

# Isotropization and QCD Plasma Instabilities

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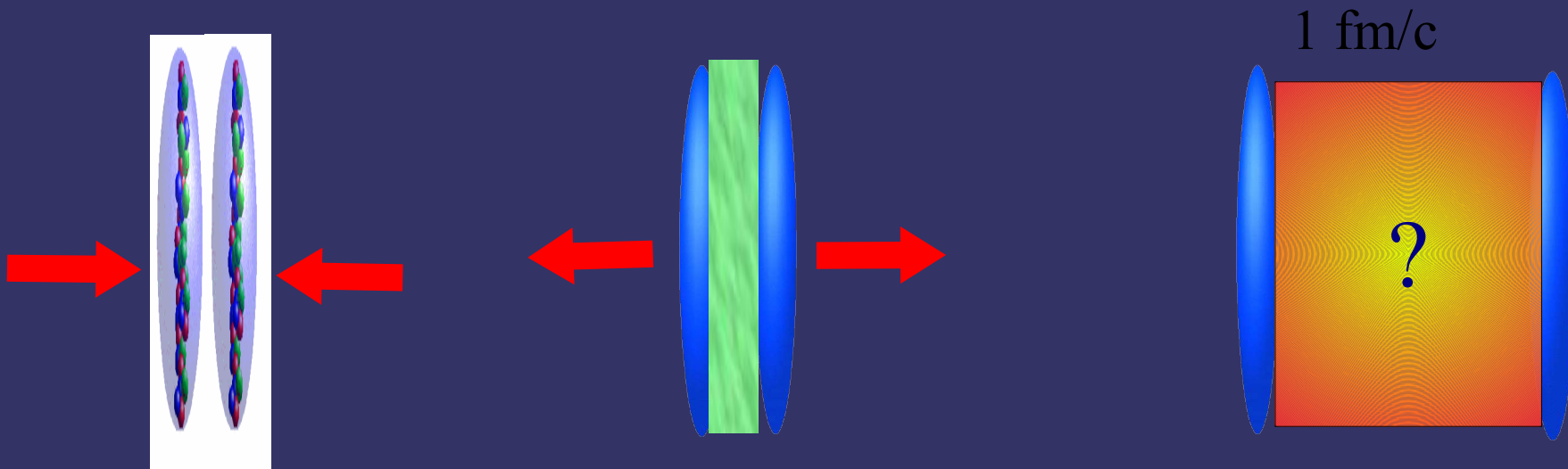
based on the collaboration with  
Adrian Dumitru ( J. W. Goethe U. )

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# *Long-term Goal*

- ⇒ Understanding the pre-equilibrium dynamics in high energy heavy ion collisions.
- ⇒ Do we have thermal deconfined matter (QGP) at RHIC/LHC?
- ⇒ If so, How? When? Which T?



# *Non-equilibrium dynamics* *within kinetic theory*

Traditional plasma physics

L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + g (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p f = C[f]$$

$$\begin{aligned} \dot{\mathbf{E}} &= \nabla \times \mathbf{B} - \mathbf{J} & \mathbf{J} &= g \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} f \\ \dot{\mathbf{B}} &= -\nabla \times \mathbf{E} \end{aligned}$$

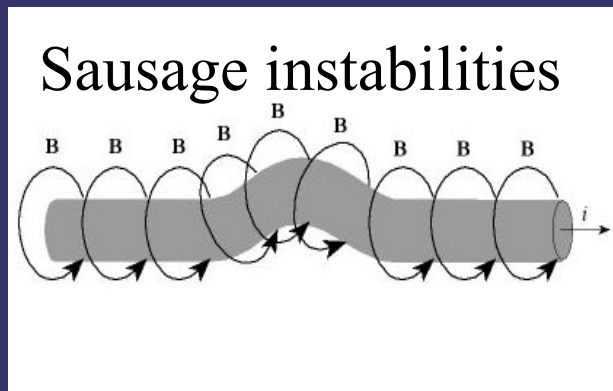
$f(\mathbf{x}, \mathbf{p})$  One-particle distribution function

- **Collision only**: Boltzmann equation (PCM: Bass&Muller, Zhang&Molnar&Gyulassy, Xu&Greiner...)
- **Mean field only**: Vlasov equation. Collective effects.

