

# Survival of Back-to-Back Correlations

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# Introduction

- Late 90's → Back-to-Back Correlations (BBC) among boson-antiboson pairs were shown to exist if the masses of the particles were modified in a hot and dense medium [Asakawa, Csörög & Gyulassy, P.R.L. 83 (1999) 4013].
- 2001 → It was shown that similar BBC existed among fermion-antifermion pairs with medium modified masses [Panda, Csörög, Hama, Krein & SSP, P. L. B512 (2001) 49].
- Some properties:
  - Similar formalism for both bosonic (**bBBC**) and fermionic (**fBBC**) Back-to-Back correlations
  - Similar (and unlimited) intensity of **fBBC** and **bBBC**
  - Expected to appear for  $p_T \leq 1-2 \text{ GeV}/c$

# Formalism

- Infinite medium

$$H = H_0 - \frac{1}{2} \int d\vec{x} d\vec{y} \phi(\vec{x}) \delta M^2(\vec{x} - \vec{y}) \phi(\vec{y}) \longrightarrow \text{In-medium Hamiltonian}$$

$$H_0 = \frac{1}{2} \int d\vec{x} (\dot{\phi}^2 + |\nabla \phi|^2 + m^2 \phi^2) \longrightarrow \text{Asymptotic (free) Hamiltonian, in the rest frame of matter}$$

- Scalar field  $\phi(x) \leftrightarrow$  quasi-particles propagating with momentum-dependent medium-modified effective mass,  $m_*$ , related to the vacuum mass,  $m$ , by

- ◆ 
$$m_*^2(|\vec{k}|) = m^2 - \delta M^2(|\vec{k}|)$$

- Consequently:  $\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2(|\vec{k}|)$

$\Omega_k \rightarrow$  frequency of the in-medium mode with momentum  $\vec{k}$

# Some comments (bosonic BBC):

- $b_k (b_k^\dagger) \rightarrow$  in-medium annihilation (creation) operator
- $a_k (a_k^\dagger) \rightarrow$  annihilation (creation) operator of the asymptotic quanta with 4-momentum  $k^\mu = (\omega_k, \vec{k})$

They are related by the Bogoliubov transformation:

$$\begin{cases} a_k^\dagger = c_k b_k^\dagger + s_{-k} b_{-k} \\ a_k = c_k^* b_k + s_{-k}^* b_{-k}^\dagger \end{cases} ; \quad c_k = \cosh[f_k] ; \quad s_k = \sinh[f_k]$$

- $f_k = \frac{1}{2} \ln(\omega_k / \Omega_k) \rightarrow$  squeezing parameter (the Bogoliubov transformation is equivalent to a squeezing operation)
- $a$ -quanta  $\rightarrow$  observed;  $b$ -quanta  $\rightarrow$  thermalized in medium

# - Full two-particle correlation (e.g., for $\pi^0$ 's)

$$\langle a_{k_1}^\dagger a_{k_2}^\dagger a_{k_1} a_{k_2} \rangle = \langle a_{k_1}^\dagger a_{k_1} \rangle \langle a_{k_2}^\dagger a_{k_2} \rangle + \langle a_{k_1}^\dagger a_{k_2} \rangle \langle a_{k_2}^\dagger a_{k_1} \rangle + \langle a_{k_1}^\dagger a_{k_2}^\dagger \rangle \langle a_{k_1} a_{k_2} \rangle$$

NOTATION

$$\begin{aligned}
 N_1(\vec{k}_i) &= \omega_{k_i} \frac{d^3 N}{d^3 k} = G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i, i) = \omega_{k_i} \langle a_{k_i}^\dagger a_{k_i} \rangle && \text{Spectra} \\
 G_c(\vec{k}_1, \vec{k}_2) &\equiv G_c(1, 2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1}^\dagger a_{k_2} \rangle && \text{Chaotic amplitude} \\
 G_s(\vec{k}_1, \vec{k}_2) &\equiv G_s(1, 2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle && \text{Squeezed amplitude}
 \end{aligned}$$

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \underbrace{\frac{|G_c(1, 2)|^2}{G_c(1, 1) G_c(2, 2)}}_{\text{HBT}} + \underbrace{\frac{|G_s(1, 2)|^2}{G_c(1, 1) G_c(2, 2)}}_{\text{BBC}}$$

$$\therefore \begin{cases} G_c(1,2) = \sqrt{\omega_k \omega_{k_2}} \left[ \langle (c_{k_1}^* b_{k_1}^\dagger)(c_{k_2} b_{k_2}) \rangle + \langle (s_{-k_1} b_{-k_1})(s_{-k_2}^* b_{-k_2}^\dagger) \rangle \right] \\ G_s(1,2) = \sqrt{\omega_k \omega_{k_2}} \left[ \langle (s_{-k_1}^* b_{-k_1}^\dagger)(c_{k_2} b_{k_2}) \rangle + \langle (c_{k_1} b_{k_1})(s_{-k_2}^* b_{-k_2}^\dagger) \rangle \right] \end{cases}$$

