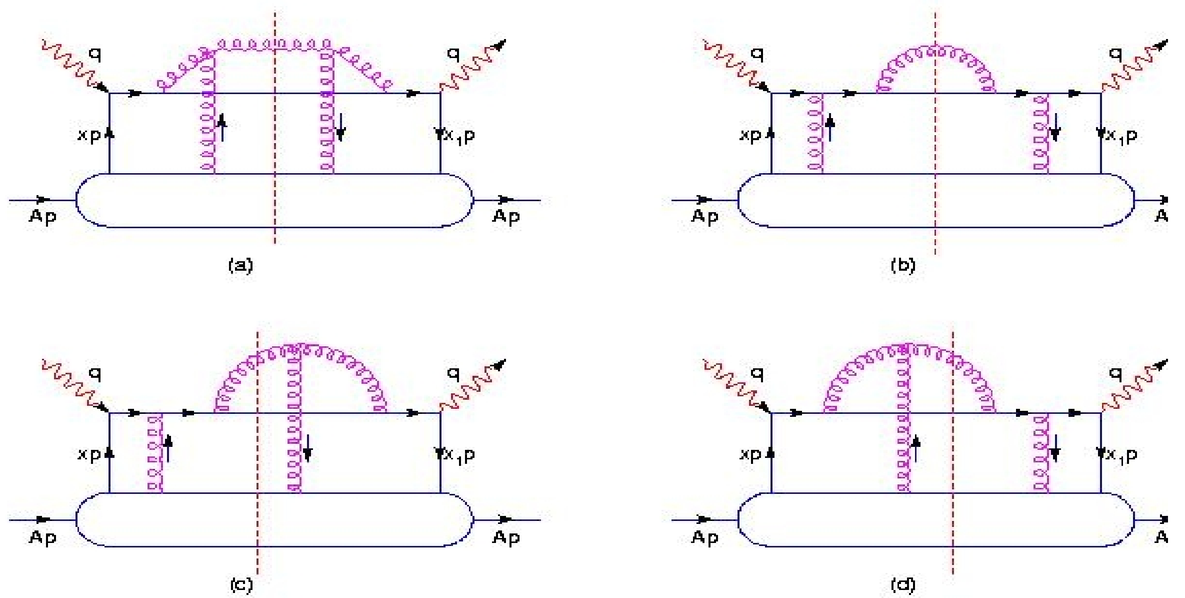
- *1 particle inclusive production, factorize from hard cross section* **F** *$D(z)$ fragmentation function*
- *Measure 2 particle distribution* **F** *$D(z_1, z_2)$*



- *Can do single inclusive measurements*
- *Can still do 2-particle measurements*
- *Select a leading particle $4<p_t<6$ GeV/c, $|\eta|<0.75$*
- *Associate all other particles ($0.15<p_t<4$ GeV/c, $|\eta|<1.1$) with the leading particle.*

Multiple higher twist diagrams need to be evaluated

Multiple scattering from soft gluons lead to LPM interference



Assume a Gaussian density distribution for nucleons in a medium sized nucleus

$$T_{qg}^{A} = C\, A^{1/3} \left(x\, G^{N}(x) \right) \left(1 - e^{-x_L^2 / x_A^2} \right)$$

$$\frac{x_L^2}{x_A^2} = \frac{R_A^2}{\gamma^2 \tau_f^2}$$

τ_f = Formation time

γ = boost

R_A = Nuclear size

OUTLINE

- MOTIVATIONS: HEAVY-ION COLLISIONS AND QGP, DIS
- VARIOUS APPROACHES: MEDIUM MODIFICATION OF FRAGMENTATION FUNCTIONS VIA PARTONIC INTR.
- DEFINITION OF DIHADRON FRAGMENTATION F e^+e^-
- DGLAP EVOLUTION: MODIFICATION IN VACUUM
- MEDIUM MODIFICATION IN COLD MEDIUM (DIS)
- MEDIUM MODIFICATION IN HOT MEDIUM (QGP)

Simpler case of Non-singlet quarks

$$D_{NS} = D_q - D_{\bar{q}}$$

Simpler: contribution from gluon fragmentation cancels out

$$D_{NS}(z_{1,} z_{2,} \mu^2) = D_{NS}(z_{1,} z_2) + \int_0^{\mu^2} \frac{dl_\perp^2}{l_\perp^2} \int_z^1 \frac{dy}{y^2} \left[\frac{1+y^2}{1-y} \right]_+ D_{NS}(z_1/y, z_2/y)$$

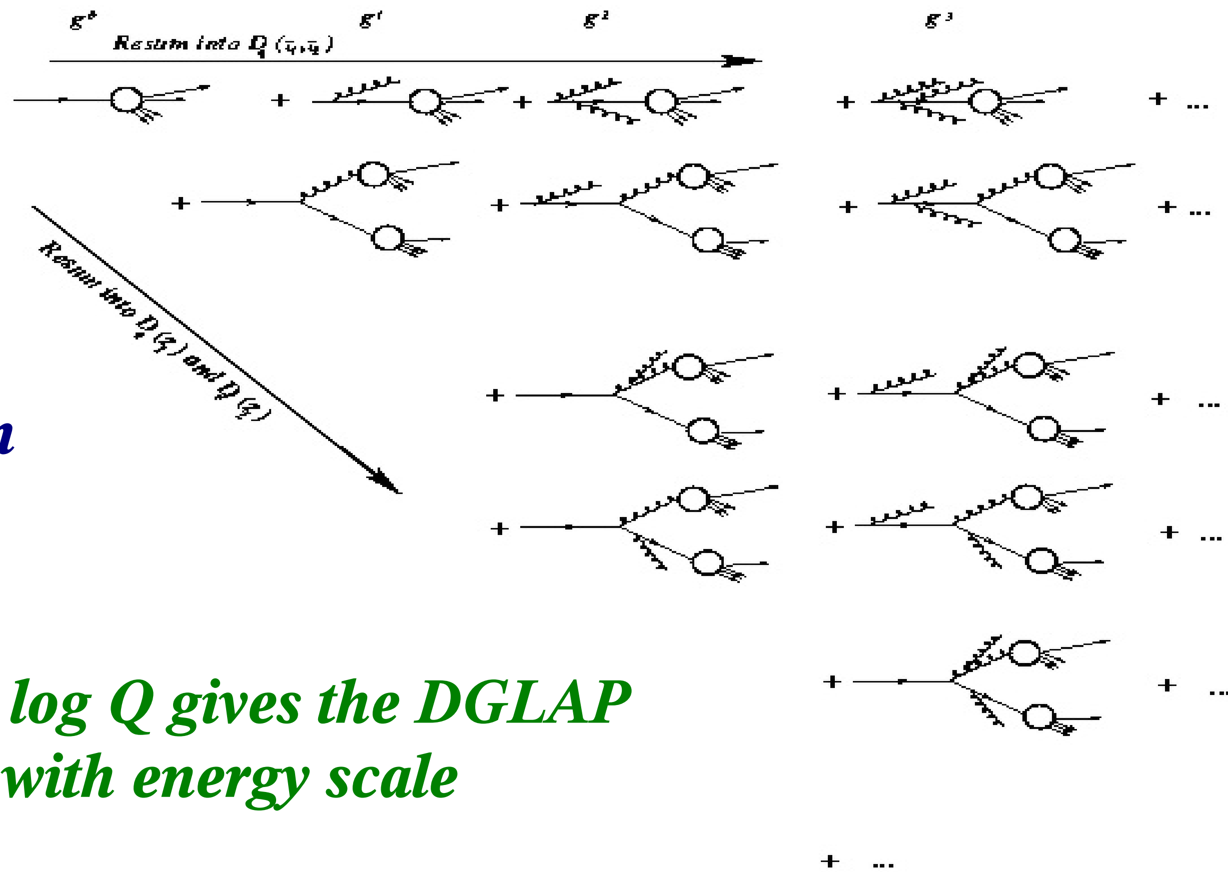
$$+ \int_0^{\mu^2} \frac{dl_\perp^2}{l_\perp^2} \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \frac{1+y^2}{1-y} D_q(z_1/y) D_g(z_2/(1-y))$$