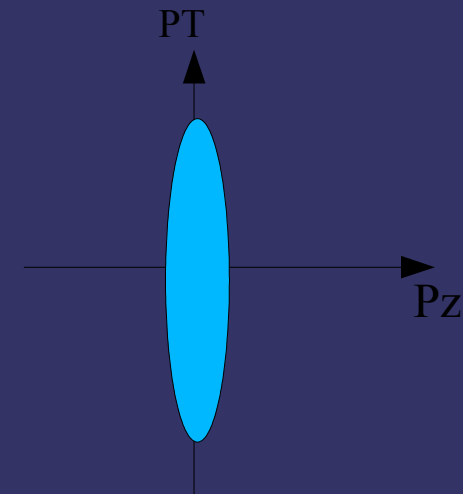
# Plasma instabilities

Exponential growth of wave solutions:  $A \sim e^{\omega t}$

- ➔ **Macroscopic instabilities:**  
coordinate space nonuniformity



- ➔ **Microscopic instabilities:**  
momentum space anisotropy

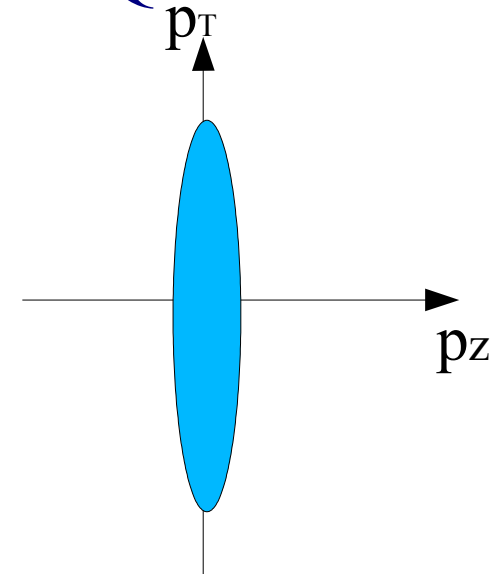
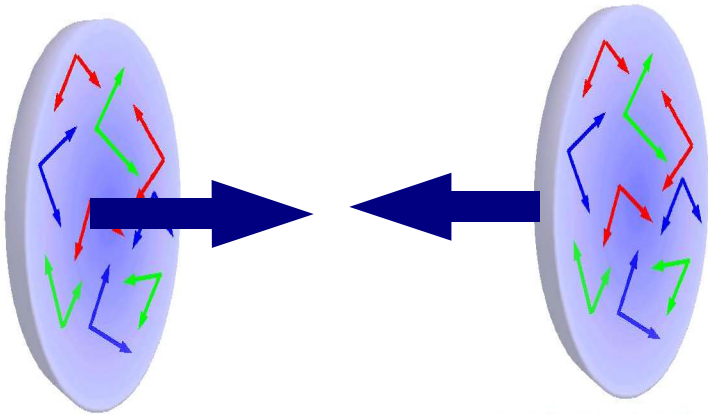


# A Picture in the limit $\alpha_s \ll 1$

“Bottom-up”, Phys. Lett. B502 (2001)51

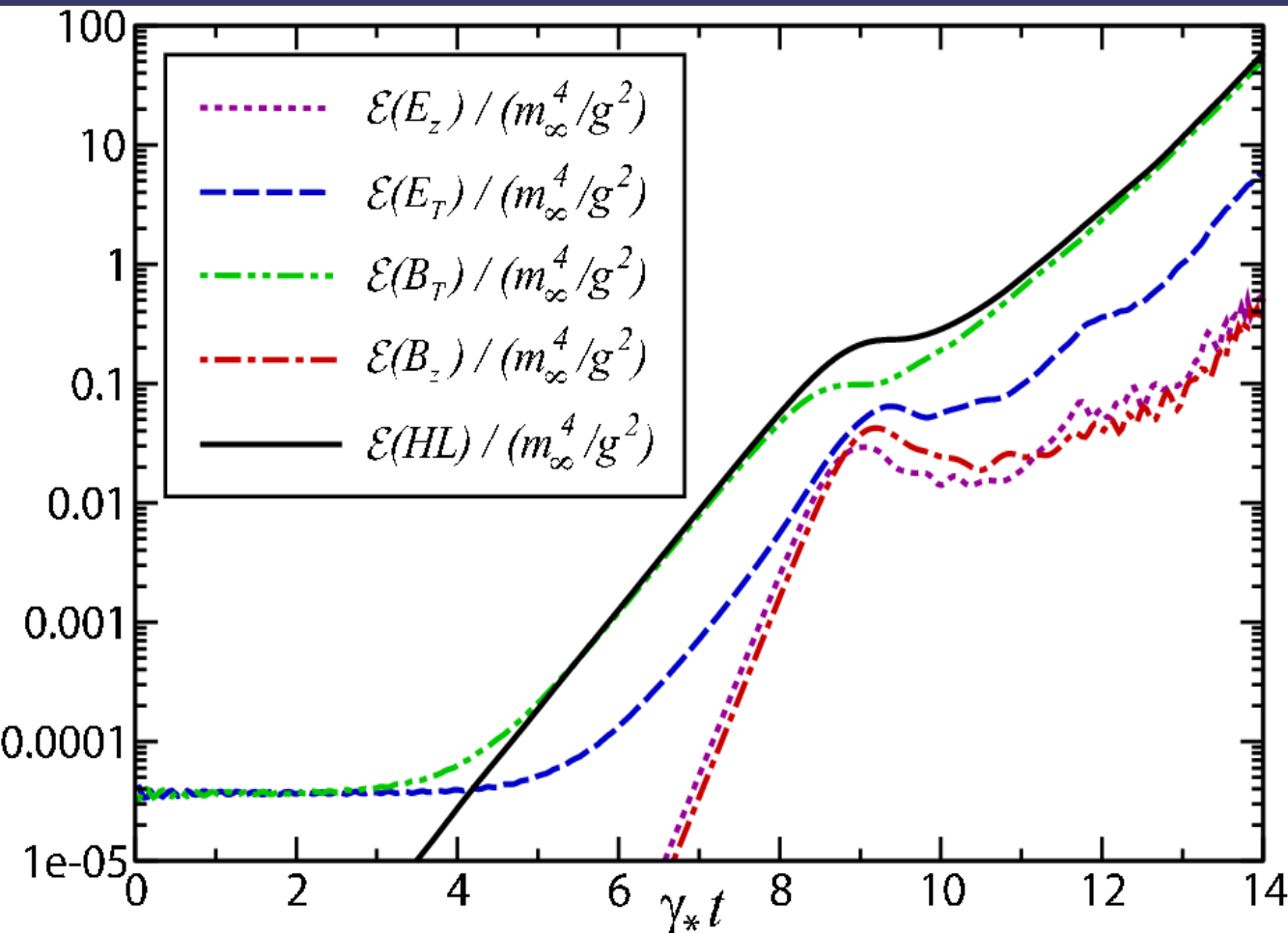
- Gluons are produced at  $\tau \sim 1/Q_s$ ,  
 $f \sim 1/\alpha_s$  typical momentum  $p \sim Q_s$
- Due to a longitudinal expansion, typical longitudinal momentum of gluons becomes

$$p_z \sim 1/\tau, \quad p_x \sim p_y \sim Q_s \quad \text{at } \tau \gg 1/Q_s$$



# 1+1D hard loop field simulations

A. Rebhan, P. Romatschke and M. Strickland, hep-ph/0412016.



Linear Vlasov  $f \simeq f_0 + \delta f$

First confirmation of instabilities in SU(2) gauge theory predicted by

Mrowczynski,  
Arnold, Lenaghan, Moore  
Romatschke, Strickland

Initial Conditions:

- random fields
- squeezed particle momentum distr.

