

# PARTICLE SPECTRA and HYDRO-INSPIRED MODELS

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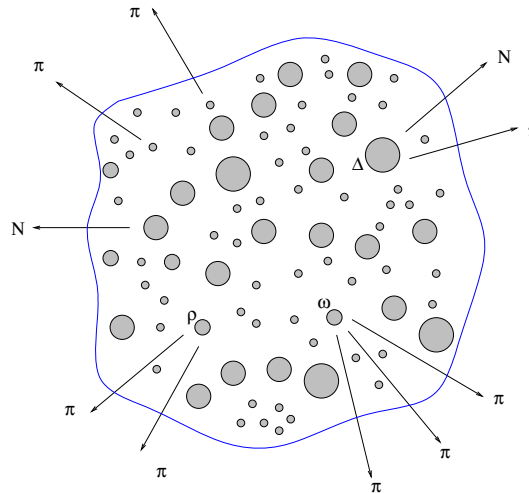
August 5, 2005

# I. Introduction

## 1. Kinetic Freeze-Out

the measured particle spectra reflect properties of matter at the stage when particles stop to interact, this moment is called the kinetic or thermal freeze-out

hydro-inspired models help us to verify the idea that matter, just before the kinetic freeze-out, is locally thermalized and exhibits collective behaviour such as, e.g., the transverse and longitudinal expansion



if this is really the case, we may infer the thermodynamic properties of matter at freeze-out (such as temperature  $T$  and flow  $u^\mu$ ) and request that the advanced hydrodynamic models reproduce this configuration

## 2. Hydro-Inspired Models vs. Advanced Hydrodynamic Calculations

the aim of the freeze-out models is to form a **BRIDGE** between sophisticated hydrodynamic calculations (**presented in the previous talk by Hama**) and the rich bulk of the experimental data describing soft phenomena ( $p_T < 2$  GeV, recall yesterday's talks)

this is an appealing idea but there are problems – one can realize that certain features of the hydrodynamic models and freeze-out models are quite different (e.g., typical shapes of the freeze-out hypersurfaces)

## 3. Attractive Features of the Hydro-Inspired Models

very effective parameterizations of the final state, that use **few parameters** possessing clear physical interpretation and give successful and simultaneous description of **many observables**

as long as we all do not have a hydrodynamic code that we are able to run on our PC (and control it), the freeze-out models form **a convenient and easy accessible tool to interpret the data**

the basic observables used in the fits are: **ratios of hadron abundances, transverse-momentum spectra, elliptic flow, (as)HBT radii, rapidity distributions, correlations between non-identical particles, balance functions**

models which reproduce successfully at least a few ( $p_T$ -spectra, HBT radii) of those observables include:

- Blast-Wave Model (Schnedermann, Sollfrank, and Heinz; Retière and Lisa)
- Buda-Lund Model (Csörgő, Lörstad, Csanád, and Ster)
- Seattle Model (Cramer, Miller, Wu, and Yoon)
- Durham Model (Renk)
- Cracow Single-Freeze-Out Model (Broniowski and WF)
- THERMINATOR (Kisiel, Tałuć, Broniowski, and WF) Monte-Carlo event generator which implements the Cracow and Blast-Wave models
- Kiev-Nantes Model (Borysova, Sinyukov, Akkelin, Erazmus, Karpenko) brand new!