- *Absorb collinear divergences up to a scale μ*
- *Defines fragmentation function at scale μ*
- *Note: only retained leading log and leading power corrections*
- *To go to a higher scale this has to be repeated to all orders*
- *Leading log corrections absorbed at each order*

To obtain the dihadron fragmentation function at a higher scale Q need to resum the LL contributions from all orders as shown...

LL contributions can be resummed

part of the LL contributions resummed into the single fragmentation functions...

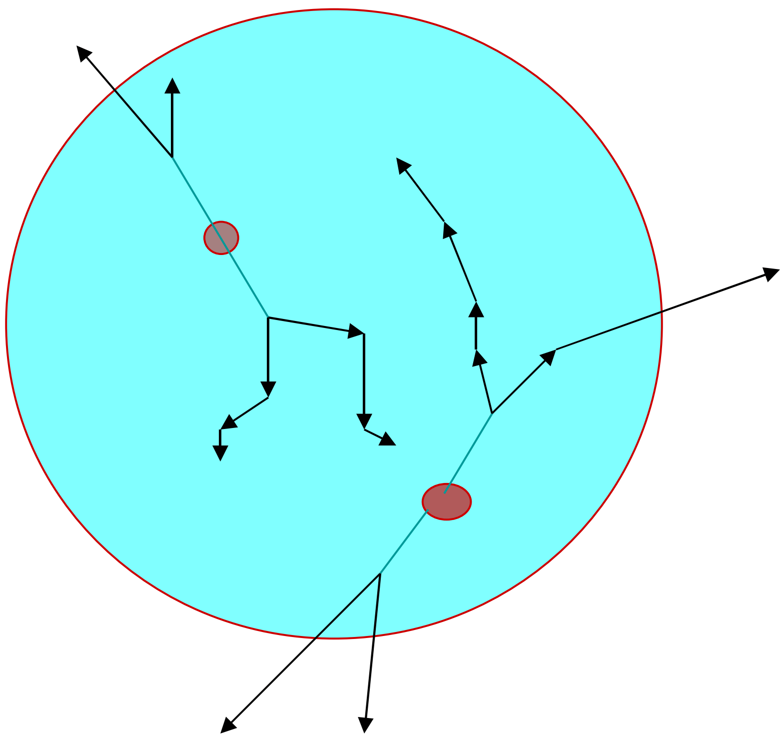


Differentiating with log Q gives the DGLAP evolution equations with energy scale

Each differentiation extracts a splitting function, and reduces the number of gluons

HADRONIC ENERGY LOSS:

- ***Fragmentation occurs inside the hot medium***
- ***Hadrons become independent due to scattering***
- ***Each hadron suffers the same Energy Loss on average***



Hadronic scattering models can explain mean single supp. !
C. Greiner et. al. @QM2004, V. Koch (unpublished!)

- *NOTE! IN THIS CASE DONT NEED A $D(z_1, z_2)$*
- *Probability of observing two hadrons factorizes $P(1,2) = P(1)P(2)$*
- *Each probability is suppressed compared to p+p: $P(h) = s\, p(h)$*

- *Thus the conditional probability is also suppressed compared to p+p collisions*

P(h) is for A+A;   p(h) is for p+p

- *Hadronic absorption models cannot explain the double inclusive spectrum*

Modification in vacuum = DGLAP evolution

Non-singlet quarks $D_{NS} = D_q - D_{\bar{q}}$

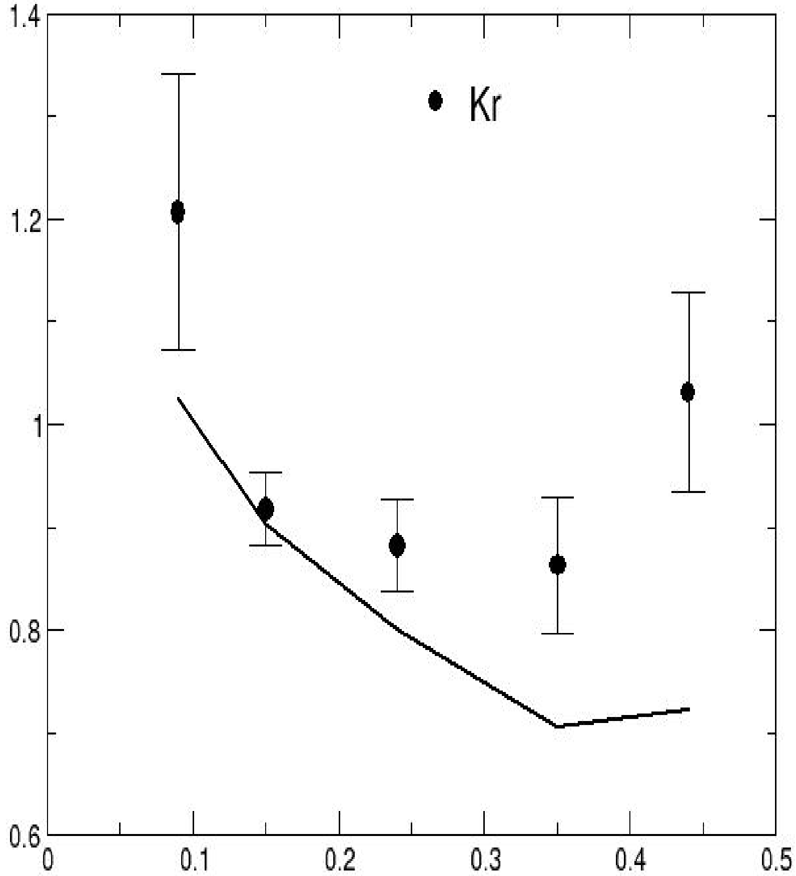
Simpler: contribution from gluon fragmentation cancels out

$$\text{Single evolution:} \frac{\partial D_{NS}(z, Q^2)}{\partial \log Q^2} = \int_z^1 \frac{dy}{y} P_{q\to qg}(y)\, D_{NS}(z/y, Q^2)$$

