Hard-loop Evolution of Nonabelian Plasma Instabilities

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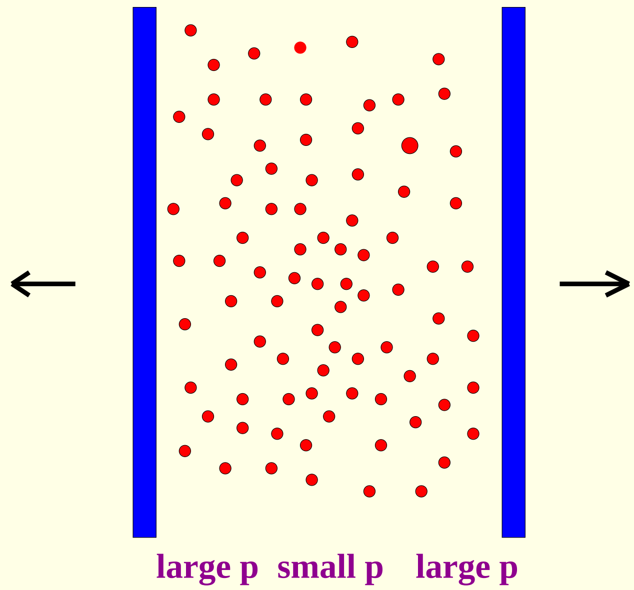
Motivation

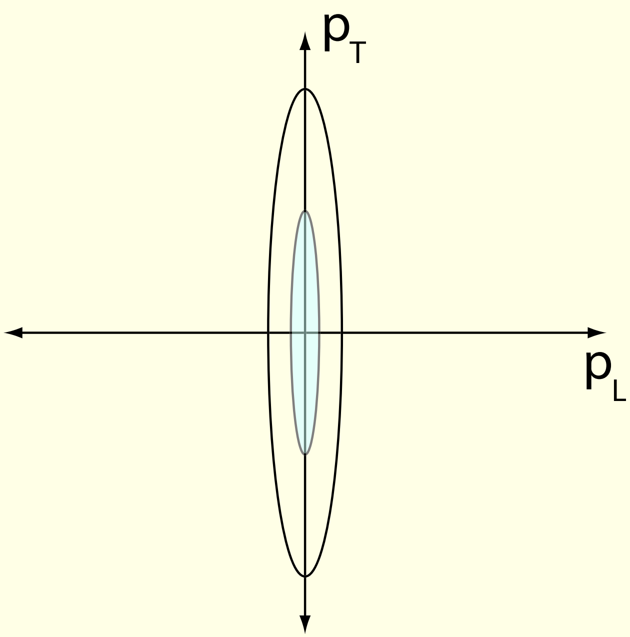
- Success of hydro models seems to imply fast isotropization (and possibly thermalization) of matter created at RHIC.
- Previous ”naive” perturbative estimates don’t seem to explain this.
- One possibility is that perturbation theory should be thrown out the window and replaced by a new calculational framework.
- However, the perturbative estimates to date have completely overlooked an important aspect of the physics, namely that in anisotropic plasmas the **collective modes** (aka mean field dynamics) are fundamentally different than in isotropic ones.
- This results in something previously not considered: the spontaneous generation of large background color fields which can cause large angle scattering of particles.

Why anisotropic distribution functions?

Because of the natural expansion of the system the gluon distribution functions created during relativistic heavy ion collisions are *generically* locally anisotropic in momentum space.

$$
\begin{aligned}
<p_T> &\sim Q_s \quad \text{(nuclear ``saturation scale'')} \\
<p_L> &\sim 1/\tau \quad \text{(including collisions} \rightarrow \sim Q_s^{2/3}/\tau^{1/3})
\end{aligned}
$$





Collective Modes of an Isotropic QGP

The isotropic hard-thermal-loop (HTL) gluon propagator is given by

$$\Delta^{ij} = (k^2 - \omega^2 + \Pi_T)^{-1}(\delta_{ij} - k^i k^j/k^2) - \frac{k^2}{\omega^2}(k^2 - \Pi_L)^{-1} k^i k^j/k^2$$

with

$$\Pi_T(\omega,k) = \frac{m_D^2}{2}\frac{\omega^2}{k^2}\left[1 - \frac{\omega^2 - k^2}{2\omega k}\log\frac{\omega+k}{\omega-k}\right],$$

$$\Pi_L(\omega,k) = m_D^2\left[\frac{\omega}{2k}\log\frac{\omega+k}{\omega-k} - 1\right],$$

and $m_D \propto gT$.

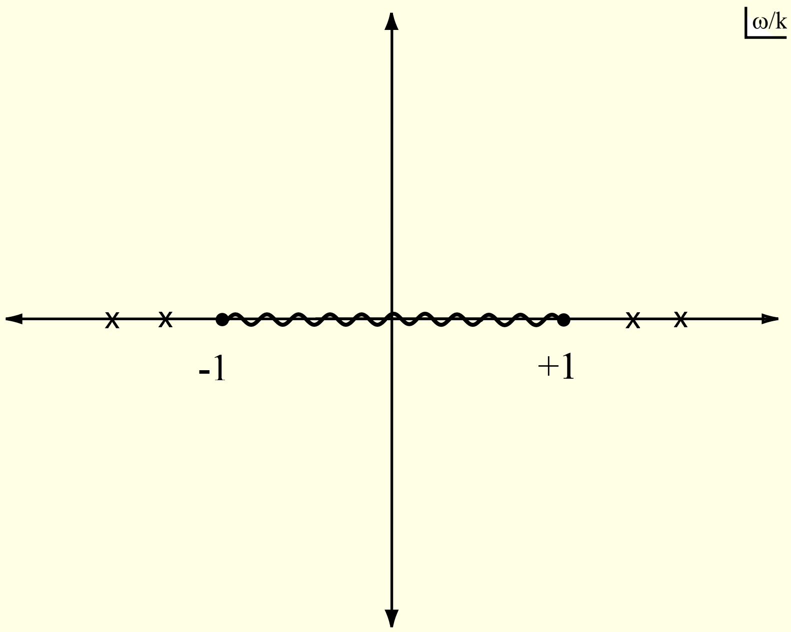
$$\lim_{\omega\to 0}\Pi_L(\omega,k) = m_D^2 \quad \text{electric screening}$$