a comparative characteristics of these models is the actual subject of this talk

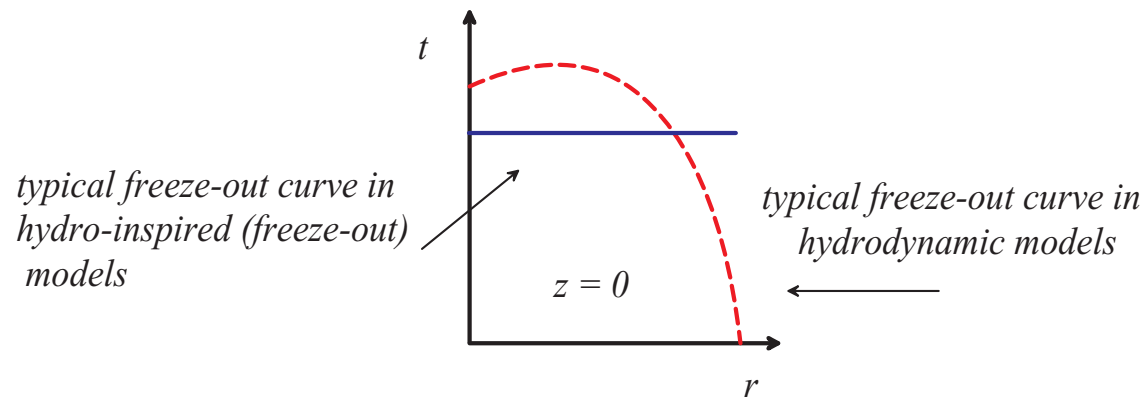
only Au + Au collisions at RHIC energies are discussed

## II. Cooper-Frye formula

$$E_p \frac{dN}{d^3p} = \int p^\mu d\Sigma_\mu(x) f_{\text{eq}}(p \cdot u(x))$$

$d\Sigma^\mu(x)$  a three-dimensional element of the hypersurface,  $u^\mu(x)$  hydrodynamic flow time-like,  $d\Sigma^\mu d\Sigma_\mu > 0$ , and space-like,  $d\Sigma^\mu d\Sigma_\mu < 0$ , parts are included

advanced hydrodynamic calculations include both parts, while hydro-inspired models include typically only the time-like parts



recent works representing the microscopic approach to the description of freeze-out (treatment of the space-like parts, sudden vs. continuous process): [Bugaev](#), [Sinyukov](#), [Akkelin](#), [Hama](#), [Magas](#), [A. Anderlik](#), [Cs. Anderlik](#), [Csernai](#), [Molnar](#), [Grassi](#), [Bravina](#), [Zabrodin](#)

## Emission/Source Function

$$E_p \frac{dN}{d^3p} = \int d^4x \int p^\mu d\Sigma_\mu(x') \delta^4(x' - x) f_{\text{eq}}(p \cdot u(x')) \equiv \int d^4x S(x, p)$$

in general  $S(x, p)$  may be modeled without any reference to the Cooper-Frye formula

**HYDRO-INSPIRED MODELS AIM TO RECONSTRUCT THE EMISSION FUNCTION**

DATA  $\longrightarrow$  EMISSION FUNCTION  $\longrightarrow$  FULL TIME DEVELOPMENT OF THE SYSTEM



### III. Blast-Wave Model

1. A bit of history, the blast-wave model of Siemens and Rasmussen, PRL 42, 880 (1979)

for spherically symmetric systems

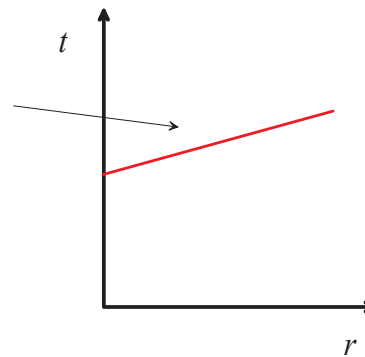
inspiration: the Bondorf-Zimanyi model

$$\frac{dN}{d^3p} = Z \exp\left(-\frac{\gamma E}{T}\right) \left[ \left(1 + \frac{T}{\gamma E}\right) \frac{\sinh(a)}{a} - \frac{T}{\gamma E} \cosh(a) \right]$$

$Z$  – normalization factor,  $E = \sqrt{m^2 + p^2}$  – hadron energy,  $T$  – temperature of the fireball (the same for all fluid shells),  $\gamma = (1 - v^2)^{-1/2}$  – Lorentz gamma factor with  $v$  – radial collective velocity (radial flow),  $a = \gamma v p / T$

distribution follows from the Cooper-Frye formula if  $t = t_0 + v r$  at freeze-out ( $d\Sigma^\mu \sim u^\mu$ )