- After performing the thermal averages  $\rightarrow \langle \mathcal{O} \rangle = \text{Tr}(\hat{\rho} \mathcal{O})$ 
  - If the thermal b gas freezes out suddenly at some time, at temperature  $T$ , the observed *single-particle distribution* for a is written as

$$N_1(\vec{k}) = \frac{V}{(2\pi)^3} \omega_k \left[ |c_k|^2 n_k + |s_{-k}|^2 (n_{-k} + 1) \right] \equiv \frac{V}{(2\pi)^3} \omega_k n_1(\vec{k})$$

- And the *squeezed correlation function* is given by

$$C_s(\vec{k}, -\vec{k}) = 1 + \frac{|c_k s_k^* n_k + c_{-k} s_{-k}^* (n_{-k} + 1)|^2}{n_1(\vec{k}) n_1(-\vec{k})}$$

# Finite size medium moving with collective velocity

- For a hydrodynamical ensemble  $\rightarrow$  amplitudes can be written as [ Makhlin & Sinyukov, N.P. A566 (1994) 598c ]:

$$G_c(1,2) = \frac{1}{(2\pi)^3} \int d^4\sigma_\mu(x) K_{1,2}^\mu e^{iq_{1,2}\cdot x} \left[ |c_{1,2}|^2 n_{1,2} + |s_{-1,-2}|^2 (n_{-1,-2} + 1) \right]$$

$$G_s(1,2) = \frac{1}{(2\pi)^3} \int d^4\sigma_\mu(x) K_{1,2}^\mu e^{i2K_{1,2}\cdot x} \left[ s_{-1,2}^* c_{2,-1} n_{-1,2} + c_{1,-2} s_{-2,1}^* (n_{1,-2} + 1) \right]$$

- where  $c_{i,j} = \cosh[f_{i,j}]$  ;  $s_{i,j} = \sinh[f_{i,j}]$

- Squeezing coefficient:

$$f(i,j,x) = \frac{1}{2} \ln \left[ \frac{(K_{i,j}^\mu u_\mu(x))}{(K_{i,j}^{*\nu} u_\nu(x))} \right] = \frac{1}{2} \ln \left[ \frac{\omega_{k_i}(x) + \omega_{k_j}(x)}{\Omega_{k_i}(x) + \Omega_{k_j}(x)} \right]$$

- » Two-particle momenta:  $K_{i,j}^\mu = \frac{1}{2}(k_i + k_j)$  ;  $q_{i,j}^\mu = (k_i - k_j)$
- »  $w^\mu \rightarrow$  local flow vector at freeze-out

- $d^4\sigma^\mu(x) = d^3\Sigma^\mu(x, \tau_f) F(\tau_f) d\tau_f \rightarrow$  product of the normal-oriented volume element depending parametrically on  $\tau_f$  (freeze-out hypersurface parameter) and the inv. distrib. of  $\tau_f$ ,  $F(\tau_f)$ .
- $\sigma^\mu \leftrightarrow$  hydrodynamical freeze-out surface
- Freeze-out possibilities:

Instant freeze-out  $\rightarrow \int dt E_{i,j} e^{-2iE_{i,j}\cdot\tau} \delta(\tau - \tau_0) d\tau_f = E_{i,j} e^{-2iE_{i,j}\cdot\tau_0}$

Finite emission interval  $\rightarrow \int dt E_{i,j} F(\tau_f) e^{-iE_{i,j}(\tau-\tau_0)} d\tau_f = \frac{E_{i,j}}{[1 + (E_{i,j}\Delta t)^2]}$

(corresponding to:  $F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau-\tau_0)/\Delta t}$  )

- $n_{i,j} \rightarrow$  Boltzmann limit of Bose-Einstein distribution:

$$n_{i,j}(x) \sim \exp\left[-\left(K_{i,j}^\mu u_\mu - \mu(x)\right)/T(x)\right]$$

Hydro solution  $\rightarrow \frac{\mu(x)}{T(x)} = \frac{\mu_0}{T(x)} - \frac{\vec{r}^2}{2R^2}$



# Non-relativistic limit

- The flow 4-velocity is written as:

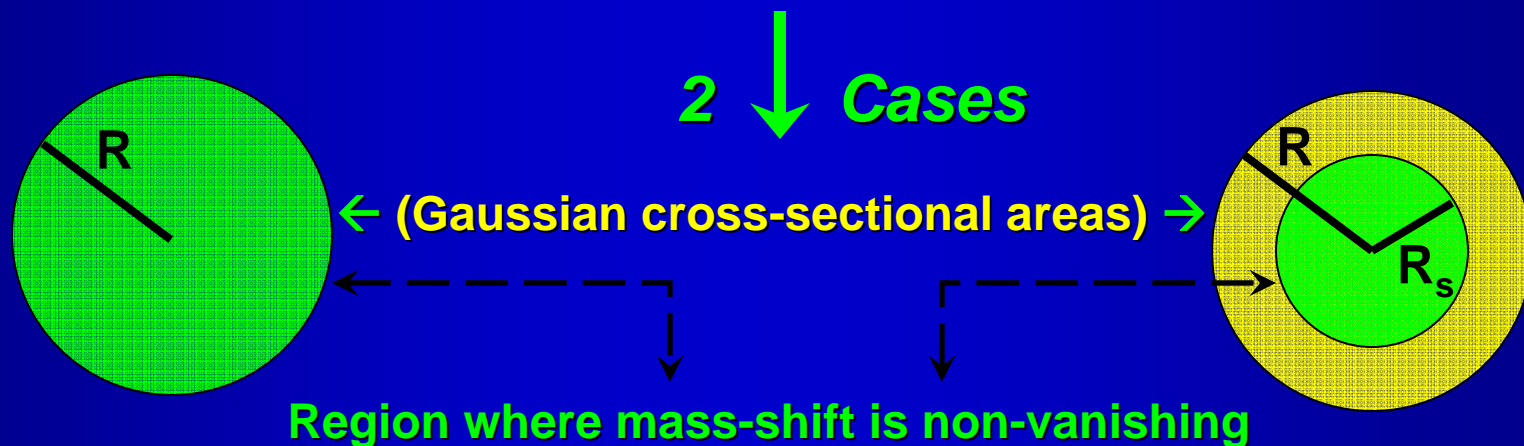
$$u^\mu = \gamma(1, \vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

- In the non-relativistic limit  $\rightarrow \gamma = (1 - \vec{v}^2)^{-1/2} \approx 1 + \frac{1}{2} \vec{v}^2$
- and keeping all moment up to  $\mathcal{O}(mv^2)$ .
  - For **large mass**  $m$  and small mass shifts  $(m_* - m)/m \ll m \rightarrow$  flow effects on squeezing parameter:

$$\mathcal{O} \left[ \left( \frac{\text{Kin. En.}}{m} \right) \left( \frac{\delta M^2}{m^2} \right) \right] \rightarrow \text{flow effects on } f_{i,j} \text{ are negligible.}$$

- So,  $c_{i,j}$  and  $s_{i,j}$  become flow independent, but could be  $r$ -dependent if  $m_*$  is (e.g., through  $T(x)$ , as in hydro)
- **Hypotheses:** mass-shift is independent on position within a fireball, which is assumed to have a sharp surface, i.e.,  $\delta M=0$ , and density also vanishes outside this volume.

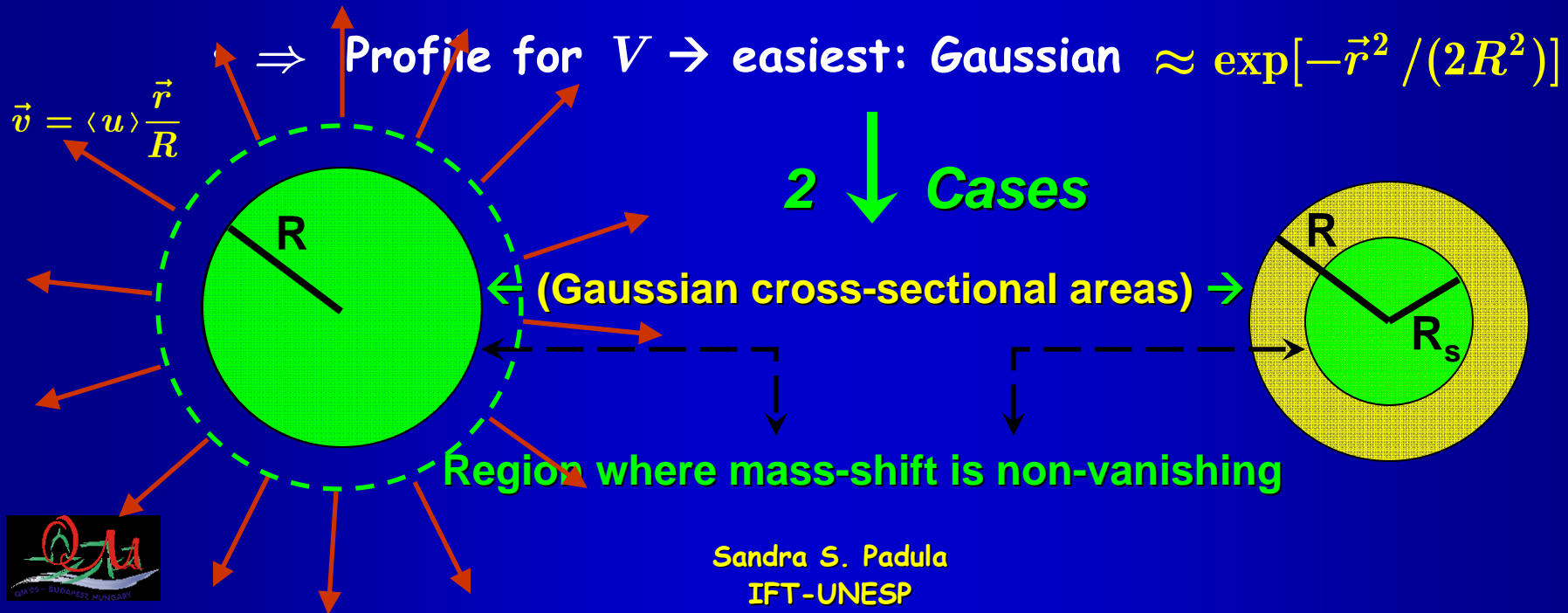
- Integration is over the region where the mass-shift is non-vanishing, which is NOT infinitely large
- In Heavy Ion Collisions:  $V \sim (R \simeq 5 - 10 \text{ fm})^3$
- What is the volume  $V$ ?
  - » The coefficient of the vacuum term in the integrands  $\rightarrow s_{i,i} = 0$  outside mass-shift region
  - » Terms proportional to  $n_{i,j}$  are finite (hydro solution)
  - »  $\rightarrow$  Integration could be extended to infinity:
- $\Rightarrow$  Profile for  $V \rightarrow$  easiest: Gaussian  $\approx \exp[-\vec{r}^2 / (2R^2)]$