# *Purpose of our Study*

*solving full Vlasov equation for gluons,  
i.e. dynamics of fields and particles.*

- ♦ non-linear effects: saturation of instabilities.
- study isotropization of the system (particles and field) by interactions between particles and field.
- weak and strong initial field.

# Particle in cell simulation for colored particles

$$\frac{d \mathbf{x}_i}{dt} = \mathbf{v}_i, \quad \frac{d \mathbf{p}_i}{dt} = g Q_i^a (\mathbf{E}_i^a + \mathbf{v}_i \times \mathbf{B}_i^a), \quad \frac{d Q_i}{dt} = ig v_i^\mu [A_\mu, Q_i].$$

$$D_\mu F^{\mu\nu} = J^\nu = g \sum Q v^\mu \delta(x - x_i(t))$$

Non-Abelian version of the charge conservation algorithm

C. R. Hu and B. Muller, Phys. Lett. B409 (1997)377

G. D. Moore, C.R. Hu and B. Muller, Phys. Rev. D58 (1998)045001.



# Yang-Mills Hamiltonian on the lattice

Numerical implementation of real time simulation in classical YM is well established. Kogut-Susskind Hamiltonian:

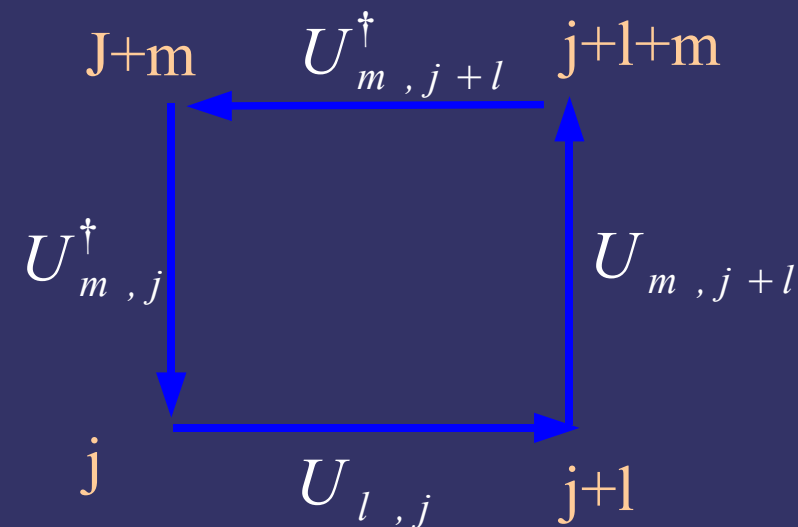
$$H_{YM} = \frac{1}{2} \sum_l E_l^2 + \frac{\tau}{2} \sum_{\square} (N_C - \Re \text{Tr} U_{\square})$$

Link  $U_{j,i} = \exp[-iga A_j(i)]$

Plaquette  $U_{\square} \equiv U_{l,j} U_{m,j+l} U_{l,j+m}^{\dagger} U_{m,j}^{\dagger}$

Equations of motion for dynamical variable  $v$

$$\frac{dv}{d\tau} = \{H_L, v\}$$



# Initial conditions for simulations

- Fields depend only on x-direction (1D), but particle motion is in 3D.
- Initial field configuration: random white noise.

1) Weak field: i.e. large separation between soft and hard modes.

