

# (Quasi) Quarks and (Quasi) Baryons in QCD at High Density

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## Motivation

There is a successful effective theory of fermionic many body systems

### Landau Fermi-Liquid Theory

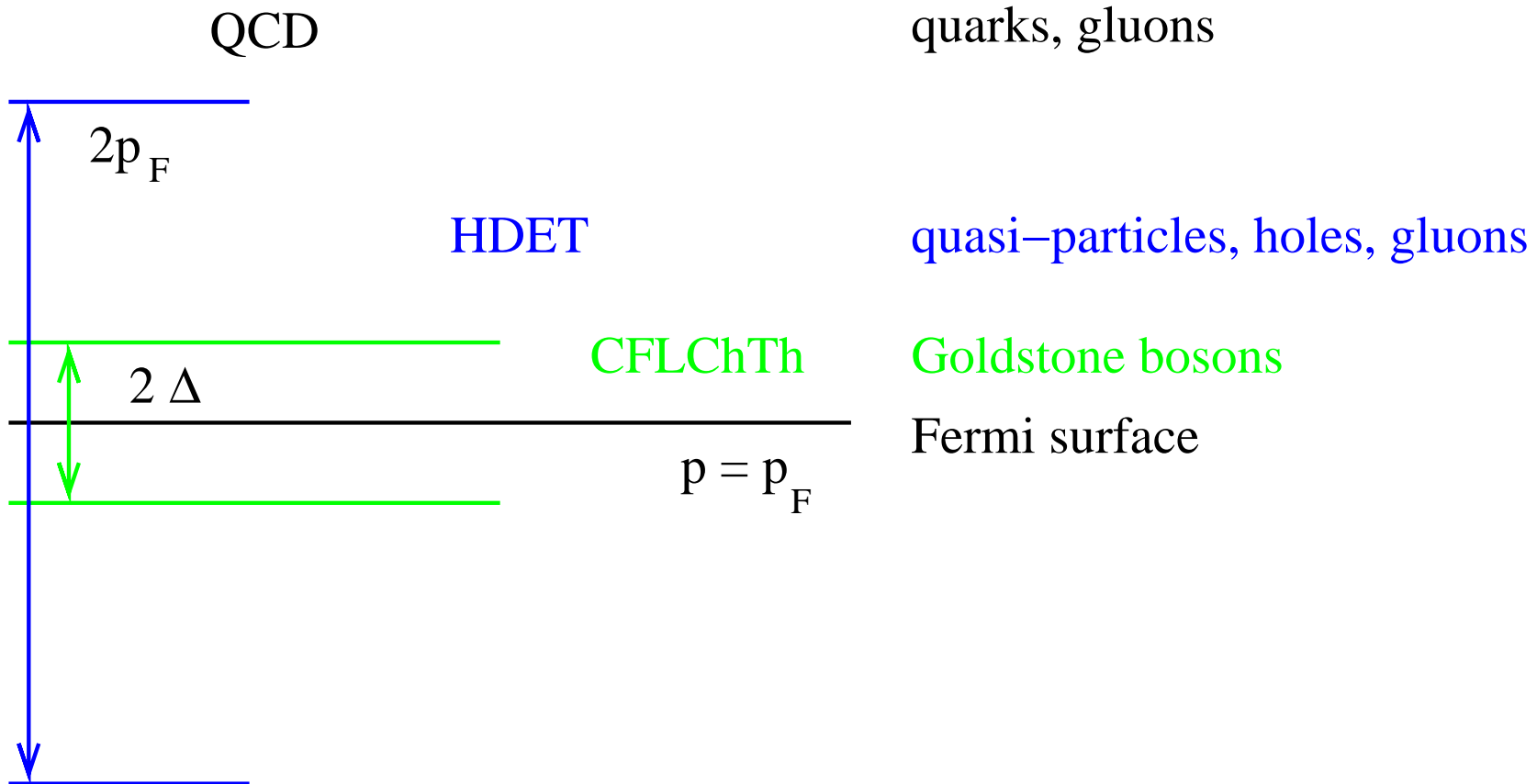
FLT theory: Quasi-particles near the Fermi surface. Interactions characterized by Fermi-liquid parameters.