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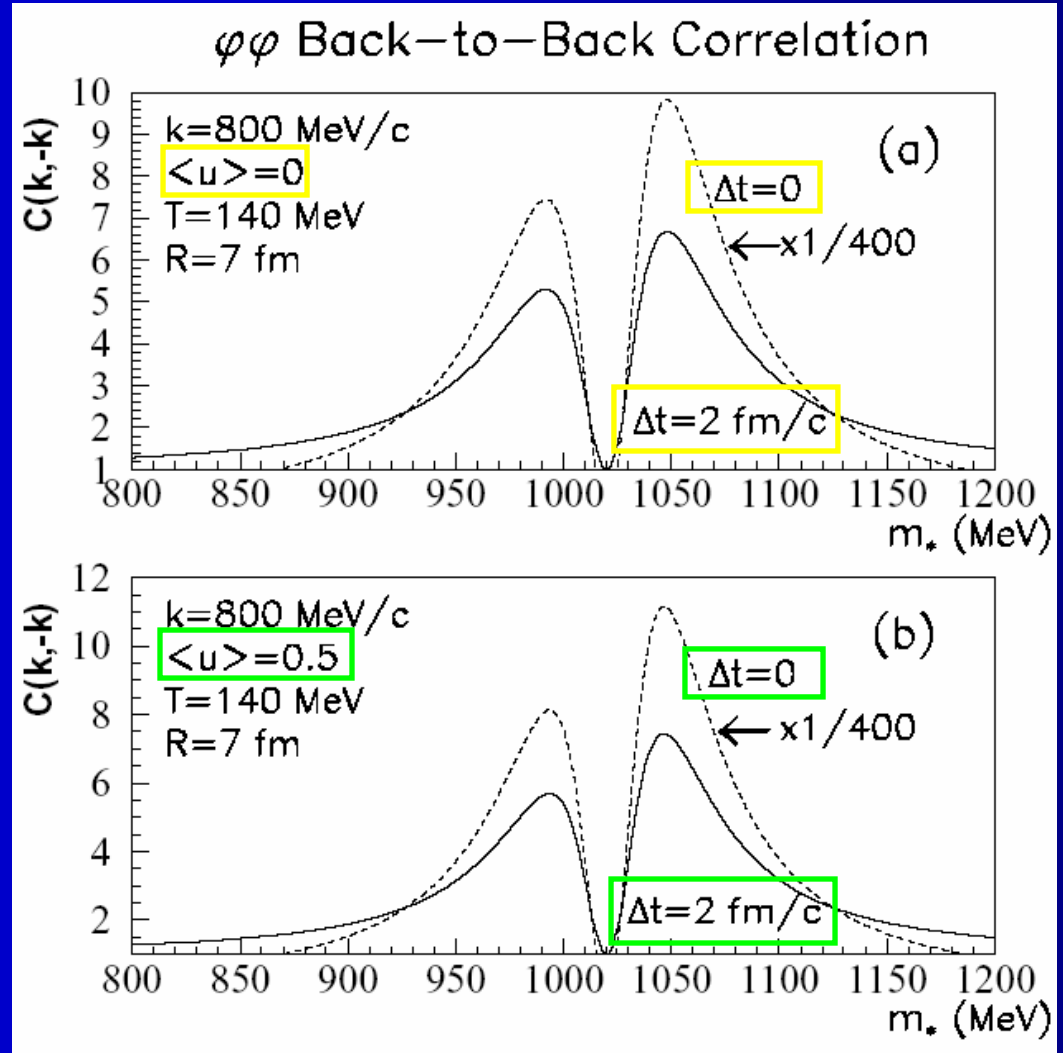
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# Squeezing region of radius $R$