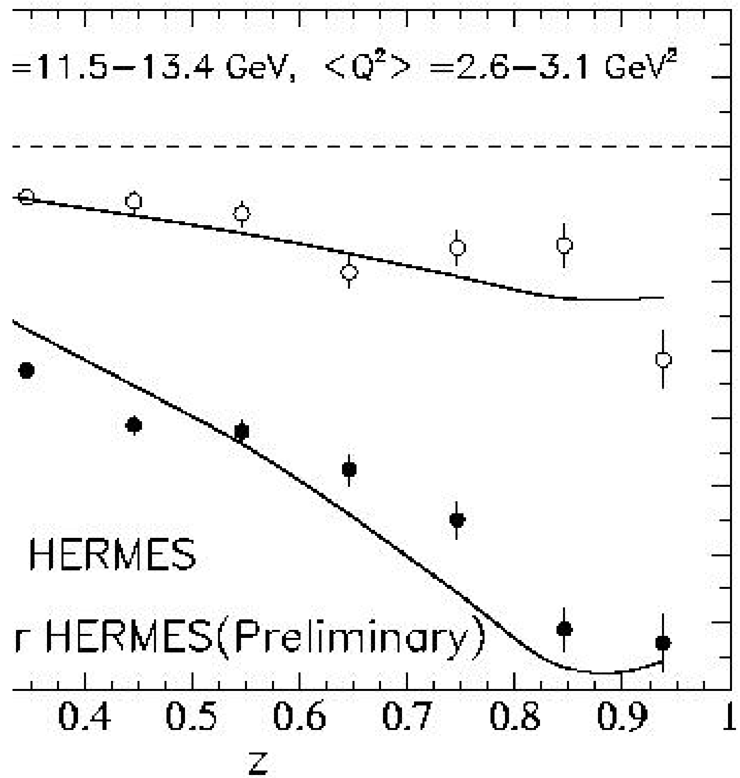
$$\text{Double evolution:} \frac{\partial D_{NS}(z_1, z_2, Q^2)}{\partial \log Q^2} = \int_{z_1, z_2}^1 \frac{dy}{y^2} P_{q\to qg}(y)\, D_{NS}(z_1/y, z_2/y, Q^2)$$

$$+ \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \hat{P}_{q\to qg}(y)\, D_q(z_1/y, Q^2)\, D_g(z_2/(1-y), Q^2)$$

$$\hat{P} = P - \text{virtual corrections}$$



1.4
1.2
1
0.8
0.6
0
0.1
0.2
0.3
0.4
0.5
Kr

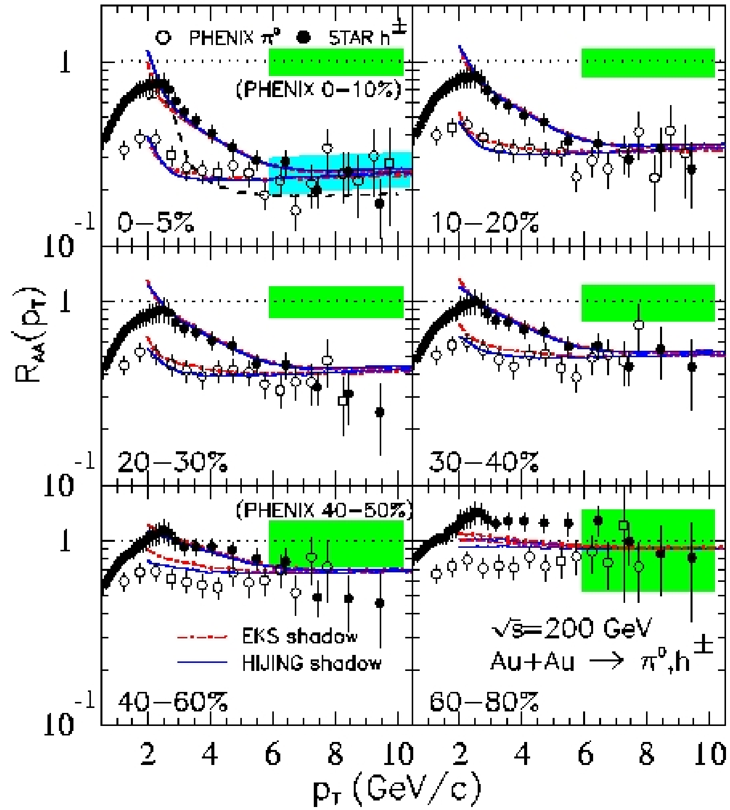


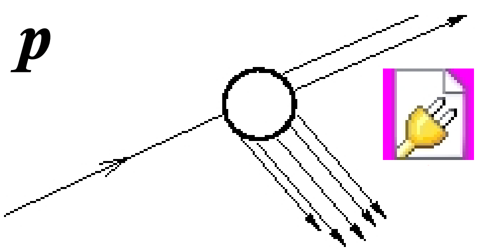
=11.5−13.4 GeV, <Q²> =2.6−3.1 GeV²
HERMES
r HERMES(Preliminary)
0.4
0.5
0.6
0.7
0.8
0.9
1
z

• *Partonic energy loss models explain single inclusive suppression pretty well !*
(GLV, BDMPS, WGZ, SW)

All models require a high density of scattering centers
High density seen as evidence of QGP.

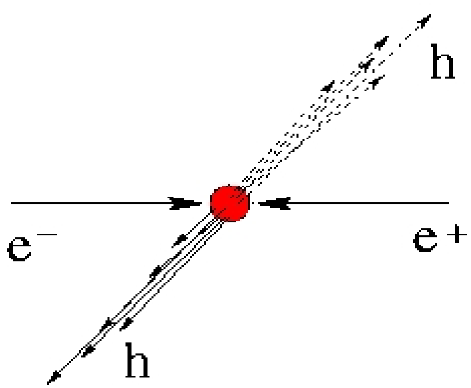
To explain double inclusive spectra requires a new phenomenological object: ***Dihadron fragmentation function!***

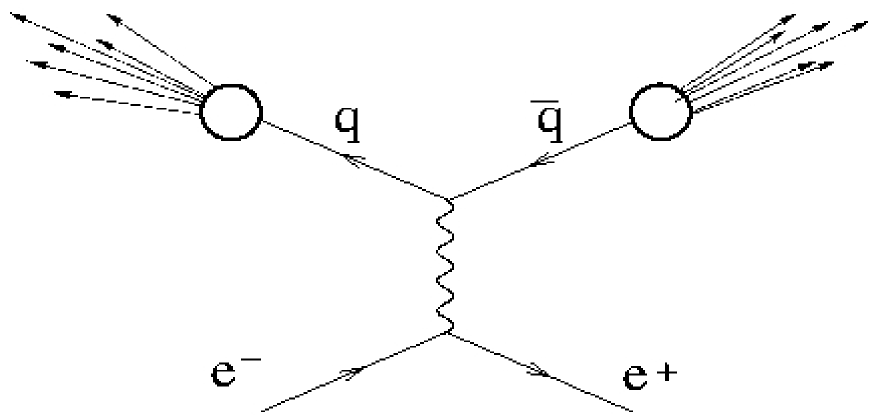




Defining the Dihadron fragmentation function!