*freeze-out curve for the  
Siemens-Rasmussen model,  
shells which are further away  
from the center decouple later*



2. The blast-wave model of Schnedermann, Sollfrank, and Heinz, PRC 48, 2462 (1993)

for boost-invariant and cylindrically symmetric systems

the CORNER STONE of the phenomenological interpretations of the  $p_T$  distributions

for constant transverse flow ( $v_T = \tanh \rho = \text{const}$ ) the Cooper-Frye formula yields

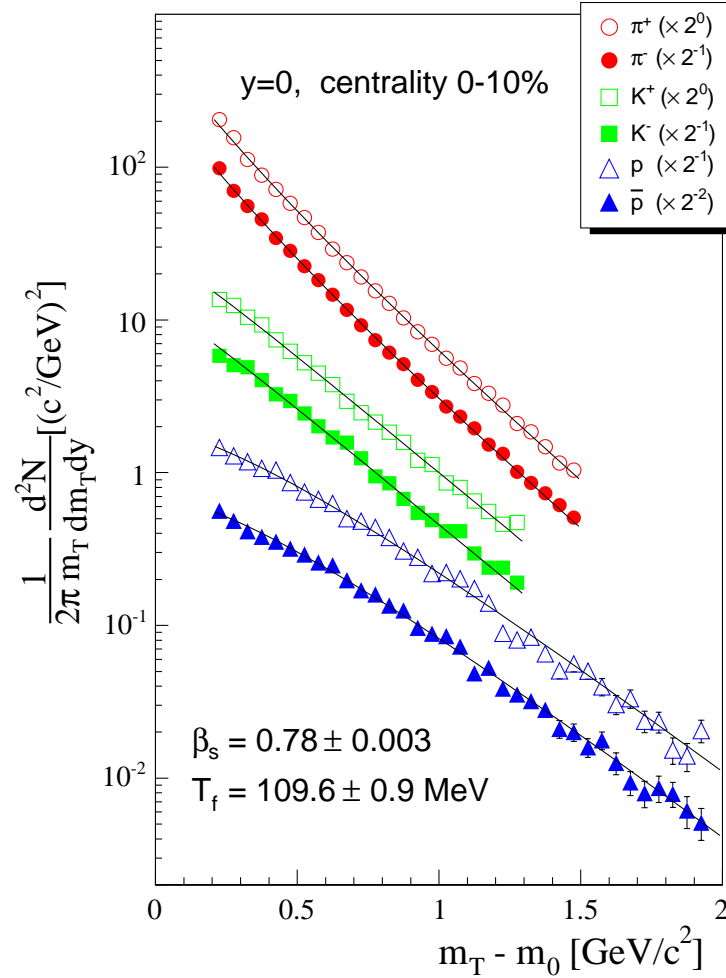
$$\begin{aligned} \frac{dN}{dy d^2 p_T} &= \frac{e^{\beta \mu}}{2\pi^2} m_T K_1 [\beta m_T \cosh(\rho)] I_0 [\beta p_T \sinh(\rho)] \int_0^1 d\zeta r(\zeta) t(\zeta) \frac{dr}{d\zeta} \\ &\quad - \frac{e^{\beta \mu}}{2\pi^2} p_T K_0 [\beta m_T \cosh(\rho)] I_1 [\beta p_T \sinh(\rho)] \int_0^1 d\zeta r(\zeta) t(\zeta) \frac{dt}{d\zeta} \end{aligned}$$