$$P_{\text{hard}} = 10 \text{ GeV}$$

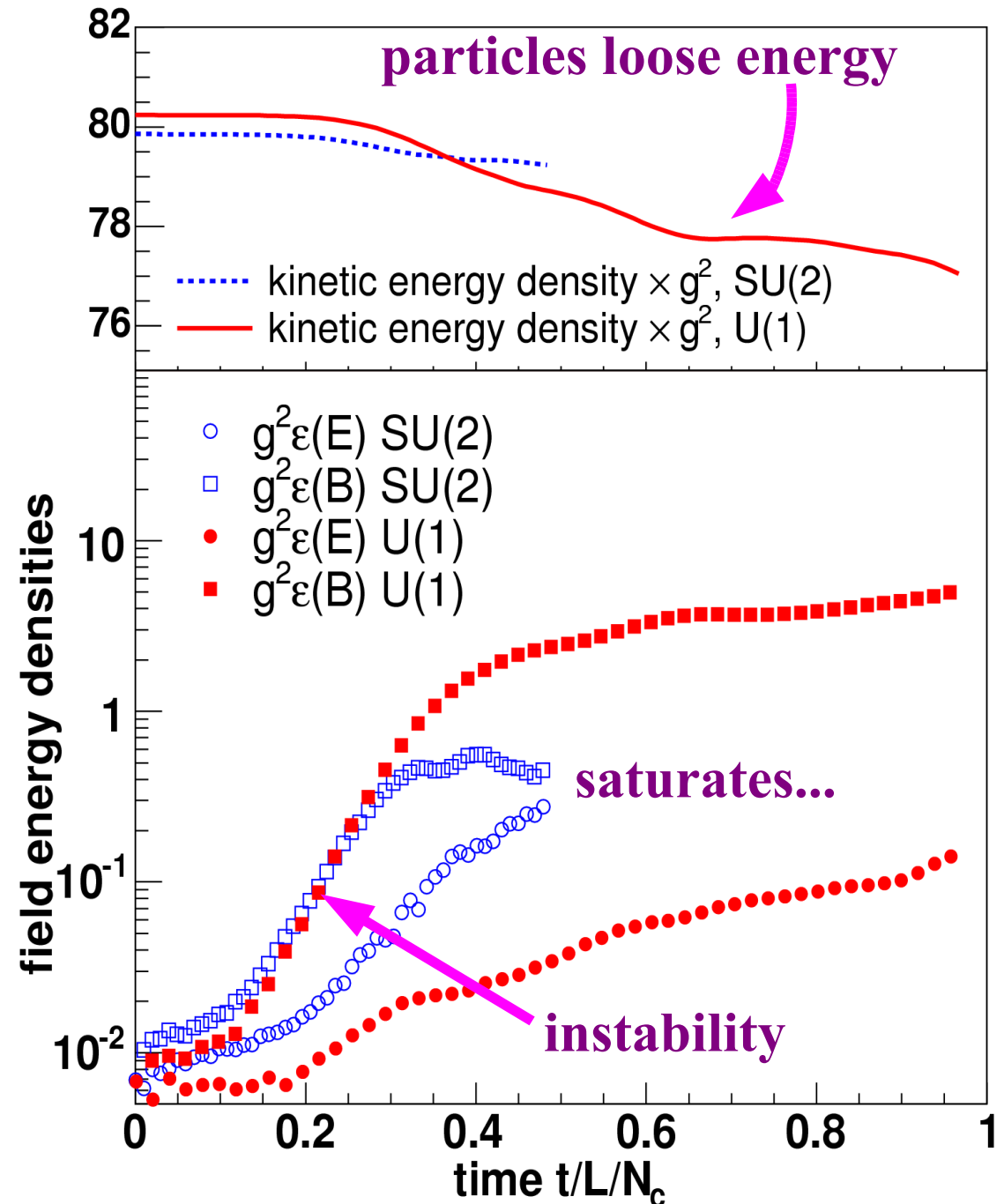
2) Strong field: small separation between soft and hard modes.