Predicts collective modes, thermodynamics, transport, ...

Gauge Theories: Unscreened long range forces

Does a quasi-particle EFT exist?

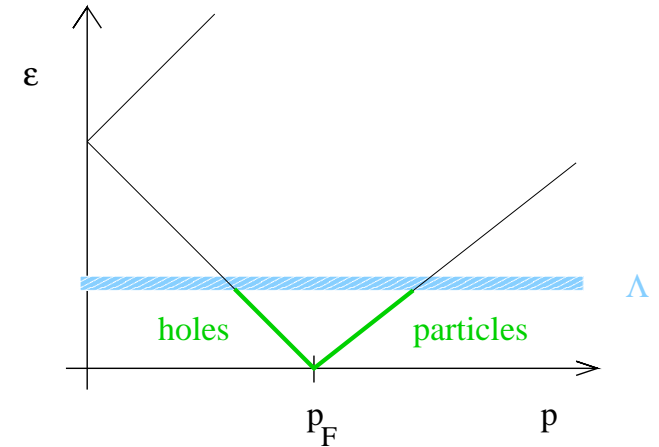
# Effective Field Theories



# High Density Effective Theory

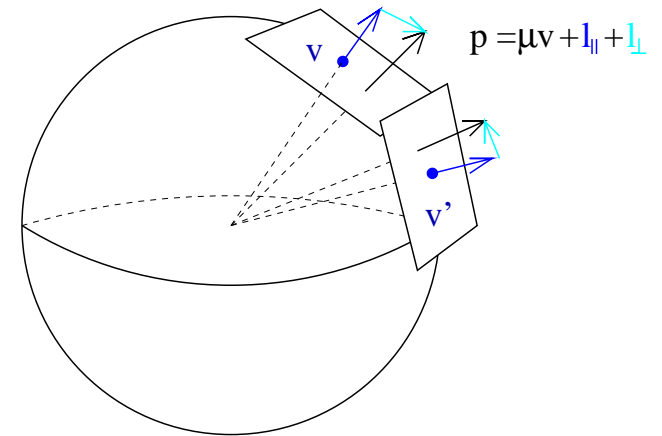
Quasi-particles (holes)

$$E_{\pm} = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$



Effective field theory on  $v$ -patches

$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left( \frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$



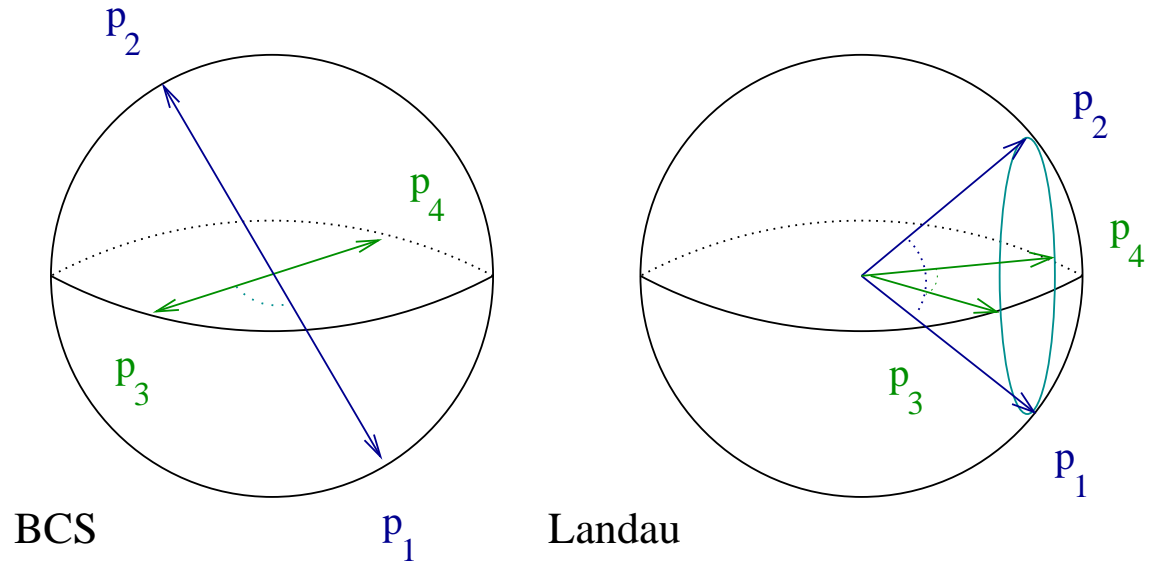
Effective lagrangian for  $\psi_{v+}$

$$\mathcal{L} = \sum_v \psi_v^\dagger (i v \cdot D) \psi_v - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + O(1/\mu)$$