$\beta = 1/T$  – inverse temperature,  $\mu$  – chemical potential,  $\rho$  – transverse rapidity,  $t$  and  $r = \sqrt{r_x^2 + r_y^2}$  – coordinates of the freeze-out hypersurface at  $z = 0$ ,  $K$  and  $I$  – modified Bessel functions in most of the phenomenological fits only the first part of this formula is used

$$\frac{dN}{dy d^2 p_T} = \text{const} \int_0^R dr r m_T K_1 [\beta m_T \cosh(\rho(r))] I_0 [\beta p_T \sinh(\rho(r))]$$

this implicitly means that the freeze-out happens at constant laboratory time ( $t = \text{const}$  at  $z = 0$ )





blast-wave fit performed by [BRAHMS \(nucl-ex/0503010\)](#), Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV, midrapidity region, most central events (0-10%),  $\tanh(\rho(r)) = v_T = \beta_s (r/R_{\max})^\alpha$   
 $R_{\max} = 13 \text{ fm}$  (input)     $T = 109.6 \text{ MeV}$ ,  $\beta_s = 0.78$ ,  $\alpha = 0.40$  (output)     $\langle v_T \rangle = 0.65$

### 3. Advanced blast-wave model of Retière and Lisa, PRC 70, 044907 (2004)

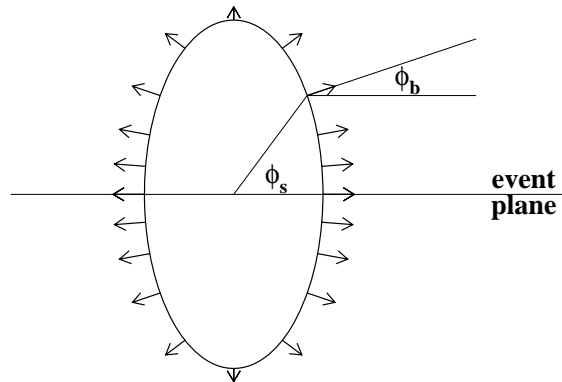
for boost-invariant and cylindrically non-symmetric systems – the parameterization takes into account the possible ellipsoidal shape of the system created in non central collisions

the emission function corresponds to the Cooper-Frye formula with  $\tau = \tau_0 = \text{const}$  replaced by a gaussian distribution

$$S(x, p) = Z m_T \cosh(\eta - y) \Omega(r, \phi_s) e^{-\frac{(\tau - \tau_0)^2}{2\Delta\tau^2}} \frac{1}{e^{p \cdot u/T} \pm 1}$$

$Z$  is an arbitrary normalization (mine),  $\Omega$  describes the spatial distribution of matter

$$\Omega(r, \phi_s) = \Omega(\tilde{r}) = \frac{1}{1 + e^{(\tilde{r}-1)/a_s}}, \quad \tilde{r}(r, \phi_s) \equiv \sqrt{\frac{(r \cos(\phi_s))^2}{R_x^2} + \frac{(r \sin(\phi_s))^2}{R_y^2}}$$



flow is orthogonal to the surface, not proportional to the position vector (more discussion of this issue and nice analytic expressions for two different models of expansion are discussed by [Tomasik, nucl-th/0409074](#))

$$p \cdot u = m_T \cosh \rho(r, \phi_s) \cosh(\eta - y) - p_T \sinh \rho(r, \phi_s) \cos(\phi_b - \phi_p)$$