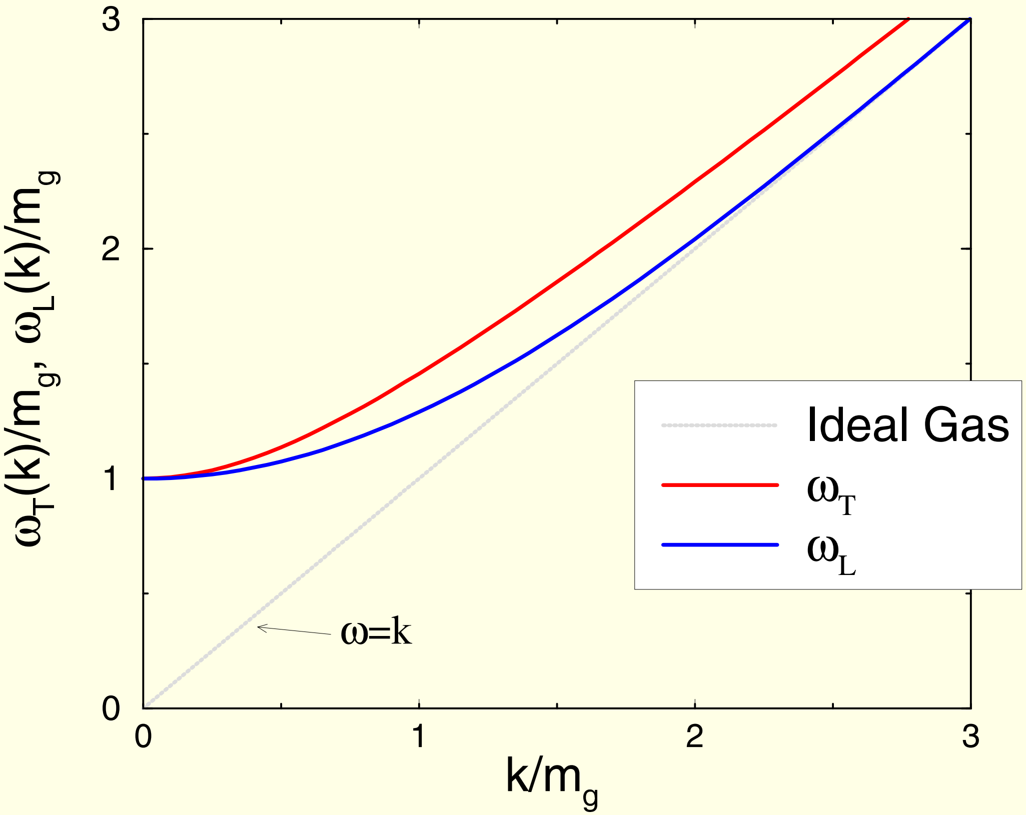
$$\lim_{\omega\to 0}\Pi_T(\omega,k) = 0 \quad \textbf{no}\ \text{magnetic screening}$$

Collective Modes of an Isotropic QGP

In the isotropic case the only poles are at real timelike ($\omega > k$) momentum. In order to determine the dispersion relations for these excitations we can then explicitly look for the poles in the propagator.

$$
\begin{aligned}
0 &= k^2 - \omega_T^2 + \Pi_T(\omega_T, k) \\
0 &= k^2 - \Pi_L(\omega_L, k)
\end{aligned}
$$





Anisotropic Gluon Polarization Tensor

In order to determine the HL gluon self-energy in an anisotropic system we can use either three-dimensional kinetic theory or diagrammatic techniques.[4,5] The result is

$$\Pi^{ij}(K) = -g^2 \int \frac{d^3p}{(2\pi)^3} v^i \partial^l f(\mathbf{p}) \left(\delta_{jl} - \frac{v_j k_l}{K \cdot V + i\epsilon} \right) \, .$$

S. Mrówczyński first pointed out that within anisotropic QCD plasmas there are unstable modes which are the equivalent of QED Weibel type instabilities.[6,7,8]

[4] H. Elze and U. Heinz, 89; J. Blaizot and E. Iancu, 94.

[5] S. Mrówczyński, 93; S. Mrówczyński and M. Thoma, 00.

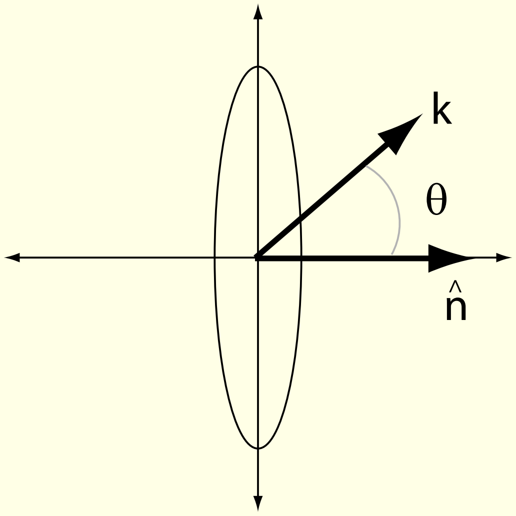
[6] E. Weibel, 59.

[7] P. Romatschke and MS, 03.

[8] P. Arnold, J. Lenaghan, and G. Moore, 03.

The nature of the anisotropy

We assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument.



$$f(p^2) \rightarrow \sqrt{1+\xi}\, f(p^2 + \xi(p\cdot n)^2)\ .$$

The polarization tensor can then be written as

$$\Pi^{ij}(K) = m_D^2 \sqrt{1+\xi} \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(v\cdot n)n^l}{(1+\xi(v\cdot n)^2)^2} \left(\delta^{jl} - \frac{v^j k^l}{K\cdot V + i\epsilon}\right)\ ,$$

where m_D is the *isotropic* Debye mass

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\, p^2 \frac{df(p^2)}{dp}\ .$$

Tensor basis

We can construct a symmetric 3d tensor basis with the following four tensors

$$A^{ij} = \delta^{ij} - k^i k^j / k^2 \qquad B^{ij} = k^i k^j / k^2 \ ,$$
$$C^{ij} = \tilde{n}^i \tilde{n}^j / \tilde{n}^2 \qquad D^{ij} = k^i \tilde{n}^j + \tilde{n}^i k^j \ ,$$

where $\tilde{n} \equiv n^i A^{ij}$. We can then decompose the propagator and gluon polarization tensor in this tensor basis.

$$\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij} \ ,$$

where

$$\begin{aligned} k^i \Pi^{ij} k^j &= k^2 \beta \ , \\ \tilde{n}^i \Pi^{ij} k^j &= \tilde{n}^2 k^2 \delta \ , \\ \tilde{n}^i \Pi^{ij} \tilde{n}^j &= \tilde{n}^2 (\alpha + \gamma) \ , \\ \mathrm{Tr}\Pi^{ij} &= 2\alpha + \beta + \gamma \ . \end{aligned}$$

Anisotropic Propagator and Static Limit

This allows us to express the propagator in terms of three functions

$$\Delta_A^{-1}(K) = k^2 - \omega^2 + \alpha \,,$$
$$\Delta_\pm^{-1}(K) = \omega^2 - \Omega_\pm^2 \,,$$

where

$$2\Omega_\pm^2 = \bar{\Omega}^2 \pm \sqrt{\bar{\Omega}^4 - 4((\alpha+\gamma+k^2)\beta - k^2\tilde{n}^2\delta^2)} \,,$$