$$P_{\text{hard}} = 1 \text{ GeV}$$

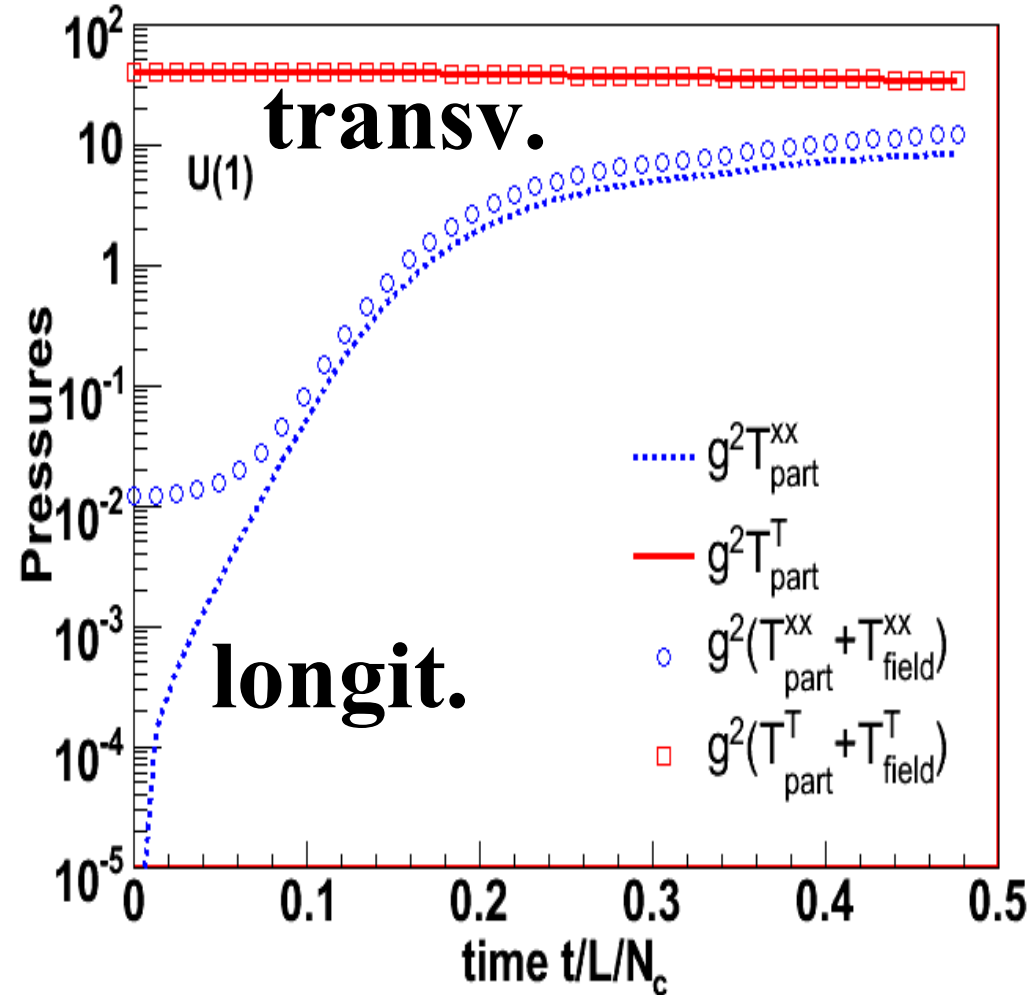
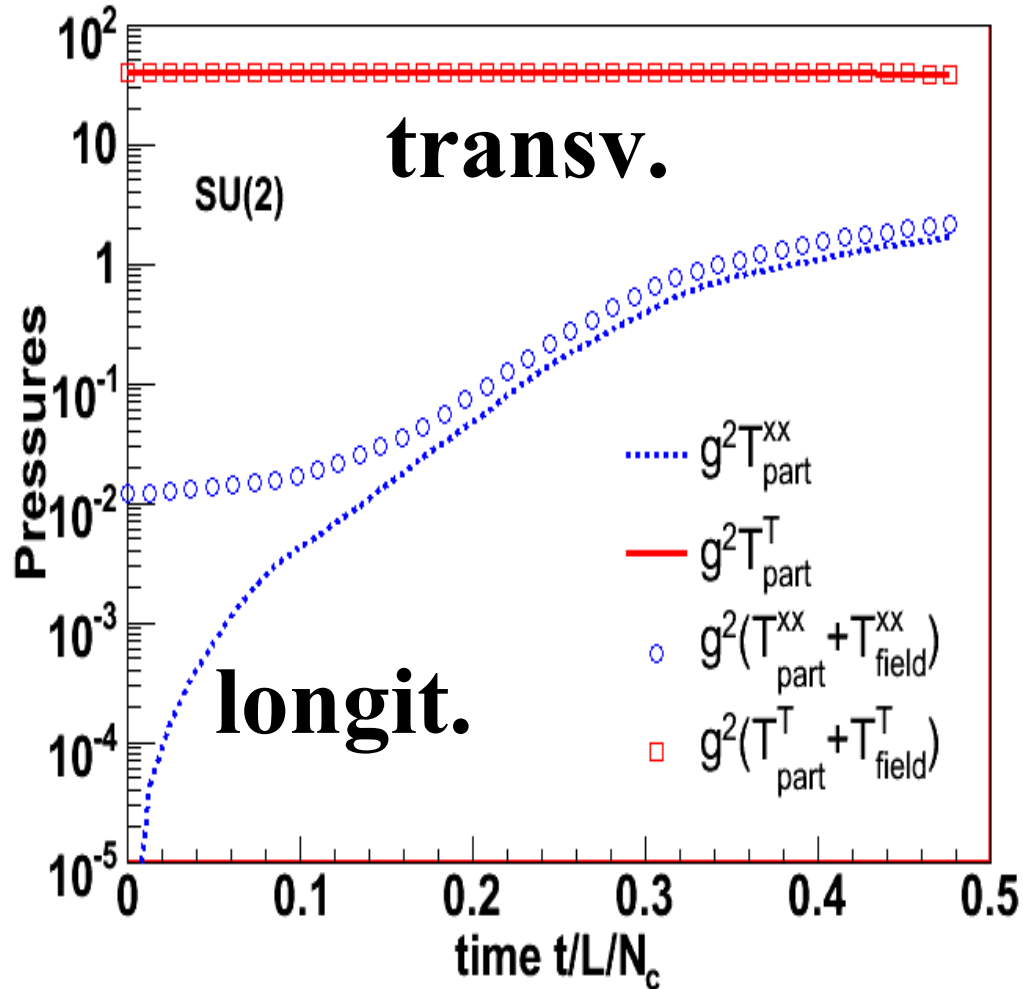
$$f(\mathbf{p}) \sim \delta(p_x) e^{-\sqrt{p_x^2 + p_y^2}/p_{\text{har}}}$$

# Weak initial fields

- ★ Confirms existence of instabilities.
- ★ Particles ARE affected
- ★ Saturation well before fields reach particle scale.