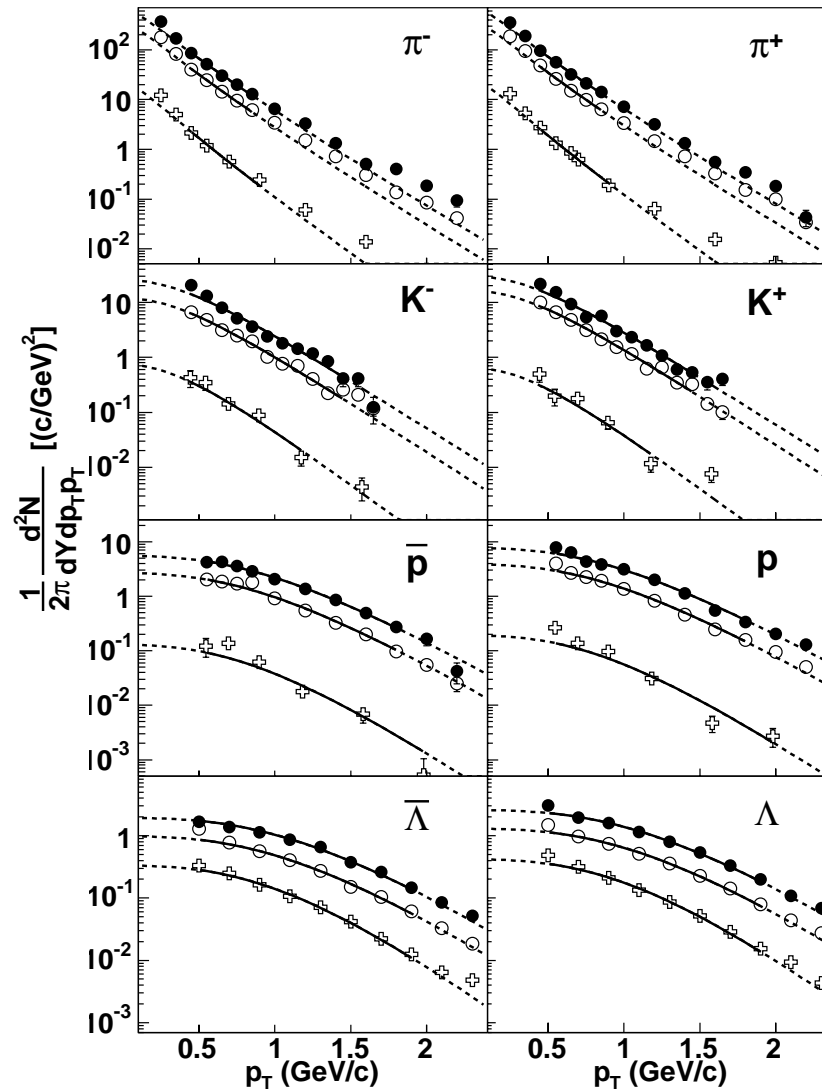
not a function of  $\phi_b - \phi_p$  anymore

transverse rapidity:  $\rho(r, \phi_s) = \tilde{r} (\rho_0 + \rho_2 \cos(2\phi_b))$

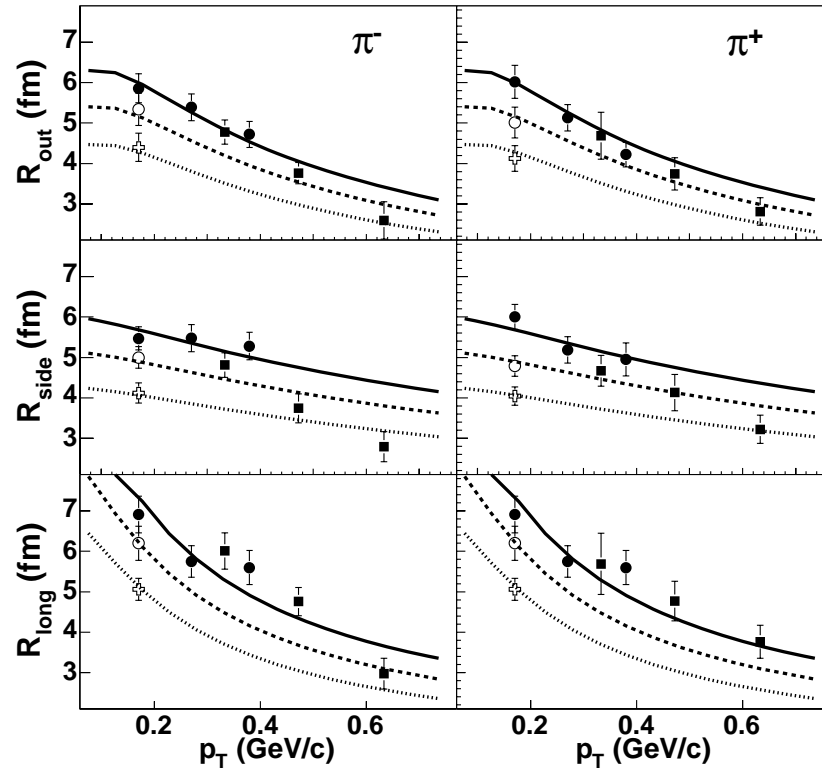
optimal parameters for Retière-Lisa Model for three different centrality classes (0-5%, 15-30%, 60-92% for pions)