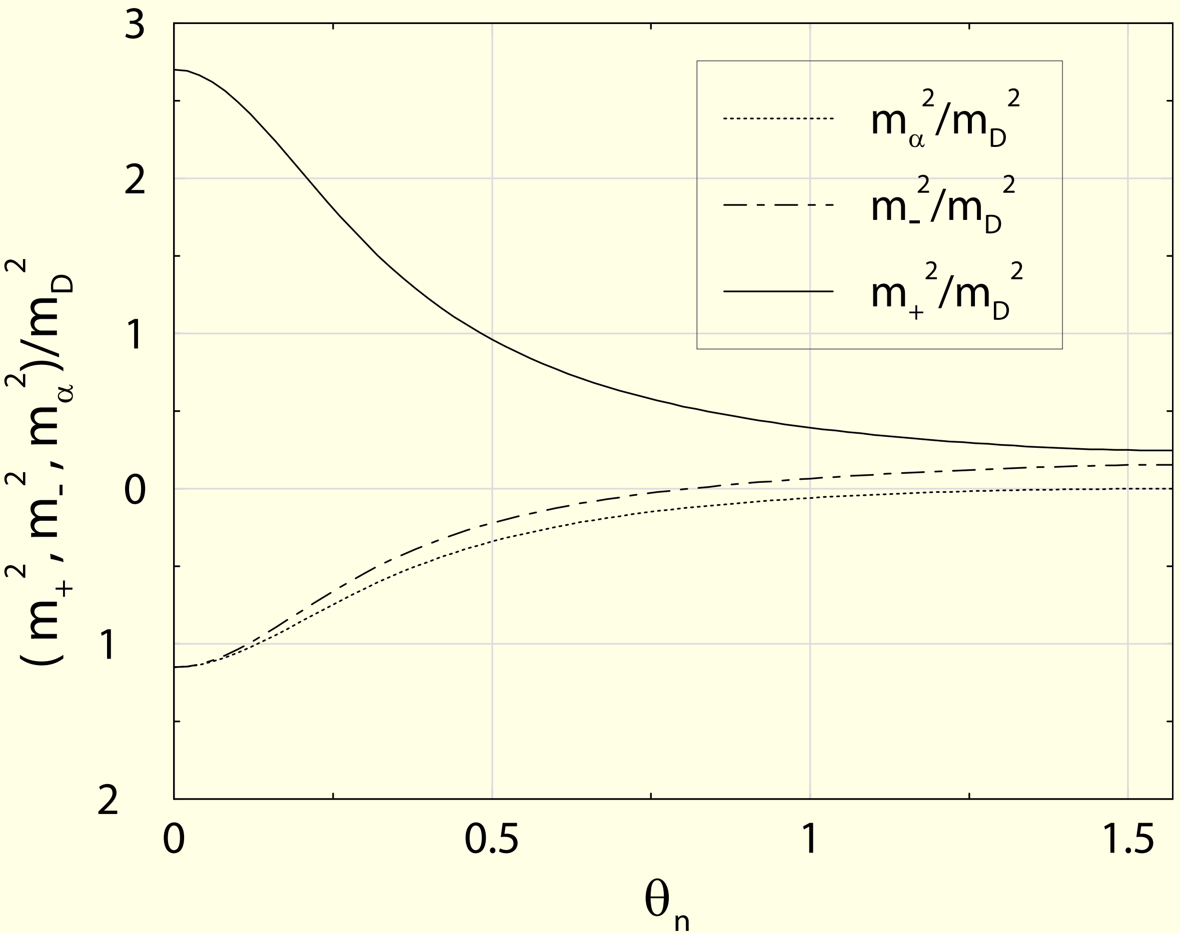
and $\bar{\Omega}^2 = \alpha + \beta + \gamma + k^2$.

Taking the static limit we can define three mass scales: $m_\pm$ and m_α.

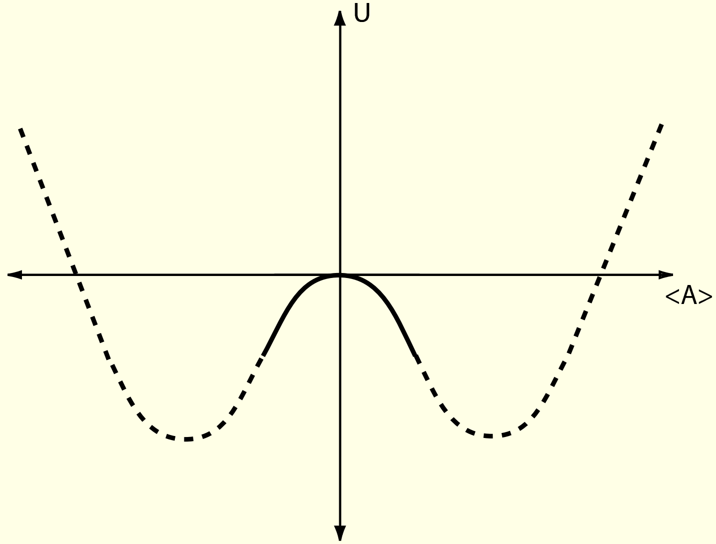
In the isotropic limit, $\xi \to 0$, $m_\alpha^2 = m_-^2 = 0$ and $m_+^2 = m_D^2$ and for finite ξ it is possible to evaluate these masses analytically.[9]

[9] P. Romatschke and MS, 03.