# Four Quark Operators

quark-quark scattering

$$(v_1, v_2) \rightarrow (v_3, v_4)$$



$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'} (\vec{v} \cdot \vec{v}') (\psi_v \Gamma \psi_{-v}) (\psi_{v'}^\dagger \Gamma' \psi_{-v'}^\dagger),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'} (\vec{v} \cdot \vec{v}') (\psi_v \Gamma \psi_{v'}) (\psi_{\tilde{v}}^\dagger \Gamma' \psi_{\tilde{v}'}^\dagger)$$

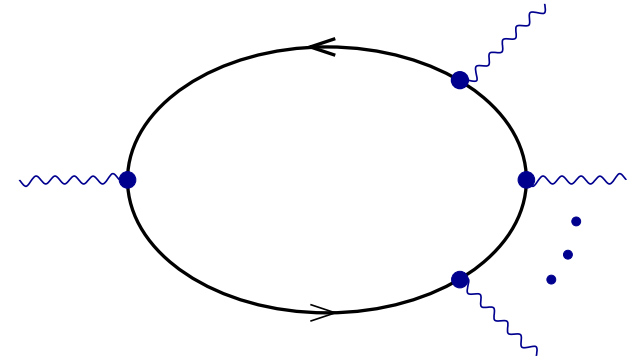
perturbative matching:  $f_0^s = \frac{C_F}{4N_c N_f} \frac{g^2}{p_F^2}, \quad f_i^s = 0 \quad (i > 1)$

# Effective Theory for $l \sim g\mu$

Integrate out hard dense loops

$$\mathcal{L} = \psi_v^\dagger (iv \cdot D) \psi_v + f_0^s (\psi_v^\dagger \psi_v) (\psi_{v'}^\dagger \psi_{v'}) - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b$$



Transverse gauge boson propagator

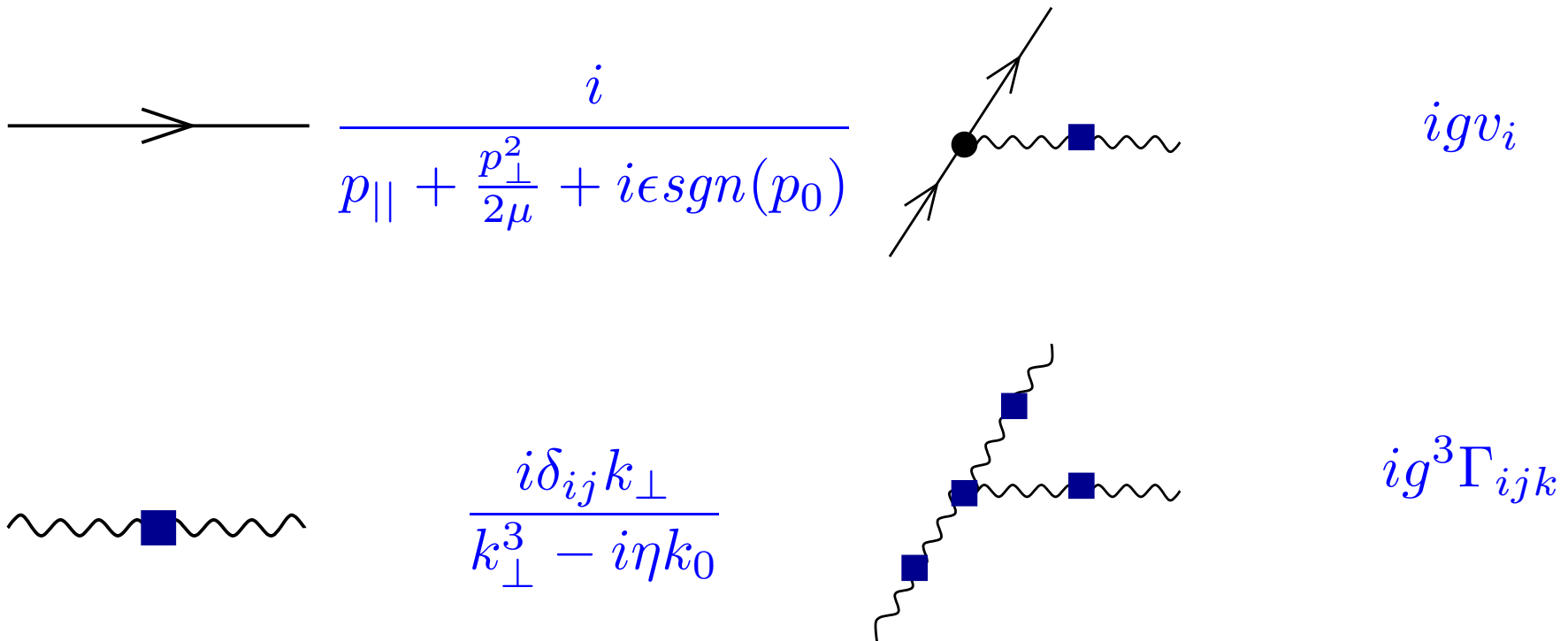
$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0| / |\vec{k}|},$$

Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} \eta^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

## Effective Theory for $l < g\mu$

Scaling:  $(p_0, p_{||}, p_{\perp}) \sim (l, l^{1/3}, l^{2/3})$

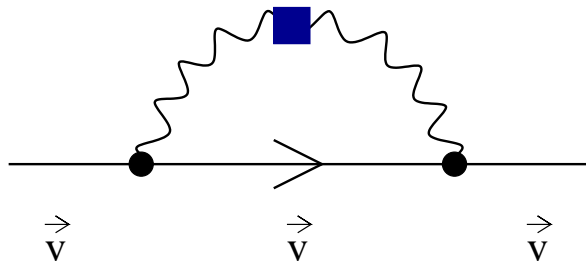


Low energy expansion contains fractional powers  $\omega^{1/3}$

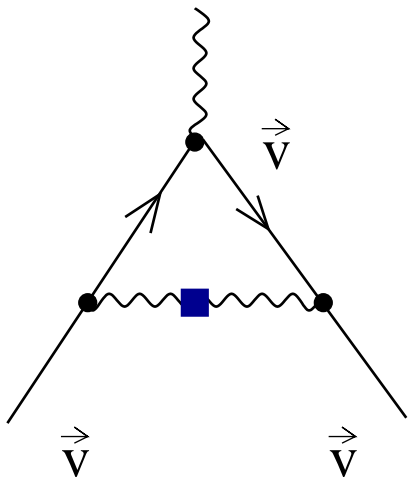
Quasi-Quarks at Large Density:  
Non-Fermi Liquid Effects



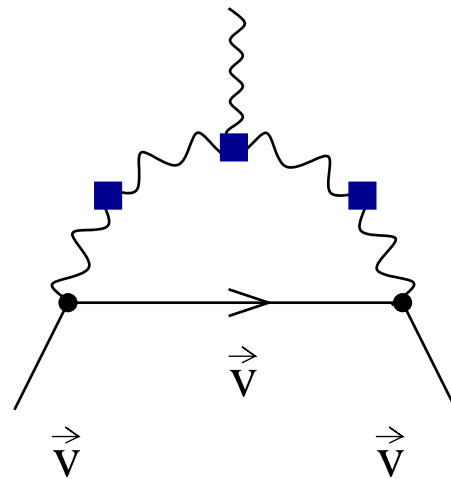
# Loop Corrections



$$\Sigma(\omega) \simeq \frac{g^2 C_F}{12\pi^2} \omega \log\left(\frac{\Lambda}{\omega}\right)$$



$$\Gamma_\alpha = \frac{g^3 C_F v_\alpha}{12\pi^2} \log\left(\frac{\Lambda}{\omega}\right) \quad \text{time like}$$



$$\Gamma_\alpha = O(g^3)$$

space like

“Migdal’s Theorem” for QCD

self energy non-perturbative for  $\omega \sim \Lambda \exp(-9\pi^2/g^2)$  ?