# (Partial) Isotropization

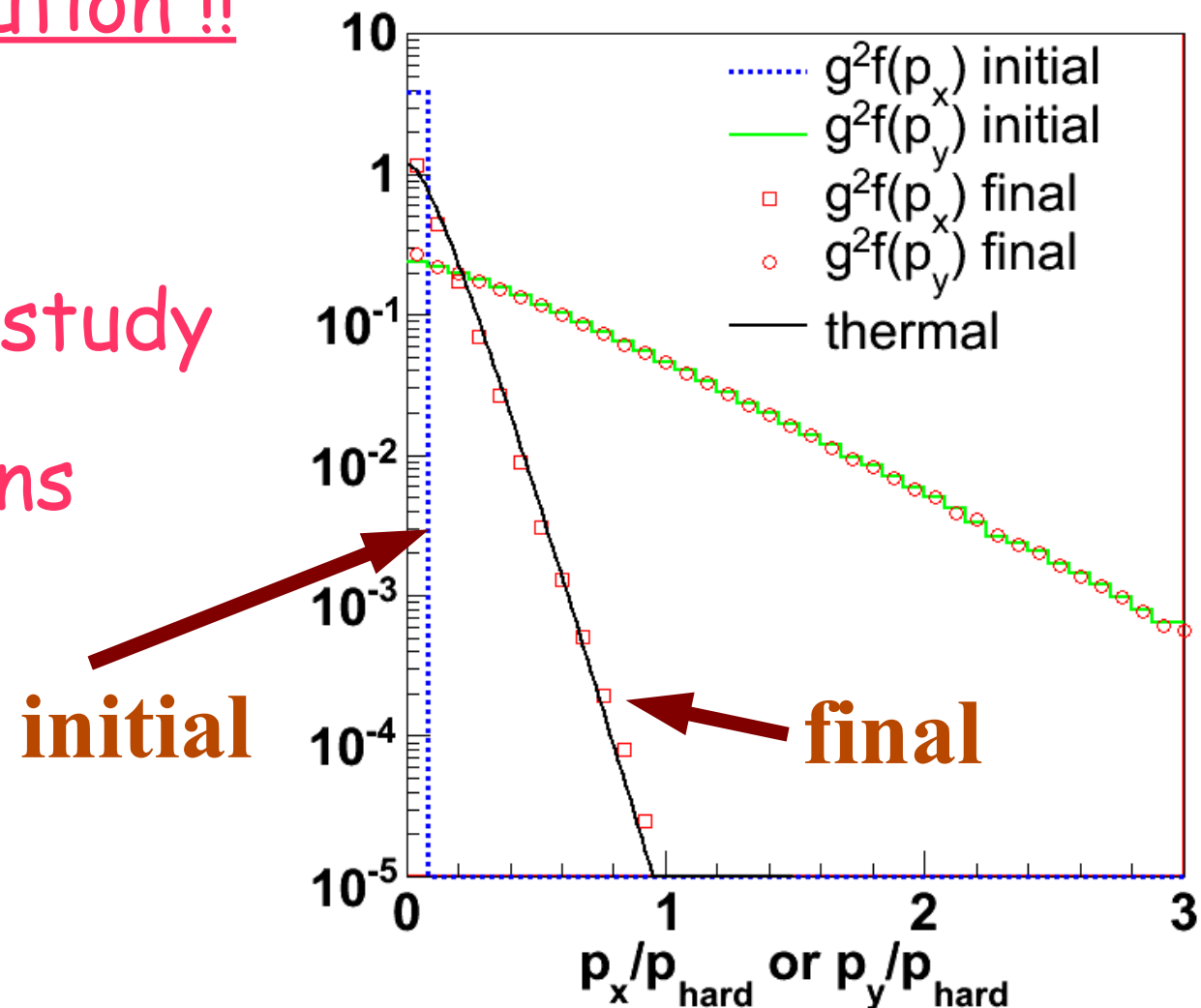


exponential growth of long. pressure

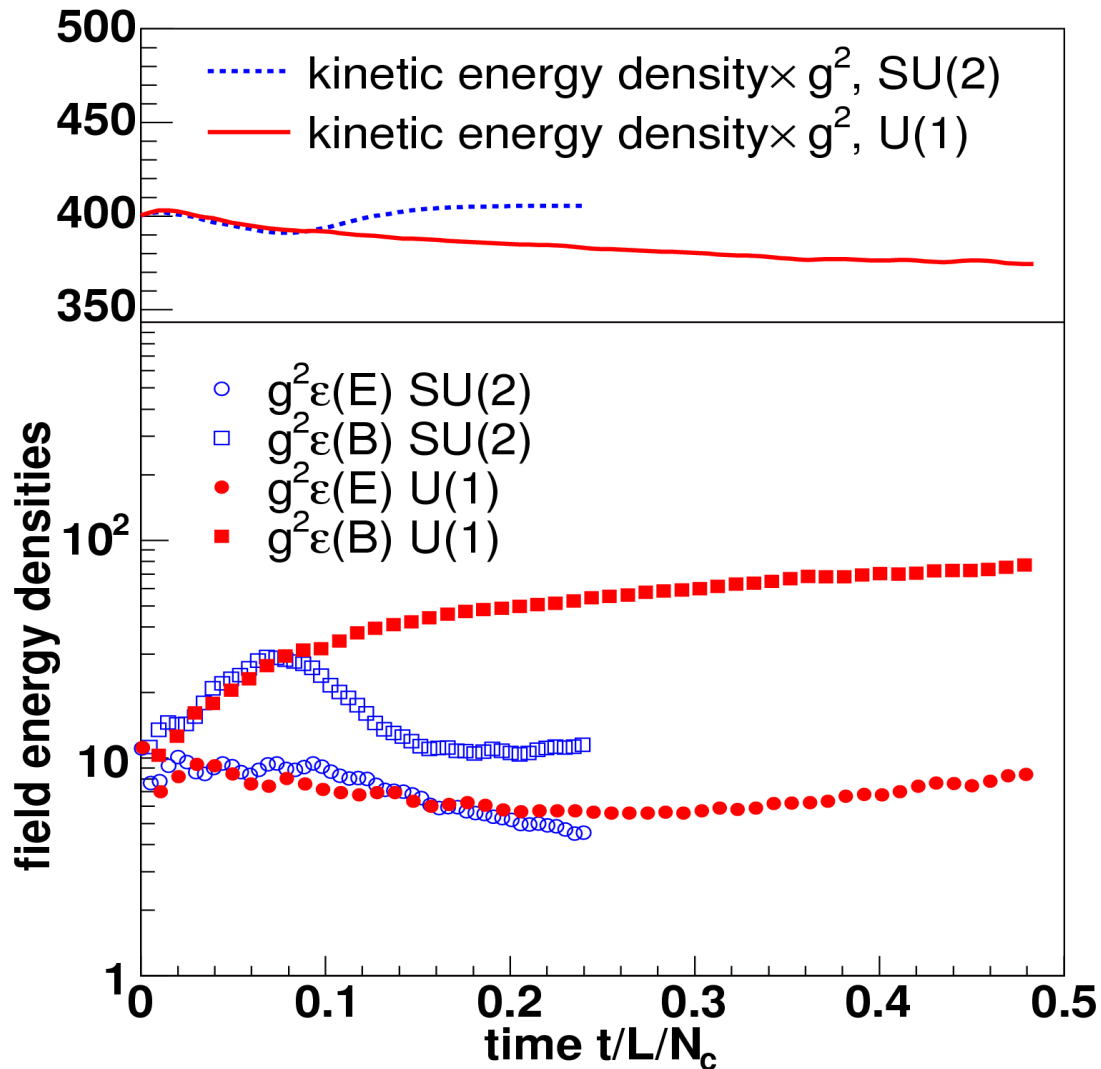
$$\text{field part} : T_{0x} = T_{\text{transverse}} = 0, \quad T_{xx} = T_{00}$$