- *Fragmentation functions have to be universal*
- *We need a definition in terms of operators*
- *Start with simple system :* $e^+ e^-$*, is factorization valid*
- *Derive evolution (vacuum splitting functions)*
- *Measure at* μ *and predict its evolution to scale Q*

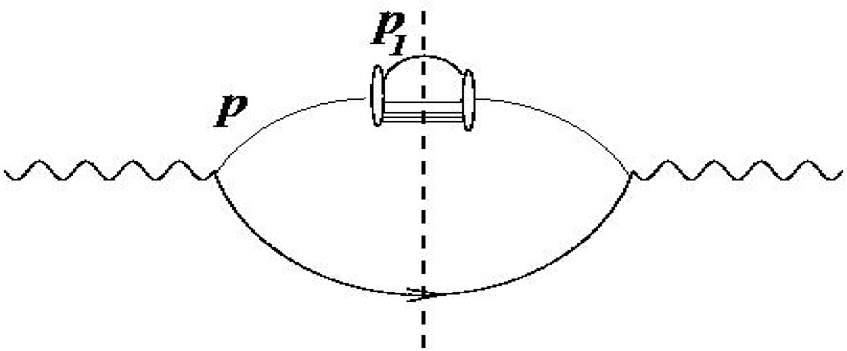


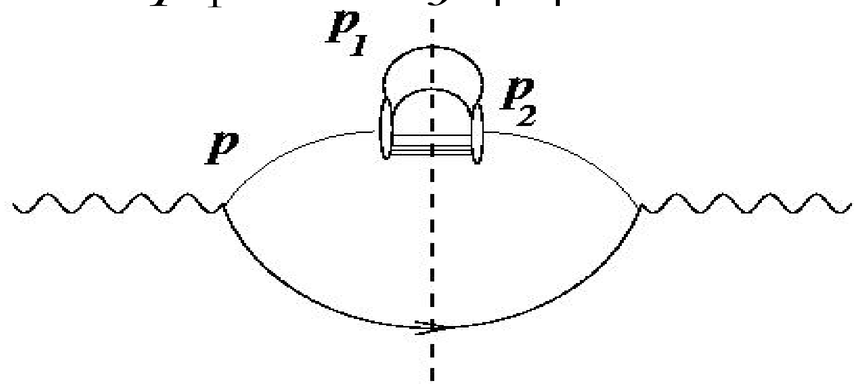


$$W^{\mu\nu} = \sum_{S-1} e_q^2 \int \frac{d^3 p_1 d^3 k}{4 E_1 E_k (2\pi)6} (2\pi)^4 \delta(q - p_1 - k - \sum_{s-1} p_f)$$

$$\bar{\Psi} \gamma^{\nu} \psi$$

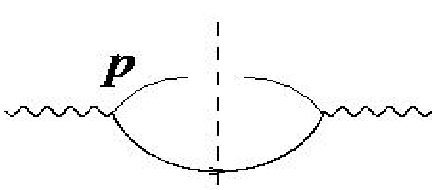
$$< 0 | \bar{\Psi} \gamma^{\mu} \psi | k \, p_1 \, S-1 > < k \, p_1 \, S-1 left | ¿ | 0 >$$

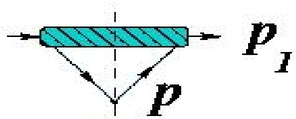


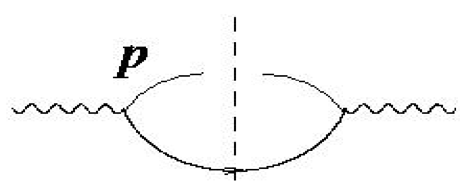


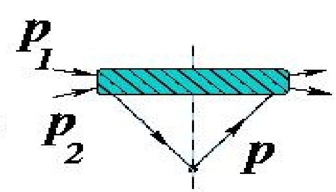
Collinear approximation: $p_h^+, p^+, p_1^+, p_2^+ >> p_h^-, p^-, p_1^-, p_2^-, p_{perp}$

Can factorize matrix elements from hard part: $Tr[\gamma^{\alpha} p_{\alpha} \gamma^{\mu} \gamma^{\beta} k_{\beta} \gamma^{\nu}] Tr[\frac{\gamma^+}{2p^+} \hat{T}]$



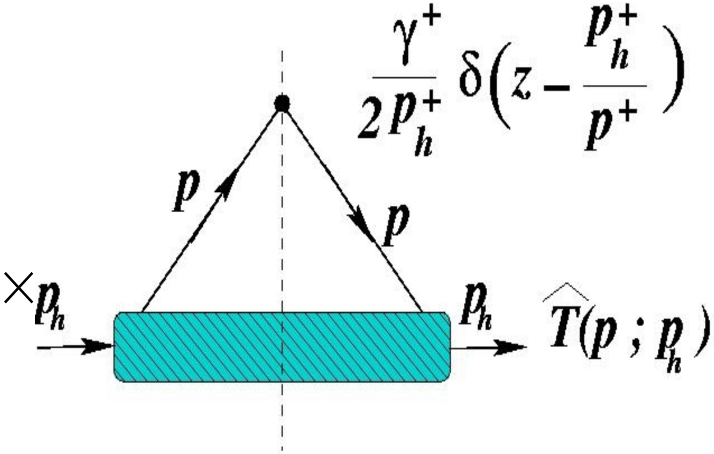






Use of universal single and double fragmentation functions defined in e^+e^-

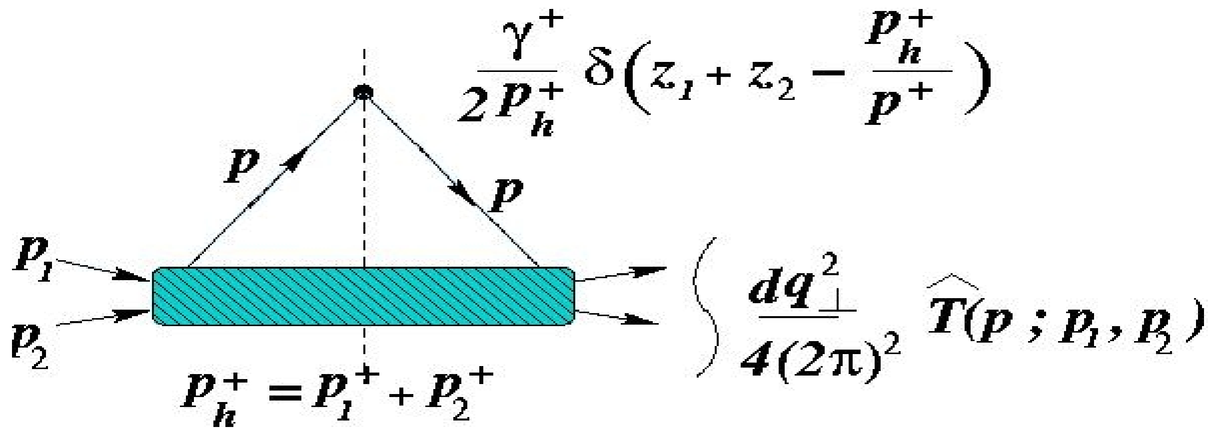
$$D_q(z) = \frac{z^3}{2}\int \frac{d^4p}{(2\pi)^4} Tr\left[\frac{\gamma^+}{2p_h^+}\delta\left(z - \frac{p_h^+}{p^+}\right)\hat{T}_q(p, p_h)\right] = \frac{z^3}{2}\times$$