$\gamma = (4\alpha/)(9\pi) \ll 1$ : RG equation can be solved exactly

$$S^{-1}(\omega, l) = \omega \left( 1 + \gamma \log \left( \frac{\Lambda}{\omega} \right) \right) - v_F l \quad \text{no } \alpha^2 \log^2(\omega), \dots \text{ terms}$$

quasi-particle velocity vanishes as

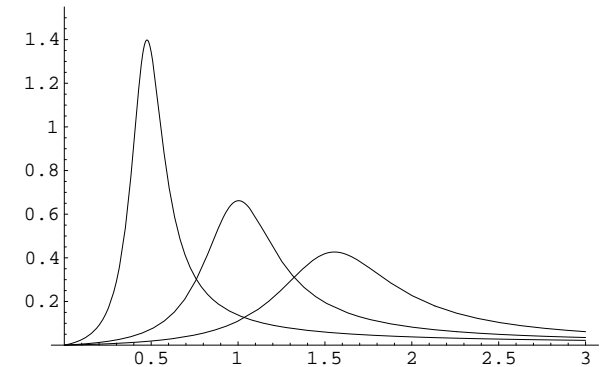
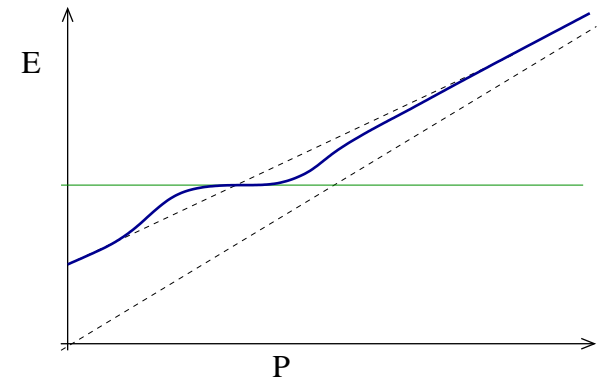
$$v \sim \log(\Lambda/\omega)^{-1}$$

anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$

enhanced corrections to the gap

$$\log(\mu/\Delta) = \log(\mu/\Delta_0)(1 - O(\gamma g))$$



## Remnants of Fermi Liquid Behavior

Perturbative corrections to matching QCD→HDET

$$v_F = 1 - \frac{C_F \alpha_s}{2\pi}, \quad \delta\mu = \frac{C_F \alpha_s}{\pi} \mu.$$

These coefficients satisfy Fermi-liquid relations (Baym&Chin)

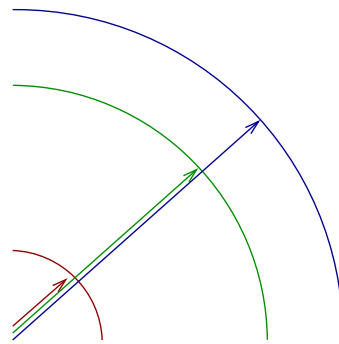
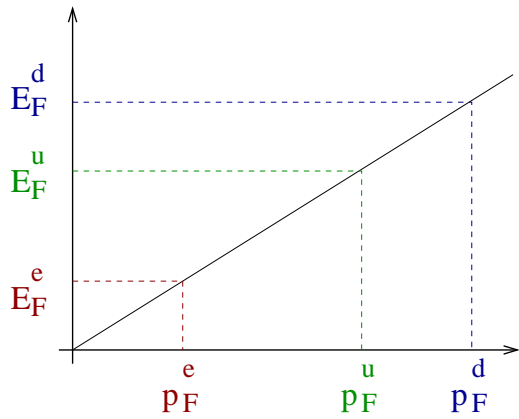
$$v_F = \frac{p_F}{\mu} - \frac{N f_1^s}{3} \quad p_F = \mu \left( 1 - \frac{N f_0^s}{2} \right)$$