- Flow, finite size & time

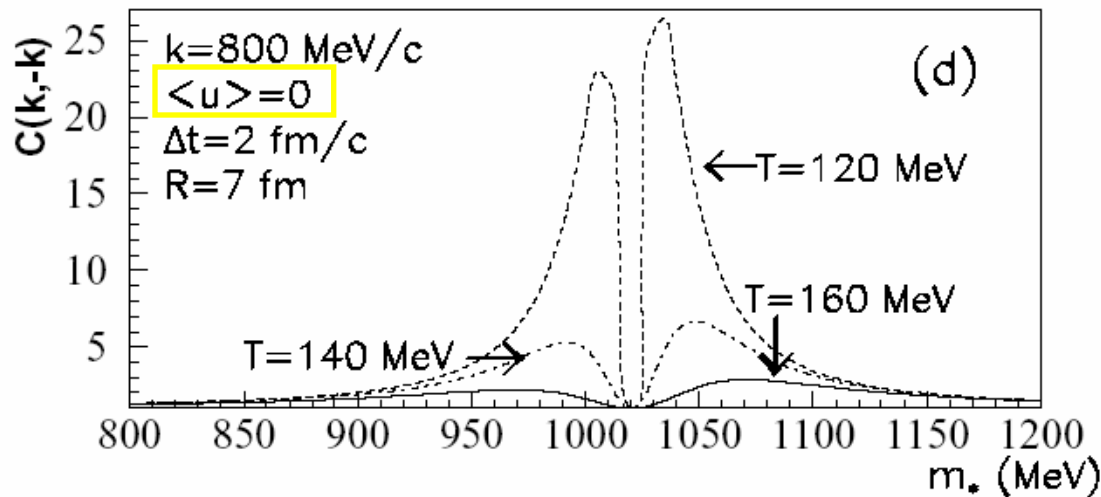
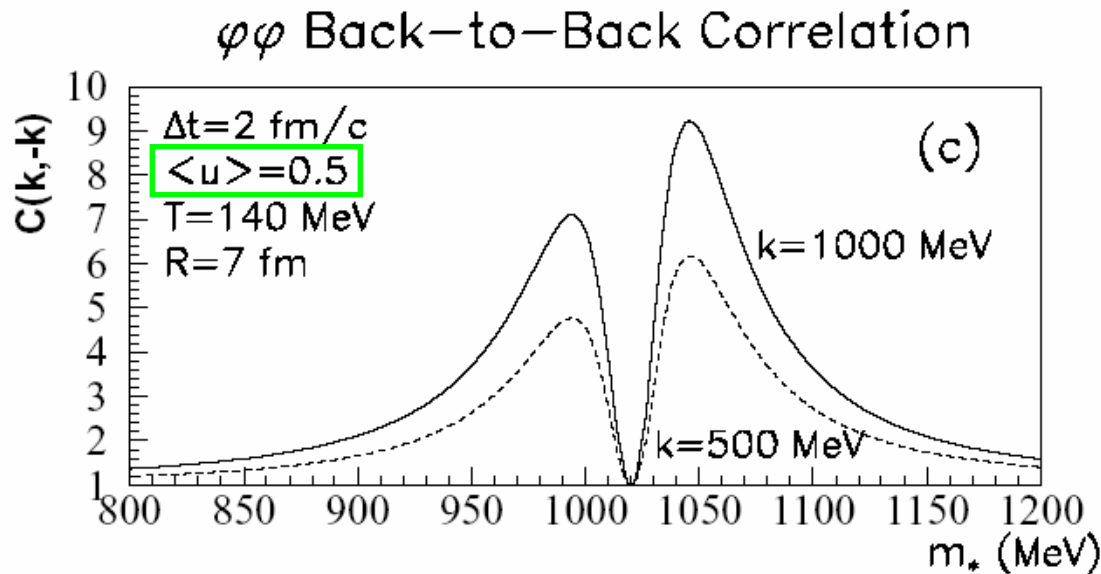
- Finite emission times  $\rightarrow$  have the effect of significantly suppress BBC signal intensity but it still is strong (already known)
- Finite size systems  $\rightarrow$  BBC signal  $\propto$  size of mass-shifting region
- For weak flow coupling  $\rightarrow$  signal strength sensitive to momenta; BBC with  $\langle u \rangle = 0.5$ : comparable, weaker or even mildly enhanced than for  $\langle u \rangle = 0$  for different  $k$



$$m_\phi = 1020 \text{ MeV}$$



# Rôle of momenta and temperatures



- Momenta ( $\langle u \rangle = 0.5$ ):
  - Increasing modulus of the back-to-back momenta of the pair enhances the effect

$$m_\phi = 1020 \text{ MeV}$$

- Temperature ( $\langle u \rangle = 0$ ):
  - BBC signal enhanced for lower emission temperatures, for finite sized systems emitting during finite time interval