New Mass Scales – $\xi > 0$

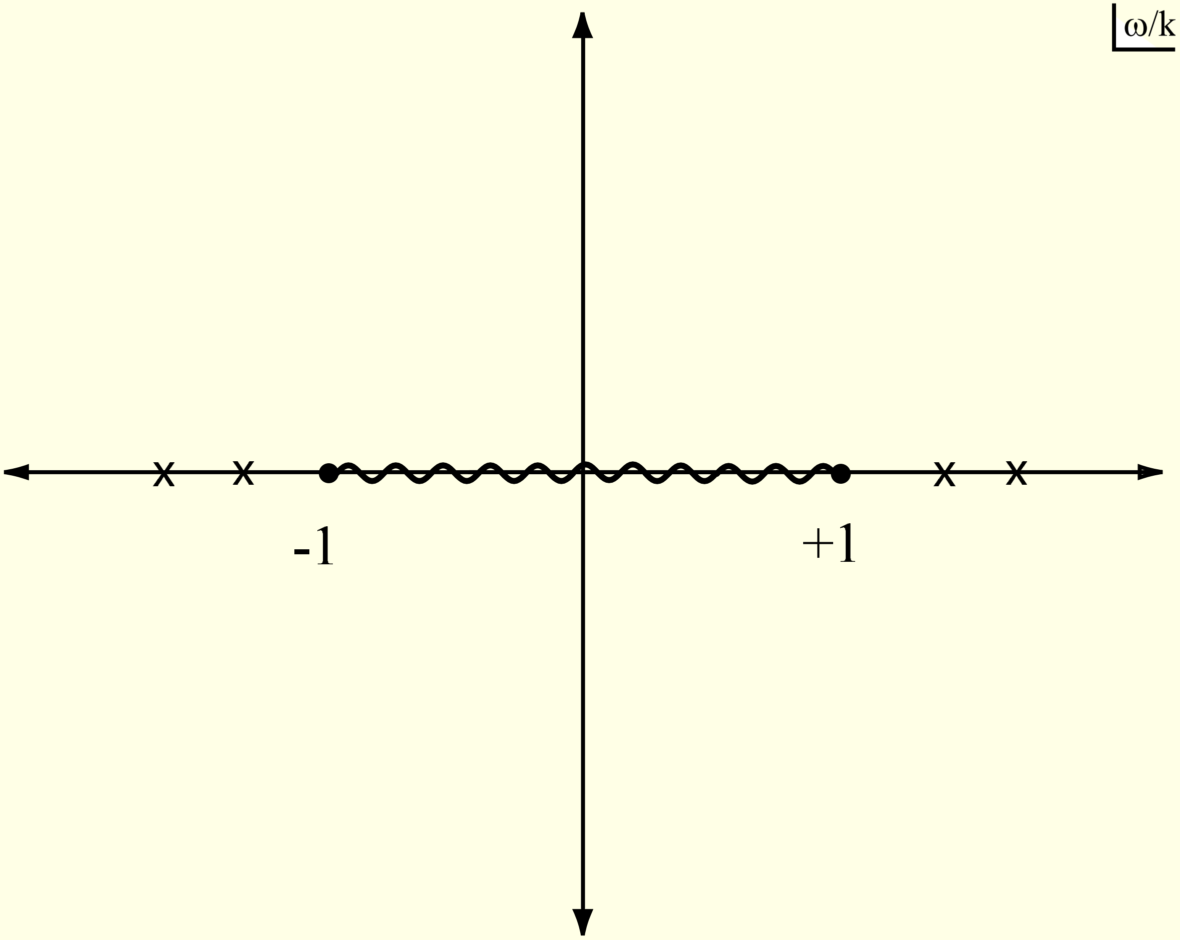


Angular dependence of m_α^2, m_+^2, and m_-^2 at fixed $\xi = 10$.



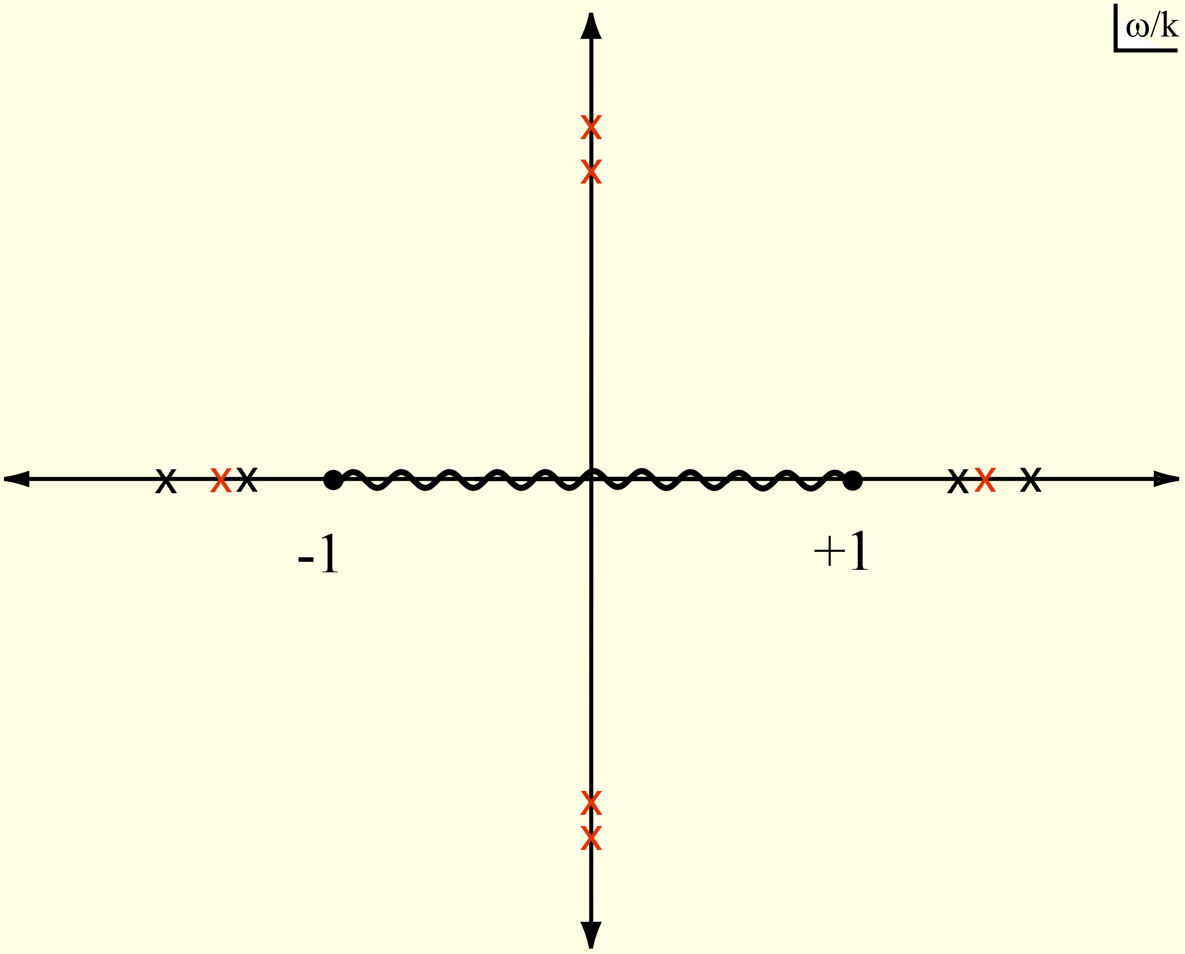
Sketch of the effective potential of an unstable mode.