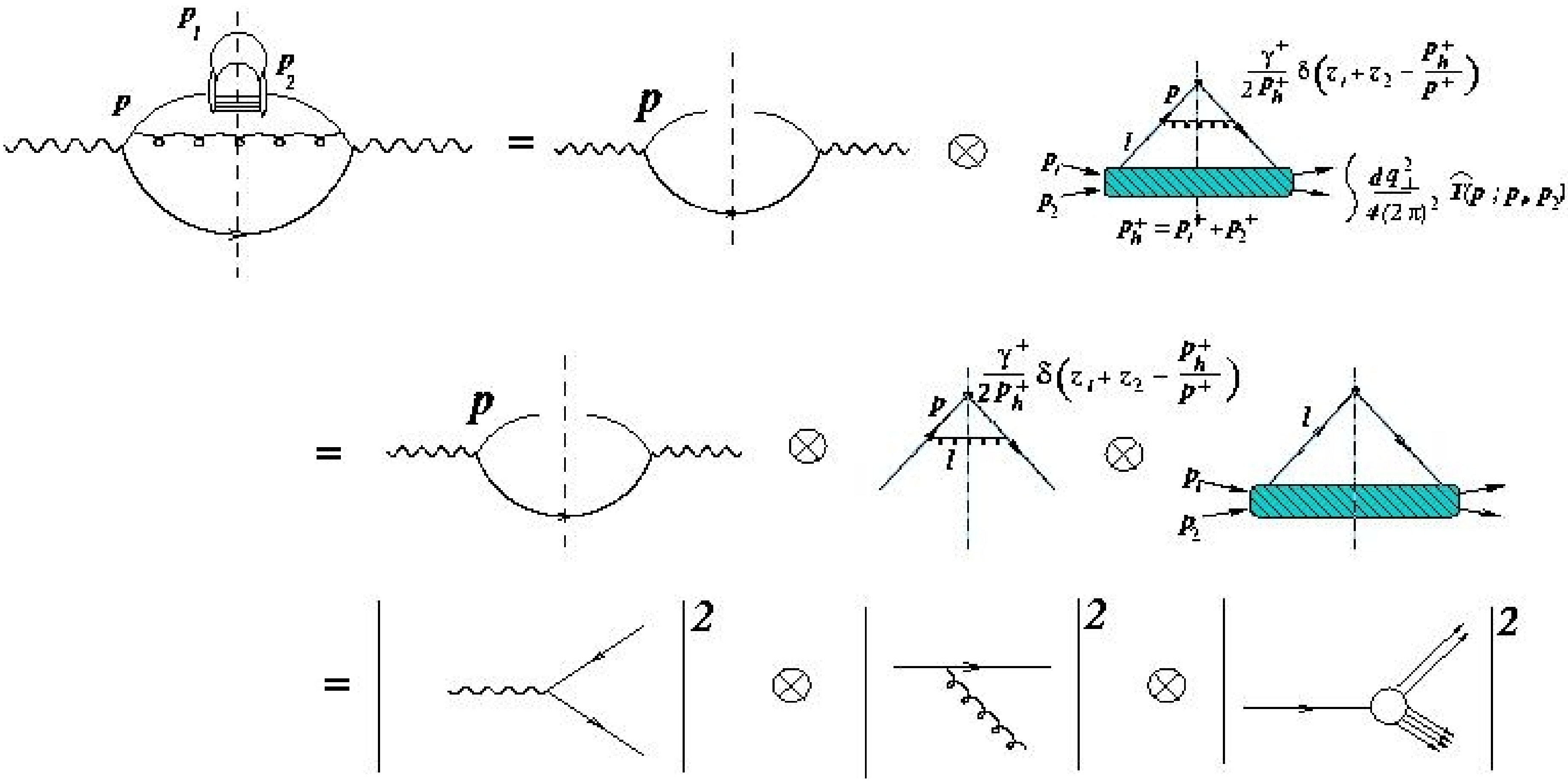
where $\hat{T}_q(p, p_h) = \int d^4x\, e^{-ip\cdot x} \sum_{S-1} \langle 0|\psi(0)|p_h, S-1\rangle\langle p_h, S-1|\bar{\psi}(x)|0\rangle$

$$D_q(z_1, z_2) = \int \frac{dq_\perp^2}{8(2\pi)^2}\frac{z^4}{4z_1z_2}\int\frac{d^4p}{(2\pi)^4} Tr\left[\frac{\gamma^+}{2p_h^+}\delta\left(z_1 + z_2 - \frac{p_h^+}{p^+}\right)\hat{T}_q(p, p_1p_2)\right]$$