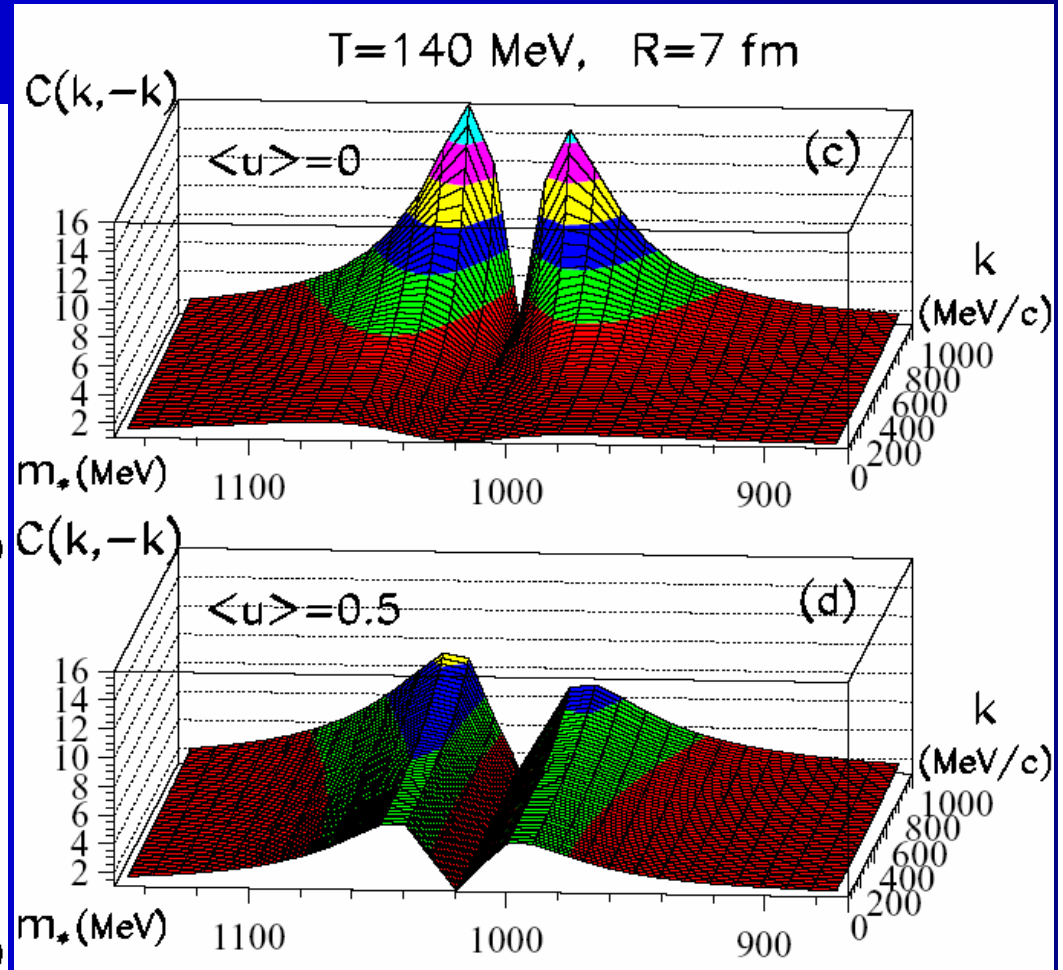
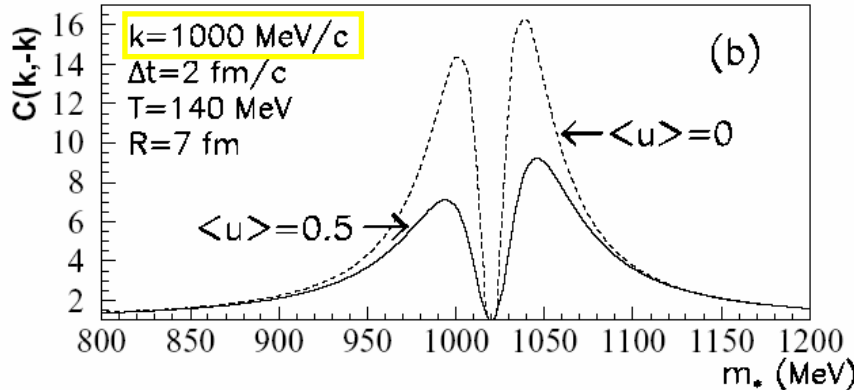
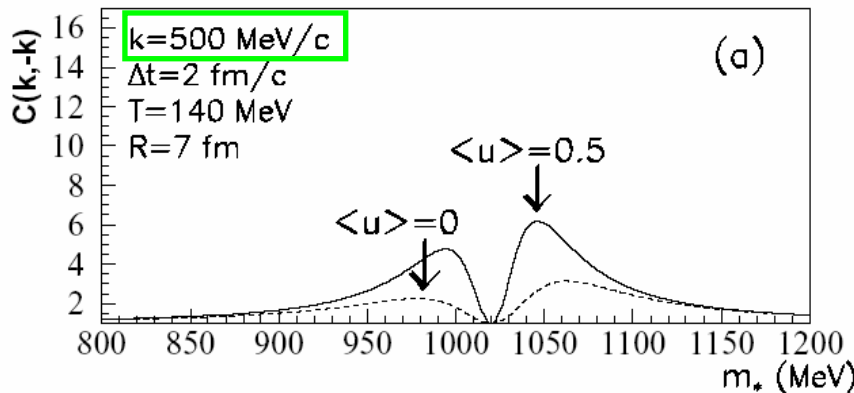
# Squeezing region of radius $R$