Isotropic Collective Modes



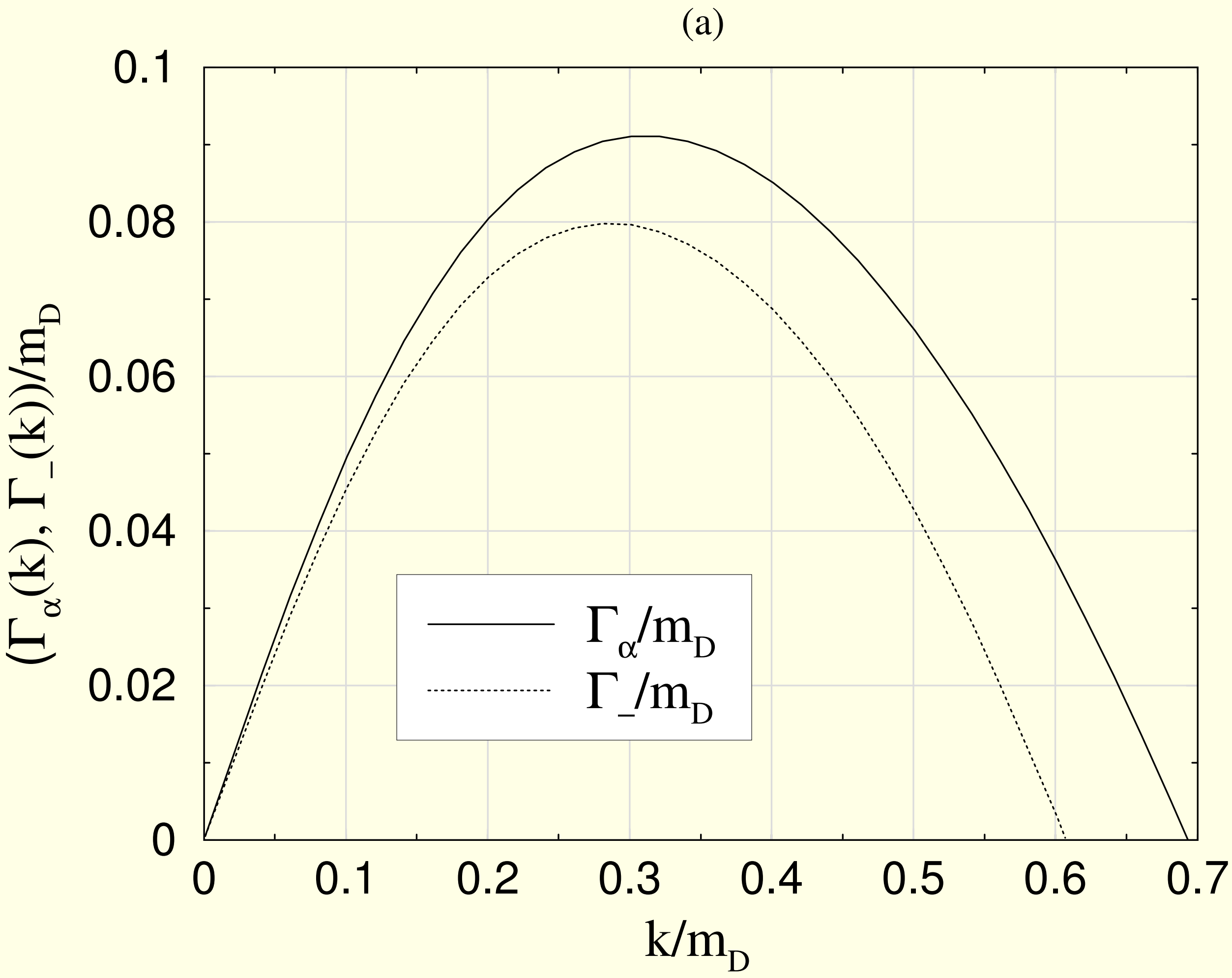
Isotropic poles ($\xi = 0$).

Anisotropic Collective Modes



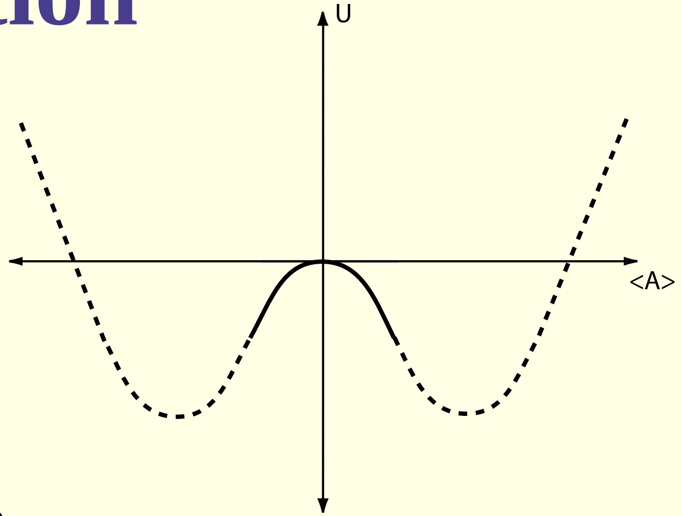
Anisotropic poles at positive ξ.

Unstable Modes – $\xi > 0$



$\Gamma_\alpha(k)$ and $\Gamma_-(k)$ as a function of k with $\xi = 10$ and $\theta_n = \pi/8$.

Anisotropic HL Effective Action



Using the requirement of gauge invariance it is possible to determine all n-point functions in the same way as in the isotropic case.[10]

$$S_{\rm HL} = -\frac{g^2}{2}\int_x \int_{\mathbf{p}} \left\{ f(\mathbf{p}) F^a_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_\rho{}^{b\,\mu}(x) + i\frac{C_F}{2}\tilde{f}(\mathbf{p})\bar{\Psi}(x)\frac{p\cdot\gamma}{p\cdot D}\Psi(x) \right\} .$$

For example, from this we can obtain the anisotropic 3-gluon vertex

$$\Gamma^{\mu\nu\lambda}(k,q,r) = \frac{g^2}{2}\int_{\mathbf{p}} \frac{\partial f(\mathbf{p})}{\partial p^\beta}\, \hat{p}^\mu \hat{p}^\nu \hat{p}^\lambda \left(\frac{r^\beta}{\hat{p}\cdot q\ \hat{p}\cdot r} - \frac{k^\beta}{\hat{p}\cdot k\ \hat{p}\cdot q} \right) .$$

[10] S. Mrówczyński, A. Rebhan, and MS, 04.

Real-Time Lattice Simulation

For QCD the presence of non-linear interactions complicates the analysis of the long-time behavior. The chief question is to what scale do the vector potentials grow.

Do they grow to the “hard scale”

$$A_{\rm hard} \sim p_{\rm hard}/g \sim Q_s/g$$

as in the case of QED or do they saturate at the parametrically smaller scale

$$A_{\rm soft} \sim p_{\rm soft}/g \sim Q_s$$

A. Rebhan, P. Romatschke, and MS, PRL 94, 102303 (2005); hep-ph/0412016.

Real-Time Lattice Simulation - contd.

In order to answer this we must numerically solve the equations of motion resulting from the HL effective action.

$$j^\mu[A] = -g^2 \int_{\mathbf{p}} \frac{1}{2|\mathbf{p}|}\, p^\mu \, \frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})$$

with

$$[v \cdot D(A)] W_\beta(x;\mathbf{v}) = F_{\beta\gamma}(A) v^\gamma$$

and $v^\mu = p^\mu/|\mathbf{p}| = (1,\mathbf{v})$.

This has to be solved with

$$D_\mu(A) F^{\mu\nu} = j^\nu$$

where $\nu = 0$ is the Gauss law constraint.