# Momentum distributions of the hard particles

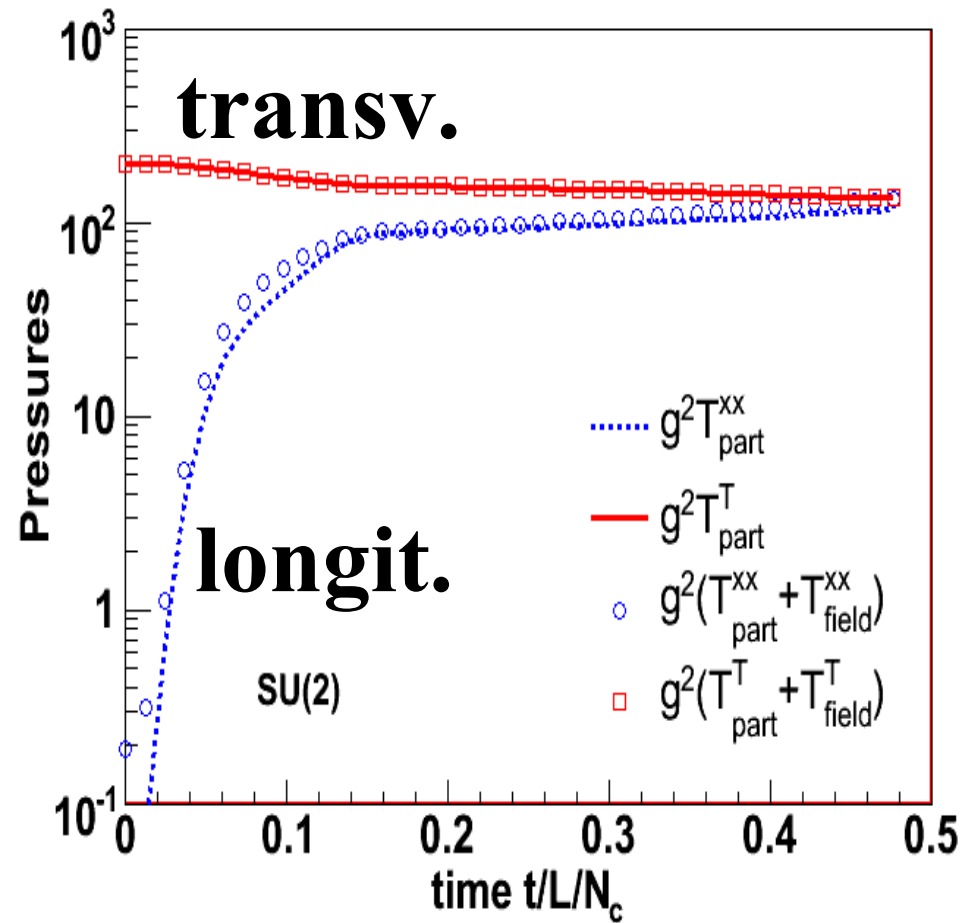
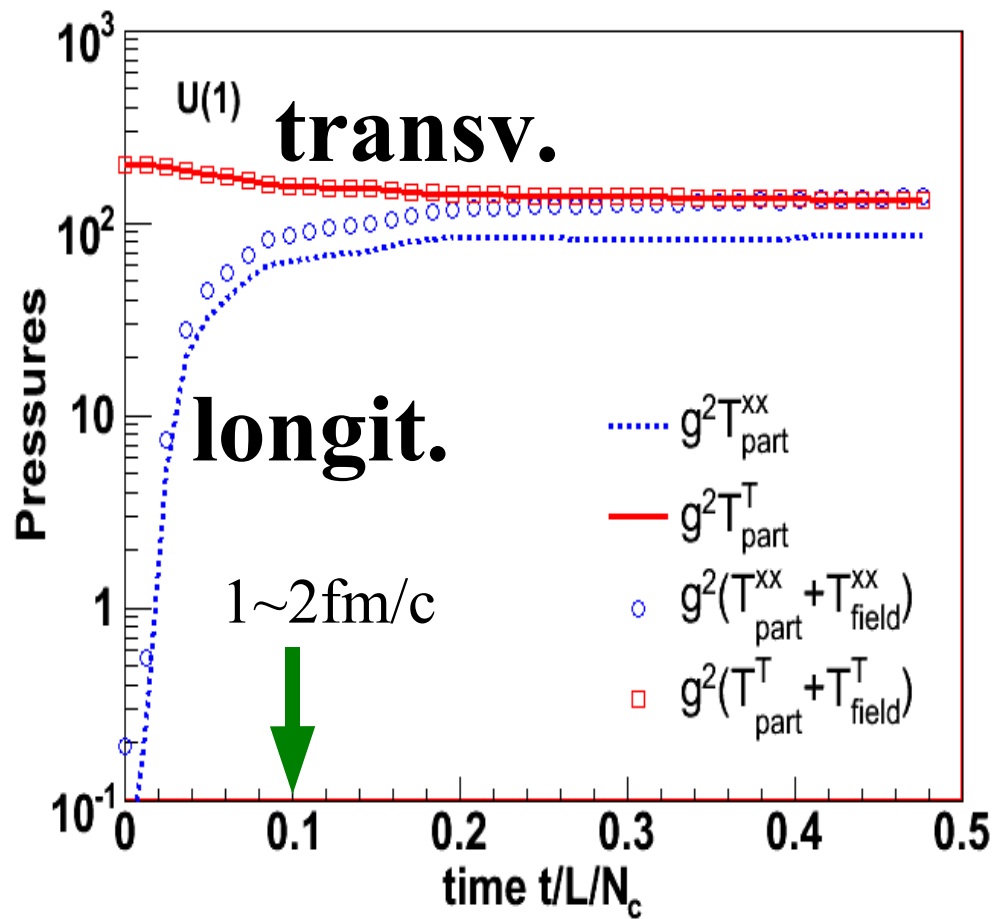
- ★ Exponential distribution !!
- ★ Longitudinal Cooler
- ★ Full 3d required to study transverse directions