Fermi-liquid parameters

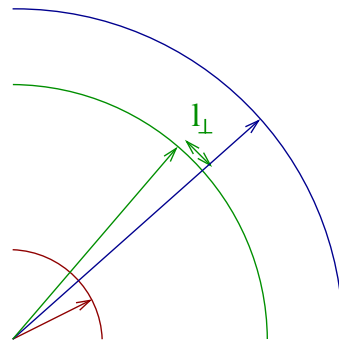
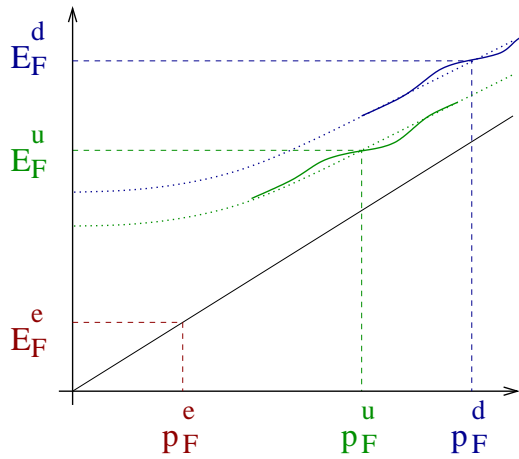
$$f_0^s = \frac{C_F}{4N_c N_f} \frac{g^2}{p_F^2} \quad f_1^s = 0$$

# Application: Neutrino Emission

Quark Direct URCA:  $d \rightarrow u + e^- + \bar{\nu}$ ,  $u + e^- \rightarrow d + \bar{\nu}$



$$\epsilon \sim G_F T^7$$



$$\epsilon \sim G_F \alpha_s^3 T^6 \log^2(T)$$

Quasi-Baryons at Large Density:

CFL Phase

# CFL Phase

Consider  $N_f = 3$  ( $m_i = 0$ )

$$\langle q_i^a q_j^b \rangle = \phi \epsilon^{abI} \epsilon_{ijI}$$

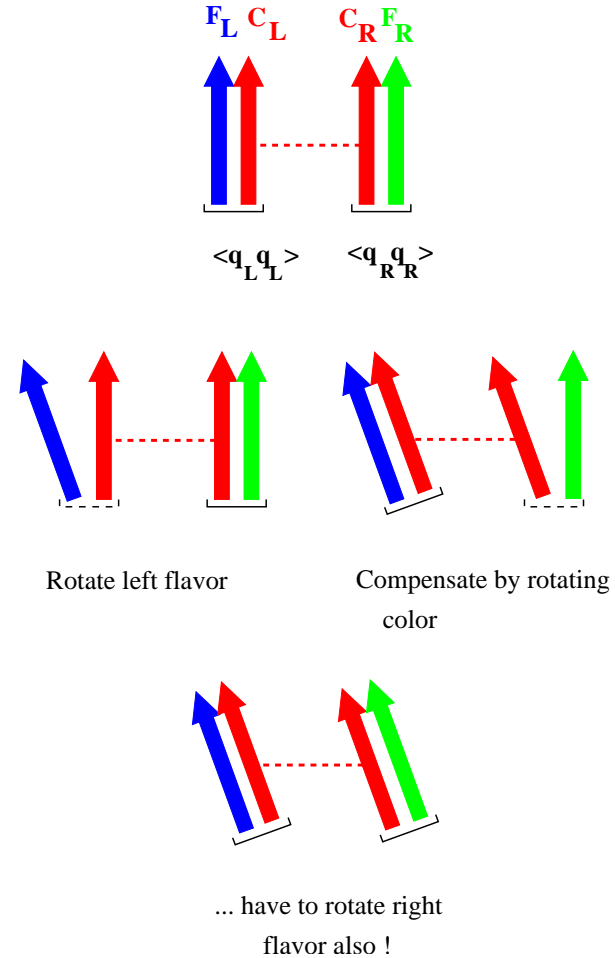
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \rightarrow SU(3)_{C+F}$$