$\vec{v}$-discretized equations of motion

Assuming cylindrically symmetric anisotropies, we can parameterize them by $f(\mathbf{p}) = f(|\mathbf{p}|, p^z)$ allowing us express the current in terms of two auxiliary W fields

$$j^\mu = -\frac{1}{2} g^2 \int_{\mathbf{p}} v^\mu \left[f_1(|\mathbf{p}|, p^z) W^0(x, v) + f_2(|\mathbf{p}|, p^z) W^z(x, v) \right]$$

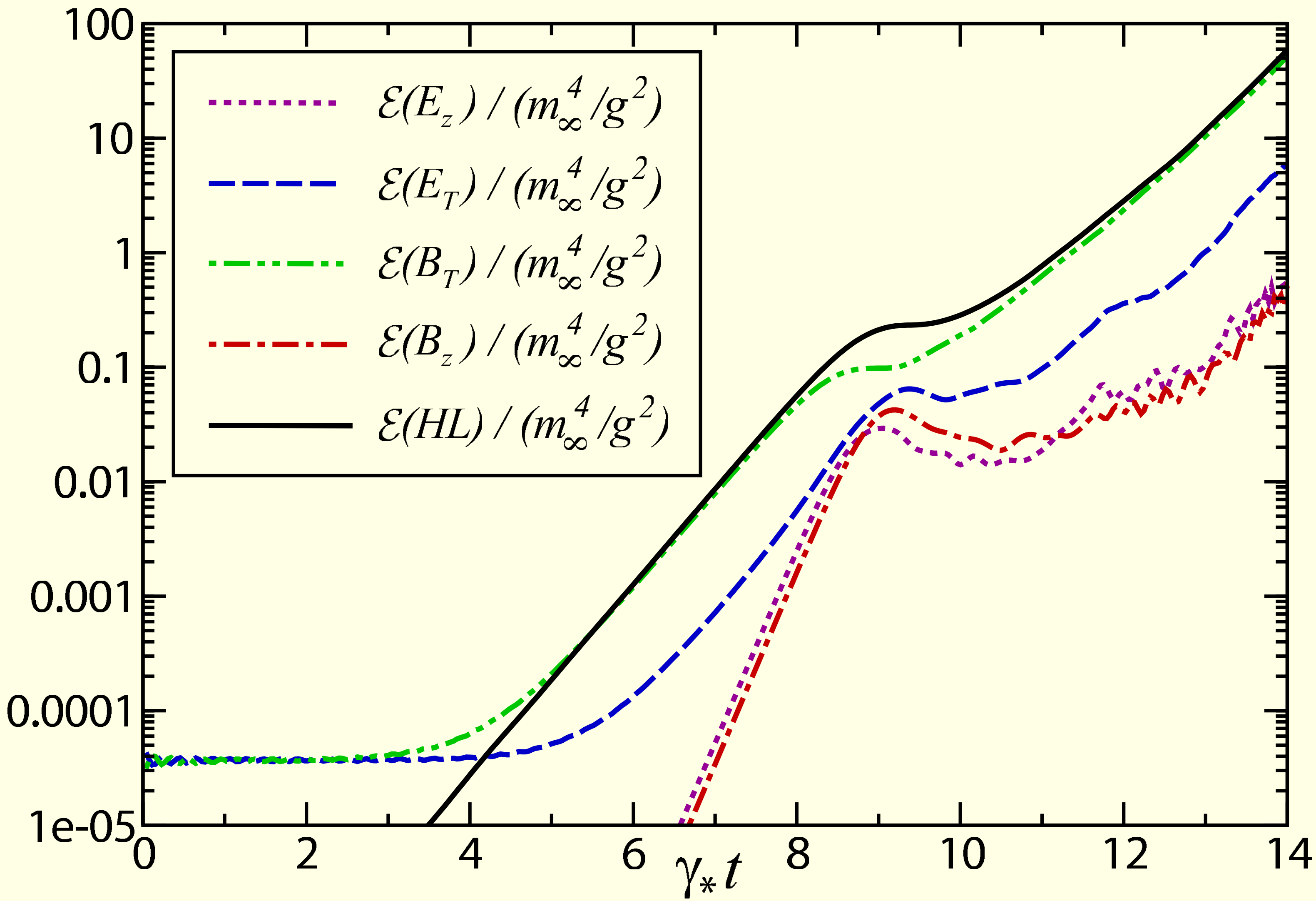
A closed set of gauge-covariant equations is obtained when the angular integral over $\mathbf{p}$ is discretized. The full HL dynamics is then approximated by the following set of equations

$$[v \cdot D(A)] \mathcal{W}_{\mathbf{v}} = (a_{\mathbf{v}} F^{0\mu} + b_{\mathbf{v}} F^{z\mu}) v_\mu$$

$$D_\mu(A) F^{\mu\nu} = j^\nu = \frac{1}{\mathcal{N}} \sum_{\mathbf{v}} v^\nu \mathcal{W}_{\mathbf{v}},$$

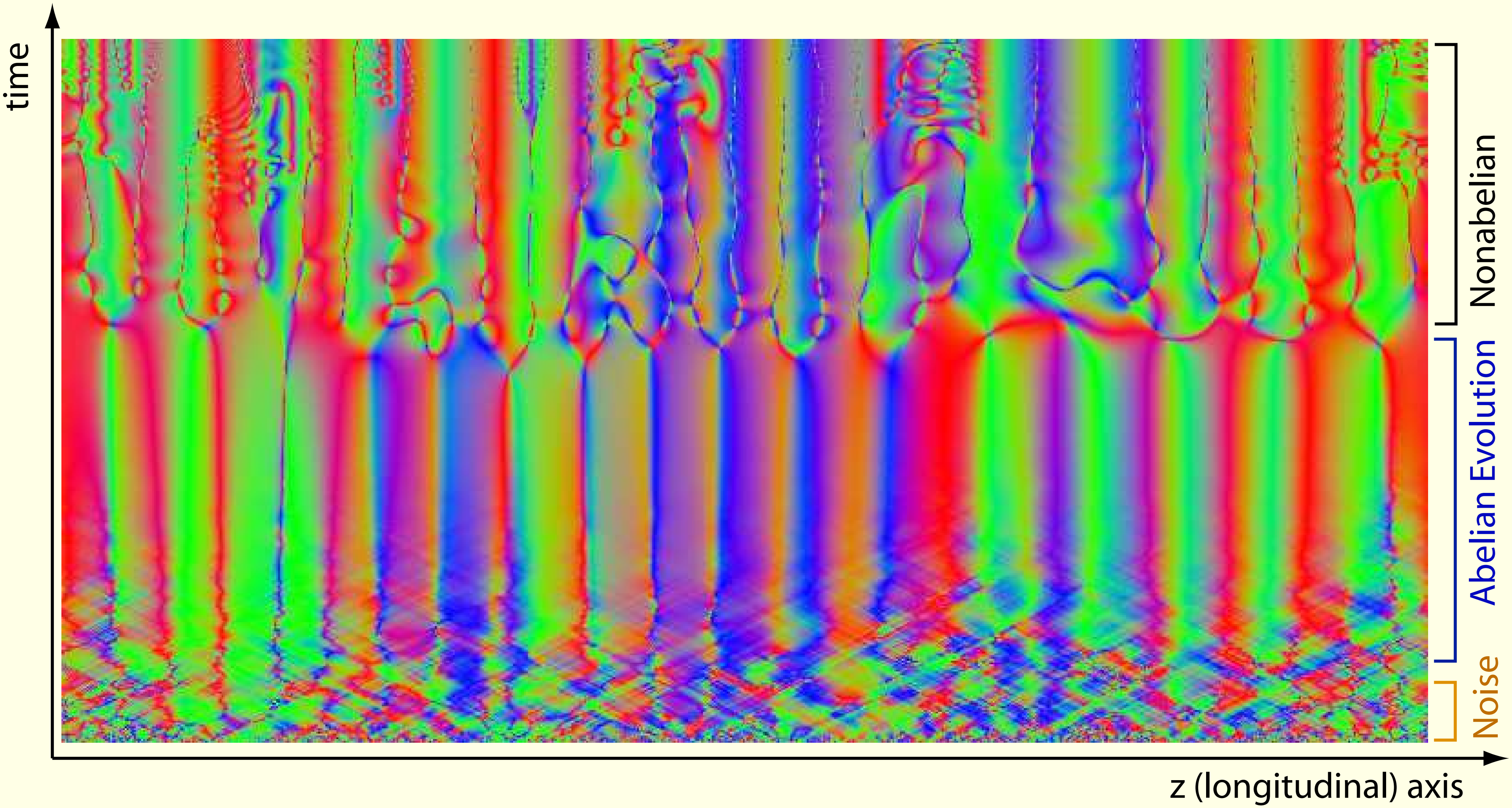
which can be systematically improved by increasing $\mathcal{N}$.