$$= \frac{z^4}{z_1z_2}$$

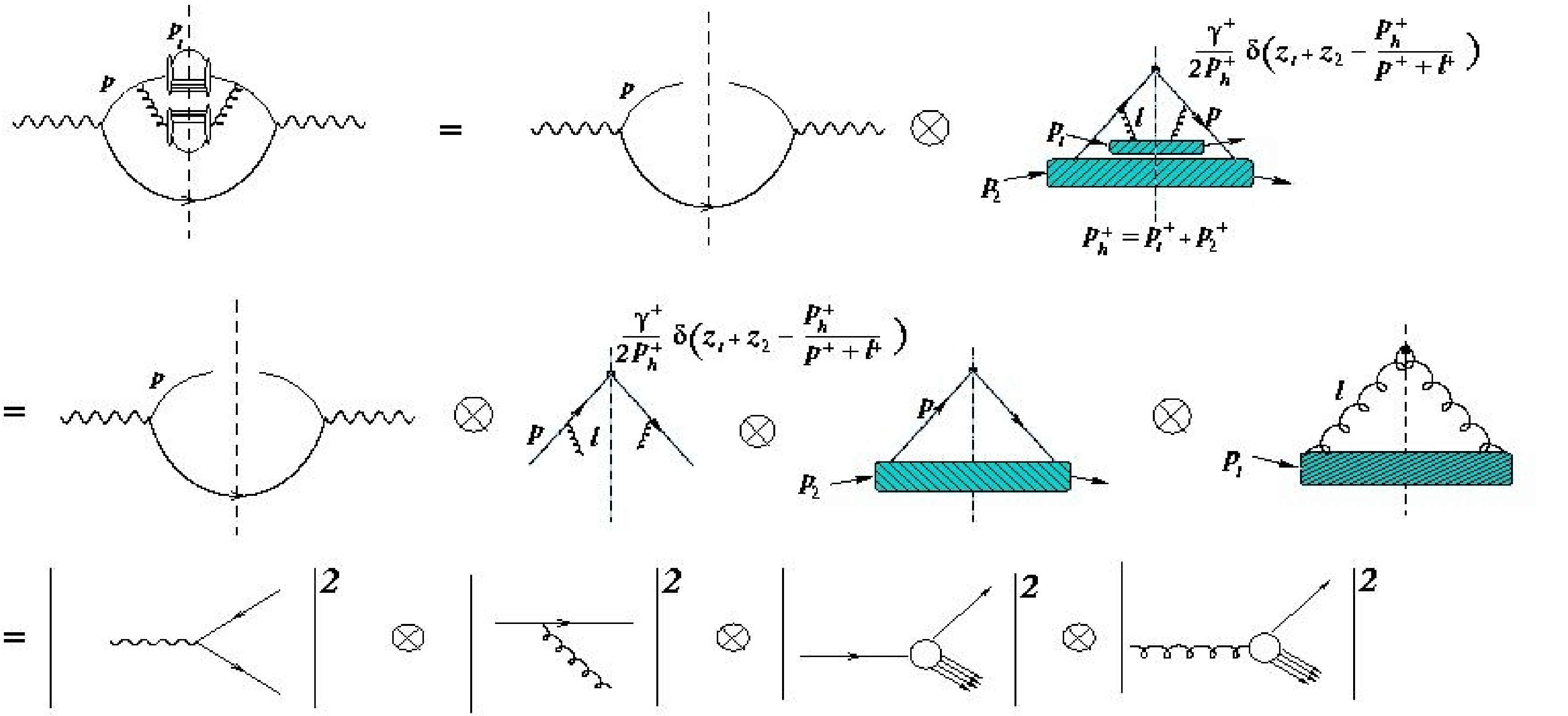


NLO contribution from quark dihadron fragmentation function



$$= \sigma_0 \otimes P_{q \to qg}(y) \otimes D_q\left(\frac{z_1}{y}, \frac{z_2}{y}\right)$$

NLO contribution from independent quark and gluon fragmentation



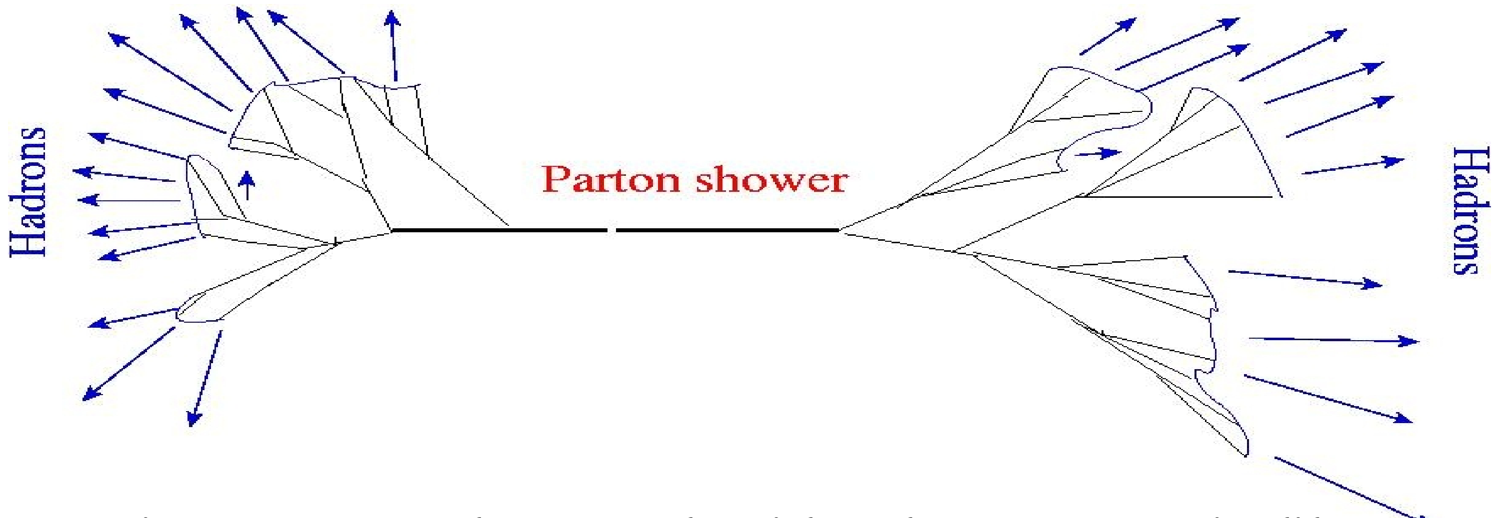
$$= \sigma_0 \otimes \hat{P}_{q\to qg}(y) \otimes D_q\left(\frac{z_1}{y}\right) \otimes D_g\left(\frac{z_2}{1-y}\right)$$

The hat indicates there is no virtual correction.
Also no infra-red divergence as hadrons from both partons detected
However, perturbative corrections under control only if $\mu_\perp^2 \gg \lambda_{QCD}^2$

Konishi, Ukawa and Veneziano, NPB157, 45 (1979).

Evolution of full quarks and gluons

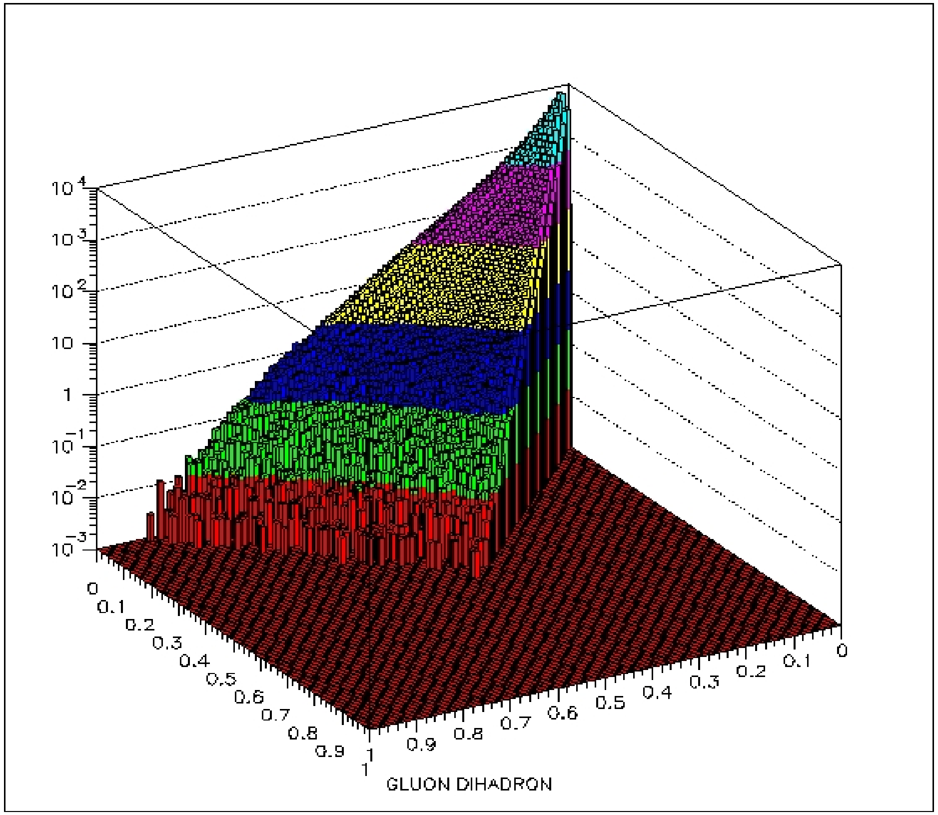
- *No experimental data to date on dihadron frag. In* e^+e^-
- *Phenomenological event generators can explain most data!*
- *Most successful of these is JETSET*
- *Use a tuned JETSET to measure dihadrons at scale* μ
- *Measure at higher scale Q and check with derived DGLAP*