$T$ [MeV]	$106 \pm 3$	$107 \pm 2$	$100 \pm 5$
$\rho_0$	$0.89 \pm 0.02$	$0.85 \pm 0.01$	$0.79 \pm 0.02$
$\rho_2$	$0.060 \pm 0.008$	$0.058 \pm 0.005$	$0.05 \pm 0.01$
$R_x$ [fm]	$13.2 \pm 0.3$	$10.4 \pm 0.4$	$8.00 \pm 0.4$
$R_y$ [fm]	$13.0 \pm 0.3$	$11.8 \pm 0.4$	$10.1 \pm 0.4$
$\tau_0$ [fm]	$9.2 \pm 0.4$	$7.7 \pm 0.9$	$6.5 \pm 0.6$
$\Delta\tau$ [fm]	$0.003 \pm 1.3$	$0.06 \pm 1.3$	$0.6 \pm 1.8$

this parameterization yields the average transverse flow  $\langle v_T \rangle = 0.52, 0.50, 0.47$



comparison of the PHENIX and STAR data, Au+Au at  $\sqrt{s_{NN}} = 130$  GeV, with the blast-wave calculations performed with optimal parameters for three different centrality classes, Retière and Lisa, PRC 70, 044907 (2004)



comparison of the pion source data measured by the STAR (circle) and PHENIX (box) Collaborations, Retière and Lisa, PRC 70, 044907 (2004)

## summary on the blast-wave model

- i. good description of  $p_T$  spectra,  $v_2$  and (low- $p_T$  STAR) HBT at midrapidity
- ii. the magnitude of the in-plane flow does not have to be stronger than the magnitude of the out-of-plane flow, more matter should flow in plane (non-zero  $v_2$  even with  $\rho_2 = 0$ )
- iii. small values of  $\tau_0$  ( $\sim 10$  fm) and  $\Delta\tau$  ( $< 1$  fm) – for central collisions practically instantaneous emission at constant laboratory time
- iv. extra parameter for the normalization of each spectrum is required  
( $\pi^+$ ,  $\pi^-$ ,  $K^+$ ,  $K^-$ ,  $p$ ,  $\bar{p}$ ,  $\Lambda$ ,  $\bar{\Lambda}$ )  
no predictive power for yields
- v. no resonances are included, although the recipe already given by SSH and many others,  
 $T \sim 100$  MeV – slope parameter rather than temperature
- vi. # of parameters = 7 (and 8 normalization constants)

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ATTENTION: more discussion on HBT and  $v_2$  in the next two talks by Lednicky and Lacey

## IV. Buda-Lund Model

standard version formulated for cylindrically symmetric systems, **no constraint of boost-invariance**, Csörgő and Lörstad, PRC 54, 1390 (1996); Csörgo, HIP 15, 1 (2002)

extension for ellipsoidally symmetric systems;  
Csanad, Csörgő, and Lörstad, Nucl. Phys. A742, 90 (2004)

the standard emission function is modeled in the form

$$S(x, p) = \frac{g}{(2\pi)^3} \frac{m_T \cosh(\eta - y)}{\exp\left(\frac{u^\mu(x)p_\mu}{T(x)} - \frac{\mu(x)}{T(x)}\right) \pm 1} \frac{1}{(2\pi \Delta\tau^2)^{1/2}} \exp\left[-\frac{(\tau - \tau_0)^2}{2\Delta\tau^2}\right]$$

where temperature and chemical potential depend on the position coordinates

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - \frac{r^2}{2R_G^2} - \frac{\eta^2}{2\Delta\eta^2}$$

$$\frac{1}{T(x)} = \frac{1}{T_0} \left(1 + \frac{T_0 - T_s}{T_s} \frac{r^2}{2R_G^2}\right) \left(1 + \frac{T_0 - T_e}{T_e} \frac{(\tau - \tau_0)^2}{2\Delta\tau^2}\right)$$