All quarks and gluons acquire a gap



$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

## EFT in the CFL Phase

Consider HDET with a CFL gap term

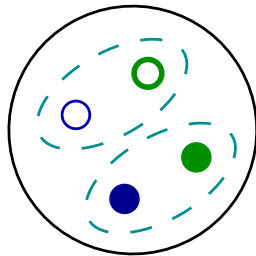
$$\mathcal{L} = \text{Tr} \left( \psi_L^\dagger (i v \cdot D) \psi_L \right) + \frac{\Delta}{2} \left\{ \text{Tr} (X^\dagger \psi_L X^\dagger \psi_L) - \kappa [\text{Tr} (X^\dagger \psi_L)]^2 \right\} \\ + (L \leftrightarrow R, X \leftrightarrow Y)$$

$$\psi_L \rightarrow L \psi_L C^T, \quad X \rightarrow L X C^T, \quad \langle X \rangle = \langle Y \rangle = \mathbb{1}$$

Quark loops generate a kinetic term for  $X, Y$

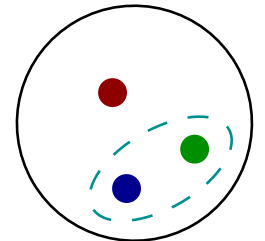
Integrate out gluons, identify low energy fields ( $\xi = \Sigma^{1/2}$ )

$$\Sigma = X Y^\dagger$$



[8]+[1] GBs

$$N_L = \xi (\psi_L X^\dagger) \xi^\dagger$$



[8]+[1] Baryons

Effective theory: (CFL) baryon chiral perturbation theory

$$\begin{aligned}
 \mathcal{L} = & \frac{f_\pi^2}{4} \left\{ \text{Tr} (\nabla_0 \Sigma \nabla_0 \Sigma^\dagger) - v_\pi^2 \text{Tr} (\nabla_i \Sigma \nabla_i \Sigma^\dagger) \right\} \\
 & + \text{Tr} (N^\dagger i v^\mu D_\mu N) - D \text{Tr} (N^\dagger v^\mu \gamma_5 \{ \mathcal{A}_\mu, N \}) \\
 & - F \text{Tr} (N^\dagger v^\mu \gamma_5 [ \mathcal{A}_\mu, N ]) + \frac{\Delta}{2} \left\{ \text{Tr} (N N) - [\text{Tr} (N)]^2 \right\}
 \end{aligned}$$

with  $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$\mathcal{V}_\mu = -\frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi)$$

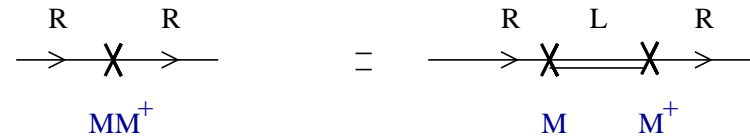
$$\mathcal{A}_\mu = -\frac{i}{2} \xi (\partial_\mu \Sigma^\dagger) \xi$$

$$f_\pi^2 = \frac{21 - 8 \log 2}{18} \frac{\mu^2}{2\pi^2} \quad v_\pi^2 = \frac{1}{3} \quad D = F = \frac{1}{2}$$

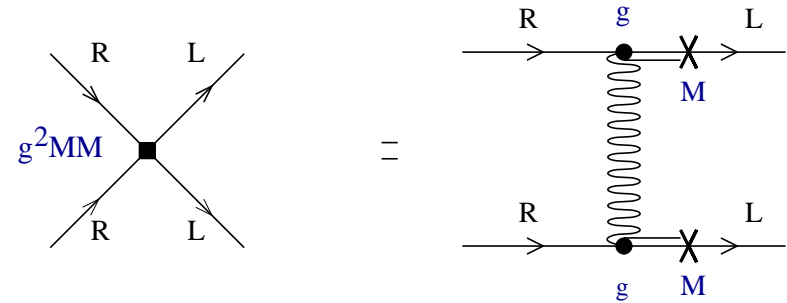


# Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L$$



$$+ \frac{C}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L) (\psi_R^\dagger M \lambda^a \psi_L)$$



mass corrections to FL parameters  $\hat{\mu}$  and  $F^0(++ \rightarrow --)$

## Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \text{Tr} (X_L \Sigma X_R \Sigma^\dagger) - A \text{Tr}(M \Sigma^\dagger) - B_1 [\text{Tr}(M \Sigma^\dagger)]^2 + \dots$$