-  $T, R, \Delta t$  fixed for two values of  $k$  and  $\langle u \rangle$

- Varying  $k$  as well:

$$m_\phi = 1020 \text{ MeV}$$

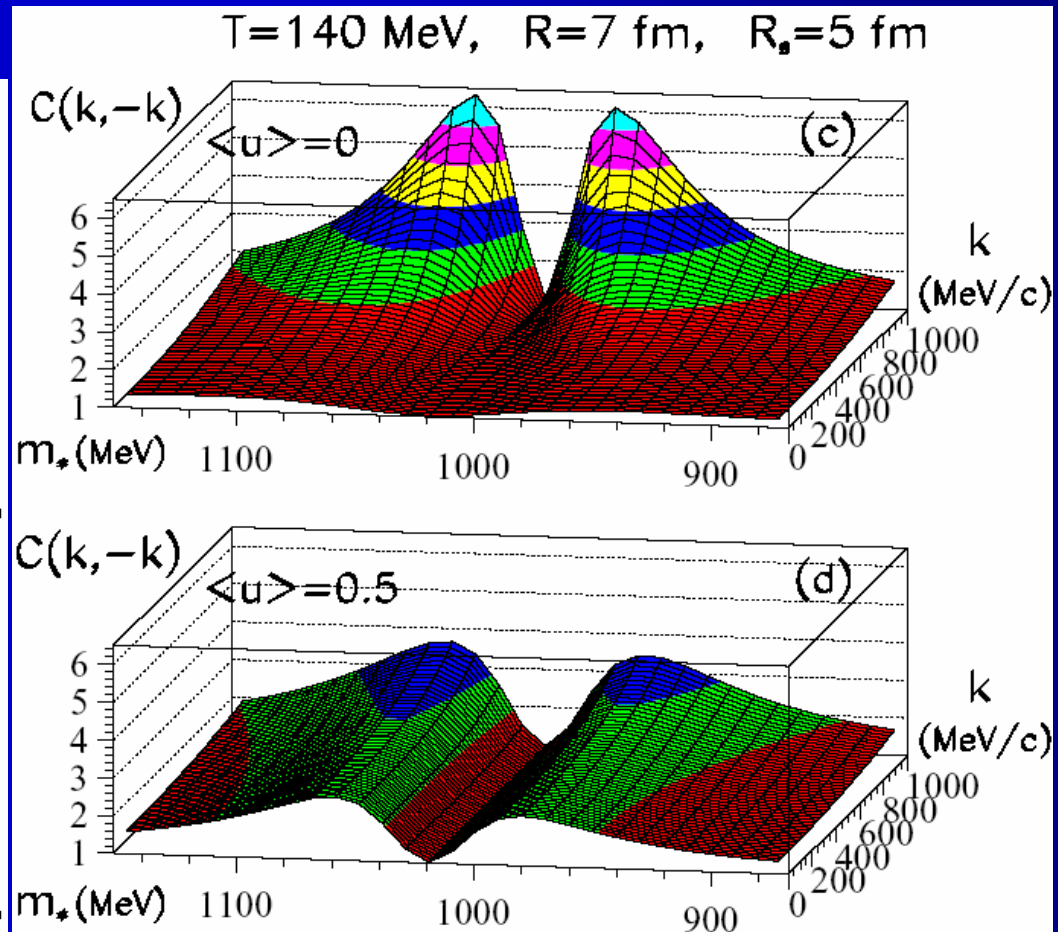
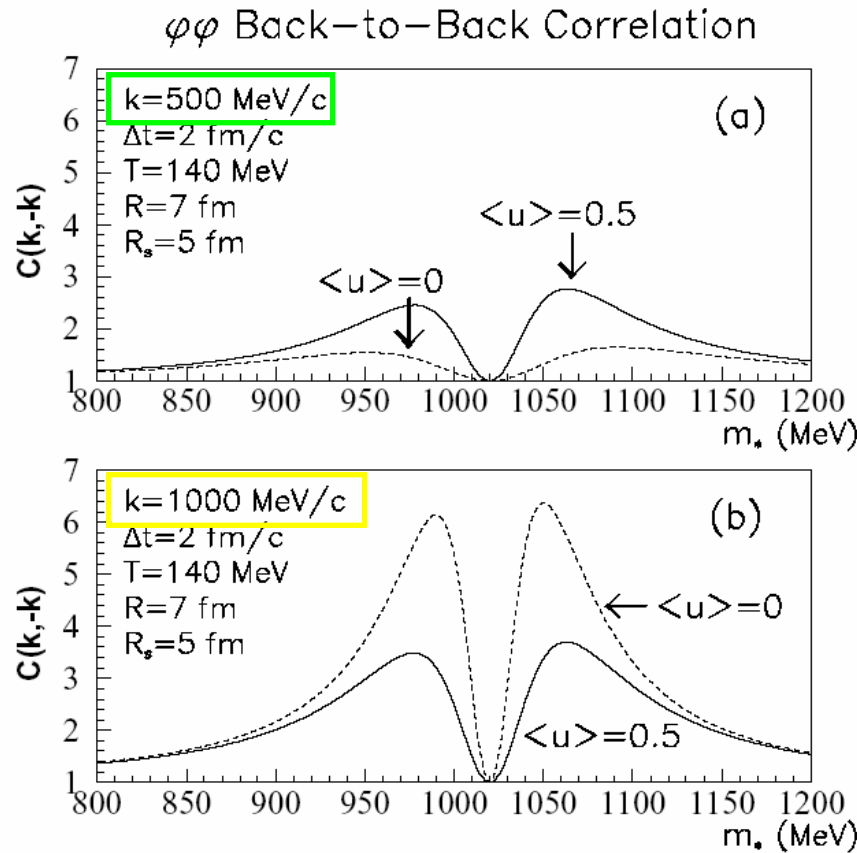
$\phi\phi$  Back-to-Back Correlation