1s × 3v Lattice results

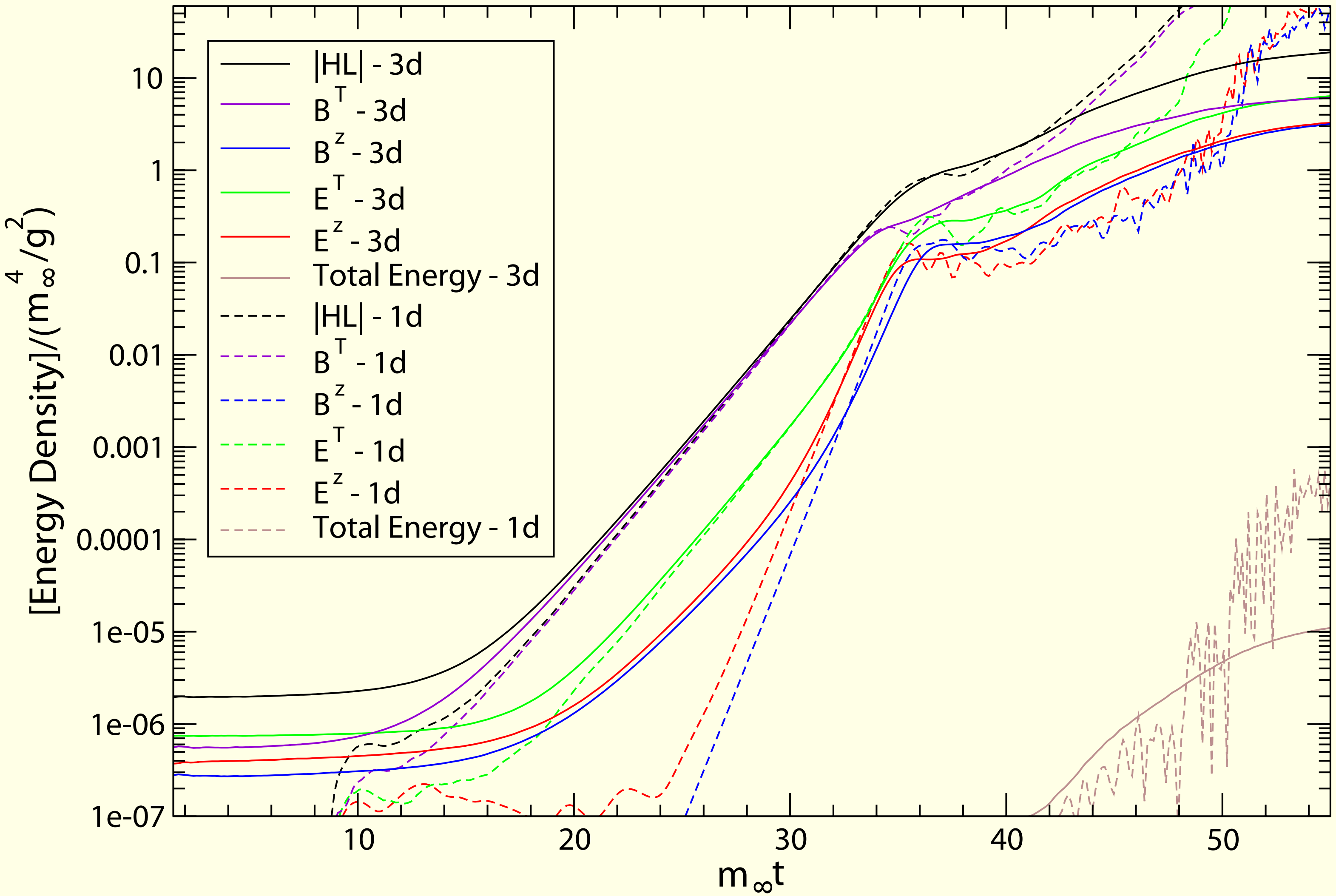


A. Rebhan, P. Romatschke, and MS, PRL 94, 102303 (2005); hep-ph/0412016.

J_x Visualization



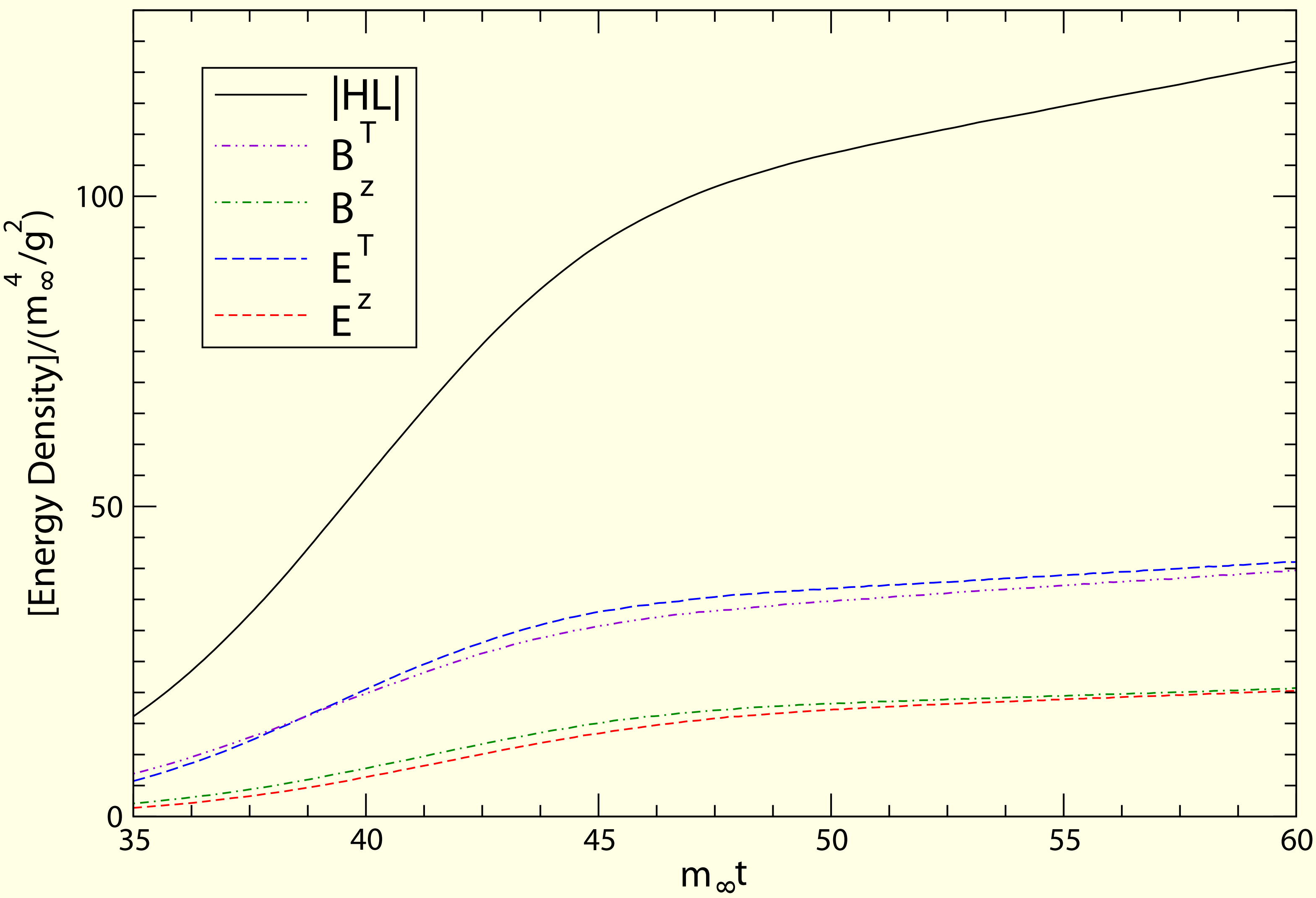
3s × 3v Lattice results



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261.

See also: P. Arnold, G. Moore, L. Yaffe, hep-ph/0505212.

3s × 3v Lattice results contd.



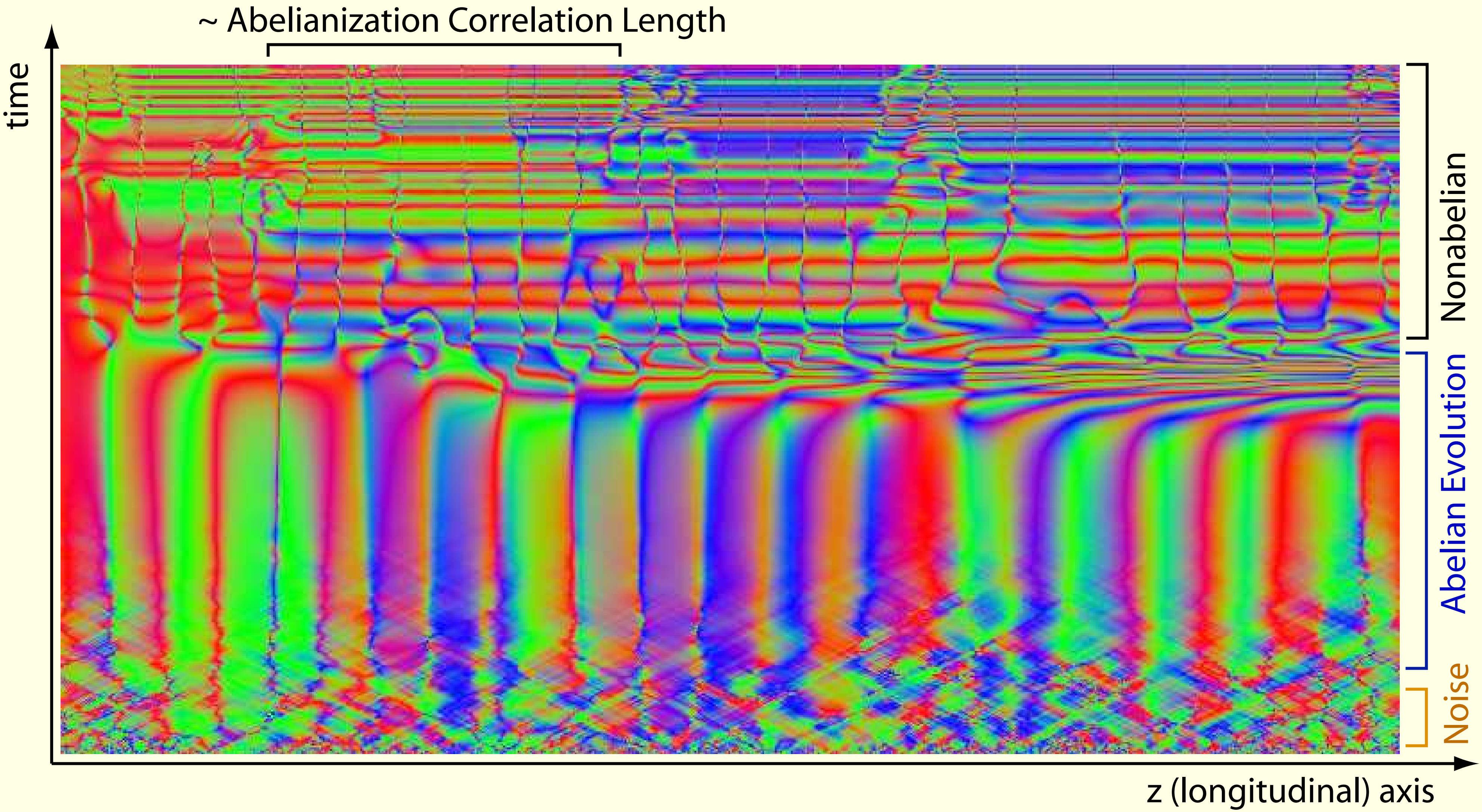
A. Rebhan, P. Romatschke, and MS, hep-ph/0505261.

See also: P. Arnold, G. Moore, L. Yaffe, hep-ph/0505212.

Conclusions

- Anisotropic plasmas are qualitatively different than isotropic ones. An entirely new phenomena associated with unstable modes appears.
- 1s$\times$3v lattice simulations soft unstable modes grow non-perturbatively large until the energy in the soft fields is on the order of the energy in the hard particles.
- 3s$\times$3v lattice simulations show however that soft unstable modes grow only to the “nonabelian” scale.
- Although this makes the method less efficient due to the reduced field amplitudes it means that the system stays within the bounds of the HL effective theory and therefore we can systematically study the effect of this soft field on the hard particles in a systematic way!
- In the next talk by Y. Nara we will hear about quite impressive attempts to include the backreaction of the hard particles using 1s $\times$ 3v nonabelian ”particle-in-cell” simulations.

Parallel transported J_x Visualization



Outlook and to do list

- Emergence of new area of study: nonabelian plasma dynamics
- Study effect of linear growth phase on particles
- Include expansion
- Study inhomogenous systems
- 3s $\times$ 3v particle-in-cell simulations including collisions
- Study system with explicit CGC initial condition (ongoing Raja Venugopalan and Paul Romatschke)
- Study role of ultrasoft modes
- Parton cascade and URQMD codes must be updated to include realistic mean field dynamics!