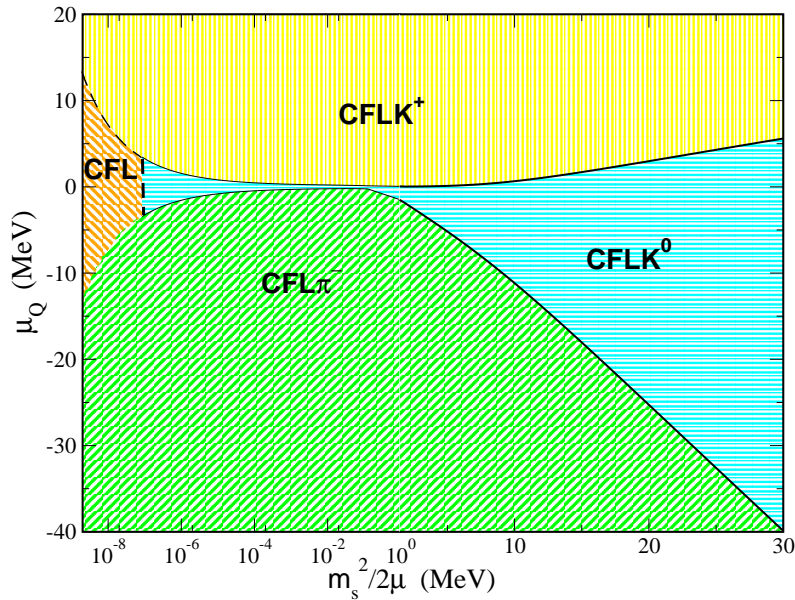
$$V(\Sigma_0) \equiv \text{min}$$

Fermion spectrum determined by

$$\mathcal{L} = \text{Tr} (N^\dagger i v^\mu D_\mu N) + \text{Tr} (N^\dagger \gamma_5 \rho_A N) + \frac{\Delta}{2} \left\{ \text{Tr} (N N) - [\text{Tr} (N)]^2 \right\},$$

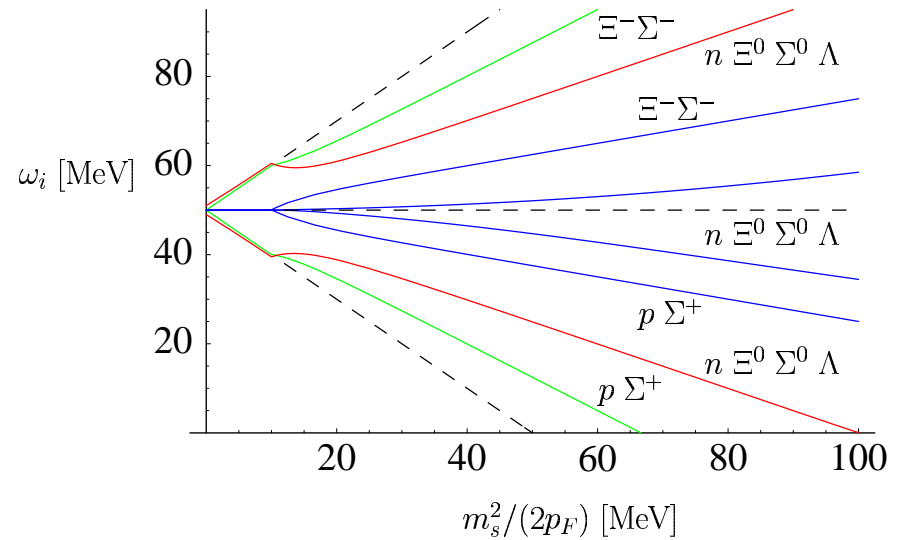
$$\rho_{V,A} = \frac{1}{2} \left\{ \xi \frac{M^\dagger M}{2p_F} \xi^\dagger \pm \xi^\dagger \frac{M M^\dagger}{2p_F} \xi \right\} \quad \xi = \sqrt{\Sigma_0}$$

# Phase Structure and Spectrum



meson condensation: CFLK

reliable: yes!



gapless modes? (gCFLK)

reliable: not clear yet

## Summary

EFT/RG methods provide powerful tools

phase structure and spectrum at large density

Normal phase: Non-Fermi liquid behavior due to unscreened transverse gauge bosons

modified quasi-particles

but: no break-down of perturbation theory

Superfluid phase: effective chiral theory with calculable coefficients

kaon condensation, possibility of gapless modes