## 2) Strong initial fields



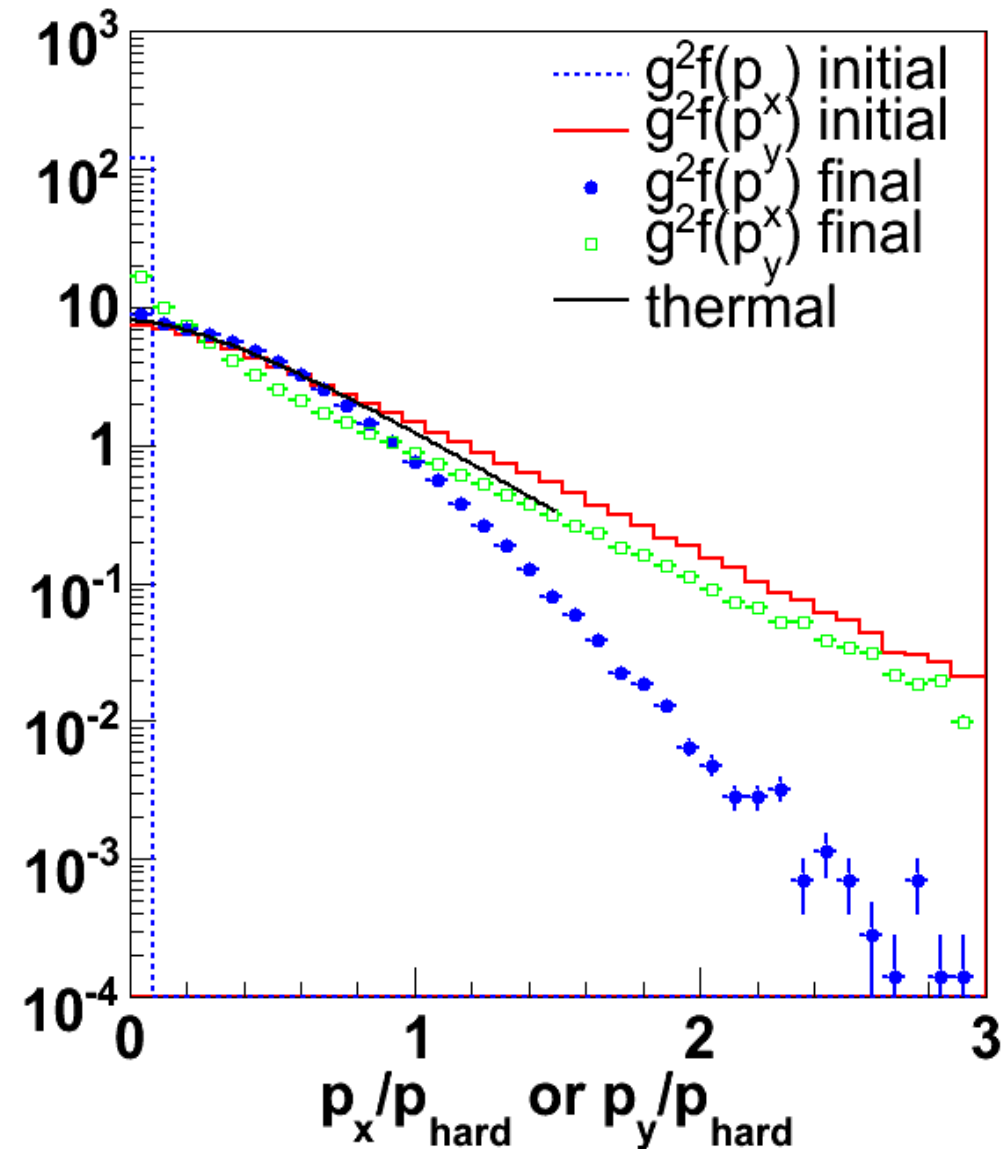
- No real instabilities seen...
- Soft fields irrelevant?



- Rapid isotropization by deflection in strong field.
- Shuts off instability.
- Even so, time scale at least as small as scattering.
- Soft fields need to be taken into account !

# Momentum distributions of the hard particles

- ★ Nearly thermal up to  $\sim p_{\text{hard}}$
- ★ ONE Temp !
- ★ Stay tuned for long. Bj expansion, 3d, collisions, ...





# Summary and Outlook

- Particle-field (Vlasov-YM) simulations in QCD.
  - So far only 1+1 D.
  - Collective effects due to color field play important role for the pre-equilibrium dynamics.
- 
- Include expansion of the system.
  - Full 3D simulation
  - Collision term

# lattice size and test particle dependence