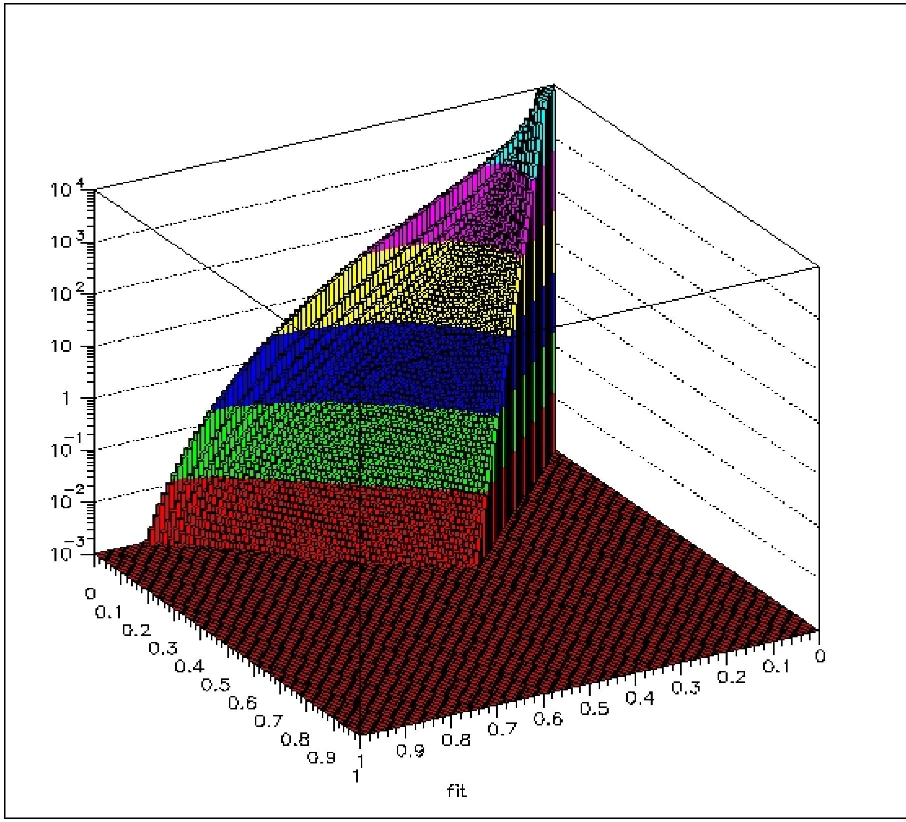


JETSET is a Monte Carlo event algorithm that generates jet like events with a parton shower followed by a string fragmentation routine to get hadrons. It has many parameters tuned to fit almost all experimental data.

Results from Event generators:

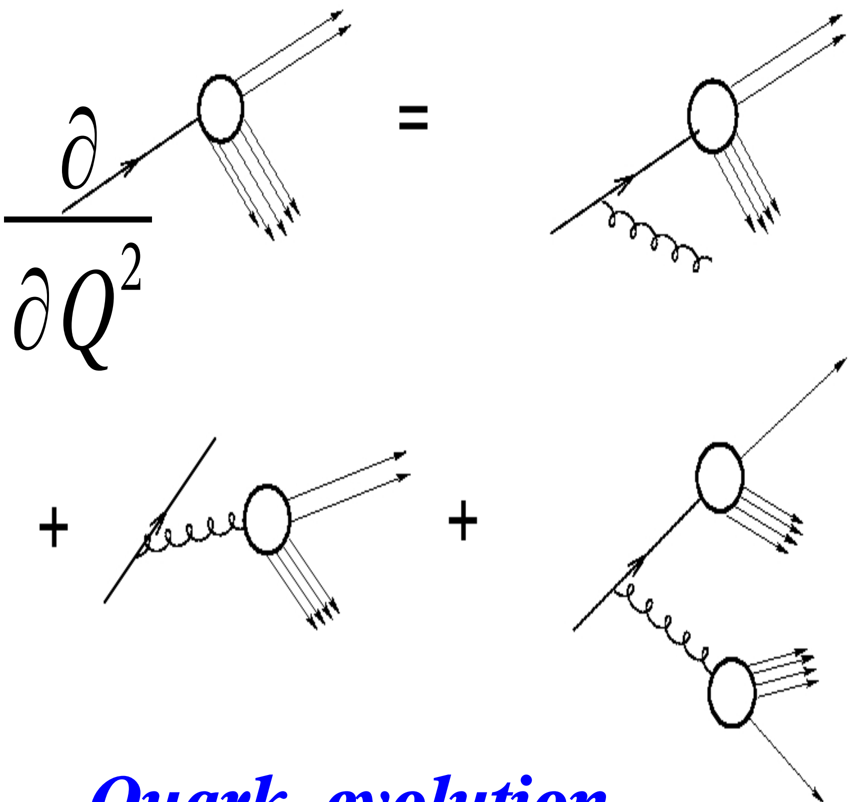
a bit ragged (Monte Carlo), fit a function to it !