# Squeezing radius $R_s$ , system radius $R$

-  $T, R, \Delta t$  fixed for two values of  $k$  and  $\langle u \rangle$

- Varying  $k$  as well:



$m_\phi = 1020 \text{ MeV}$

Sensitive to size of squeezing region



# Comments and Conclusions

- **Main motivation: revive the discussion on BBC search**
- For this → estimated strength of BBC signal for:
  - Finite size systems emitting during finite time intervals
  - Expanding system with radial flow & non-relativistic limit
  - Illustration → Simplified situation of weak flow dependence of the BBC pair (close to central region) →  $\phi\phi$  correlation
- For single ( $R$ ) and two volumes ( $R$  &  $R_s$ ) cases:
  - **BBC vs.  $m_*$  and BBC vs.  $m_*$  vs.  $k$** 
    - Pronounced maxima around  $m_* \approx m$
    - Similar behavior with and without flow
    - BBC signal:
      - » increases for increasing size of mass-shifting region
      - » sensitive to spread in emission interval
      - » sizeable effect for particular cases studied here



# Next

- Introduce model-based mass-shift
- Perform more realistic calculations with flow
- Search for kinematical regions optimizing BBC signal
- Experimental feedback wanted for exp. acceptance
- Estimate shape and width of the BBC around the direction  $k_2 = -k_1 = k$
- ...

# EXTRAS



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# BBC function

- Momenta of the pair

$$k_2 = -k_1 = k$$

- Back-to-Back correlation function

$$C_s(k_1, k_2) = 1 + \left\{ |c_0| |s_0| \left[ R^3 + 2 \left( \frac{R^2}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right)} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T}\right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + (|c_0|^2 + |s_0|^2) \left( \frac{R^2}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right)} \right)^{\frac{3}{2}} \exp\left(-\frac{m_*}{T} - \frac{k^2}{2m_* T} - \frac{R^2 (m \langle u \rangle k / m_*)}{\left(1 + \frac{m^2 \langle u \rangle^2}{m_* T}\right) R T}\right) \right\}^{-2}$$

# Correspondences

## • Bosonic BBC

$$c_k = \cosh[f_k] \quad ; \quad s_k = \sinh[f_k]$$

$$\begin{cases} a^\dagger_k = c_k b^\dagger_k + s_{-k} b_{-k} \\ a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2} \log \left( \frac{\omega_k}{\Omega_k} \right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

## • Fermionic BBC

$$c_k = \cos[f_k] \quad ; \quad s_k = \sin[f_k]$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}_{\lambda',-k}^\dagger \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^\dagger & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}_{\lambda',-k}^\dagger \end{pmatrix}$$

$$A = [\chi_\lambda^\dagger (\sigma \cdot \hat{k}) \tilde{\chi}_{\lambda'}] \quad ; \quad A^\dagger = [\tilde{\chi}_{\lambda'}^\dagger (\sigma \cdot \hat{k})^\dagger \chi_\lambda]$$

$\chi_{\lambda'}$  → is a Pauli spinor

$$\tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} \quad ; \quad \hat{k} = \vec{k}/|\vec{k}|$$

$$\tan(2f_k) = -\frac{|k| \Delta M(k)}{\omega_k^2 - \Delta M(k) M}$$

$$m_*(k) = m - \Delta M(k)$$

$$\omega_k^2 = m^2 + \vec{k}^2 \quad ; \quad \Omega_k^2 = m_*^2 + \vec{k}^2$$