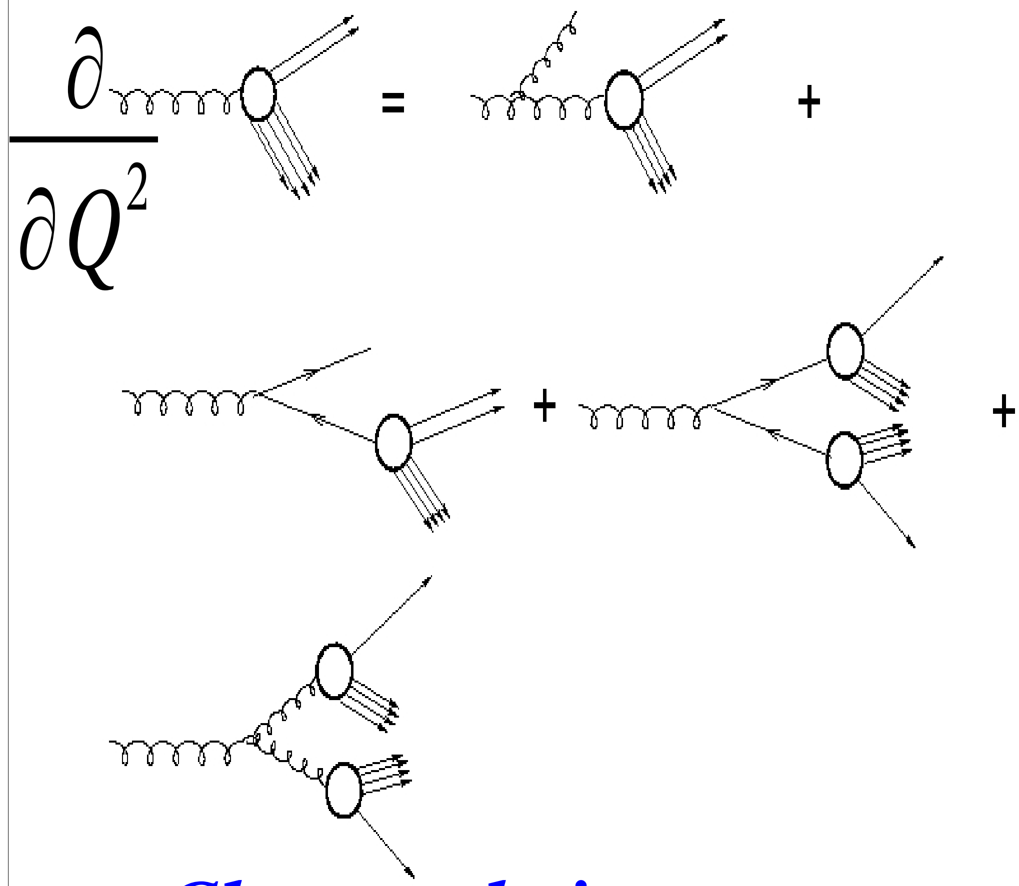


$$D(z_{1,} z_2) = N\, z_1^{\alpha_1} z_2^{\alpha_2} (z_1 + z_2)^{\alpha_3} (1 - z_1)^{\beta_1} (1 - z_2)^{\beta_2} (1 - z_1 - z_2)^{\beta_3}$$

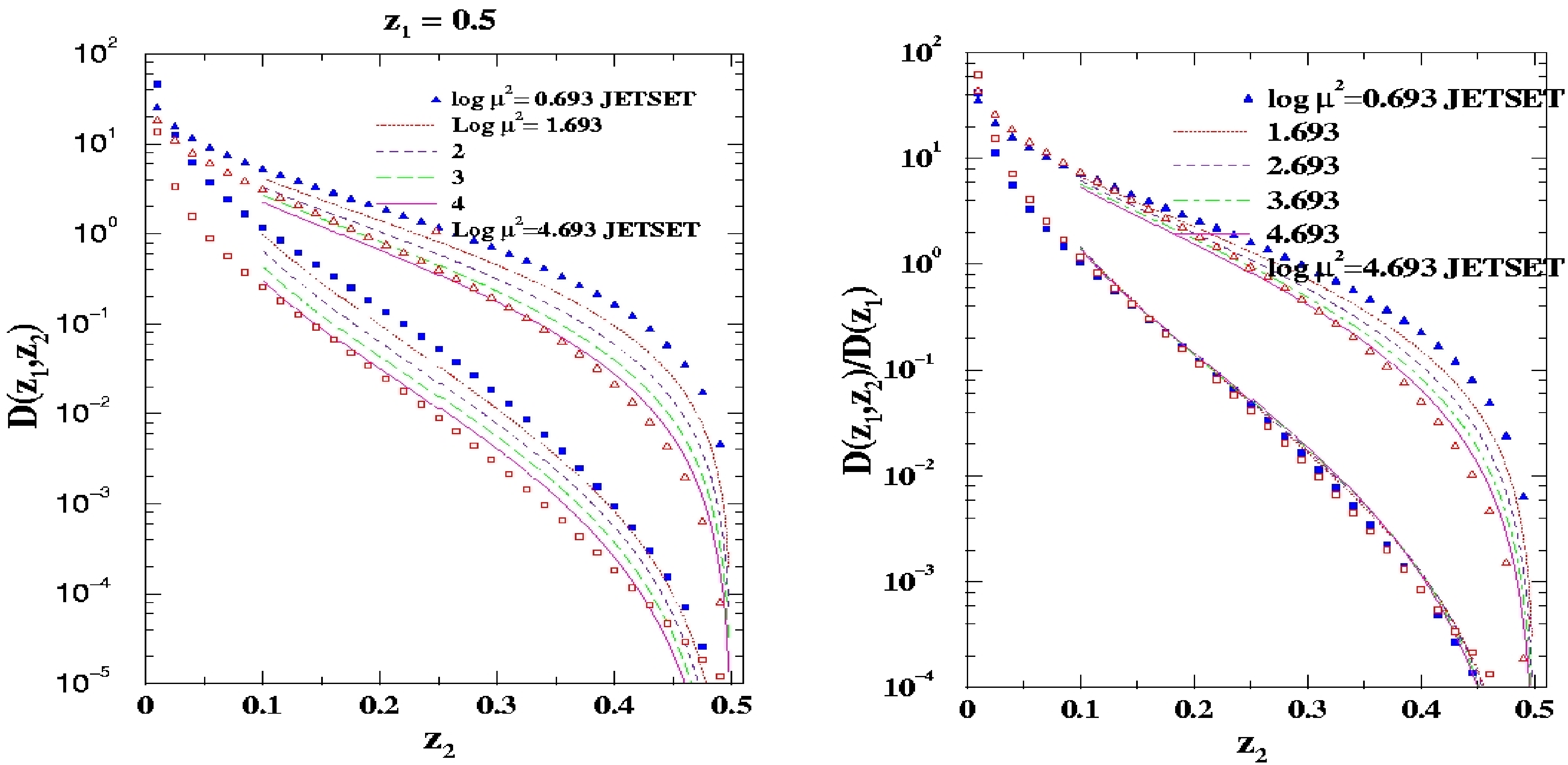
- *Evolving to a higher scale Q   solving DGLAP equations*
- *Set of coupled differential equations containing the following processes: for quarks and gluons..*



Quark evolution



Gluon evolution



Quark and Gluon evolution fits event generator data very well!

Thus we can understand evolution of DFF from QCD.

Note: the double to single ratio shows little change

Dihadron fragmentation in e^+e^- Collisions

The basic process may be factorized as:

$$\frac{d^2\sigma}{dz_1 dz_2} = \sigma_0 [D_q(z_1, z_2, \mu) + D_{\bar{q}}(z_1, z_2, \mu)]$$

$$\sigma_0 = Hard\ Cross\ section$$

$$D_q(z_1, z_2, \mu) = Dihadron\ fragmentation\ function$$

Can be factorized from hard process if $\lambda_{QCD}^2 \ll \mu^2 \ll Q^2$

Measure the function at the scale μ, can be done in 2 ways

$$Factorized\ Distribution: D(z_1, z_2, \mu) = D(z_1, \mu) D(z_2, \mu)$$

$$Event\ generator\ distribution: D(z_1, z_2, \mu) = \frac{1}{N_{events}} \frac{dN}{dz_1 dz_2}$$