$T_0$  – temperature in the center,  $T_s$  – temperature at the surface,  
 $T_e$  – temperature in the center in the end of the particle emission

the flow has Hubble-structure (velocity proportional to the distance from the center)

$$u^\mu(x) = \left( \cosh(\eta) \cosh(\rho), \sinh(\rho) \frac{r_x}{r}, \sinh(\rho) \frac{r_y}{r}, \sinh(\eta) \cosh(\rho) \right)$$

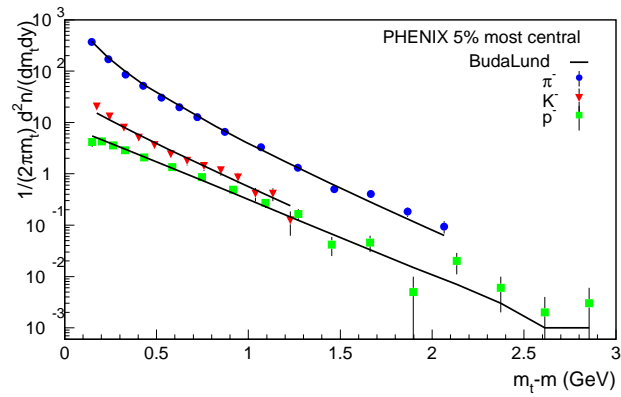
the transverse flow strength determined by the parameter  $\langle u_t \rangle$ ,  $\sinh(\rho) = \langle u_t \rangle r / R_G$ , other dependent variables

$$R_s^2 = R_G^2 \frac{T_s}{T_0 - T_s}, \quad \langle u'_t \rangle = \langle u_t \rangle \frac{R_s}{R_G} \quad \langle v_t \rangle \sim 0.55 \text{ (200 GeV)}$$

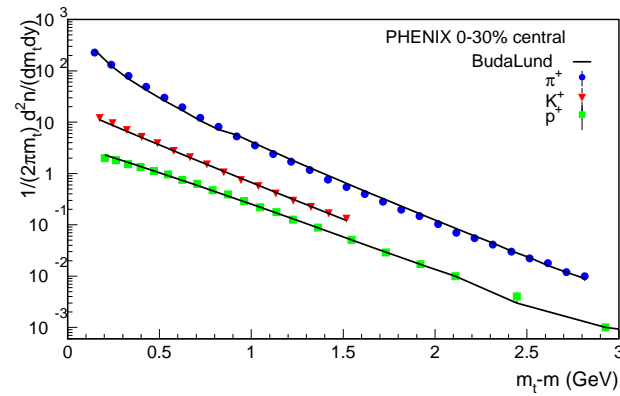
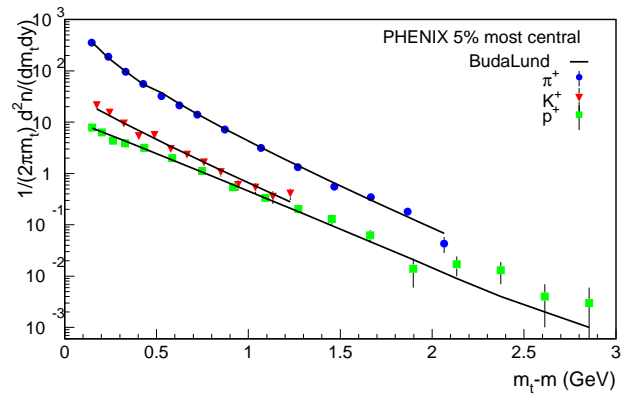
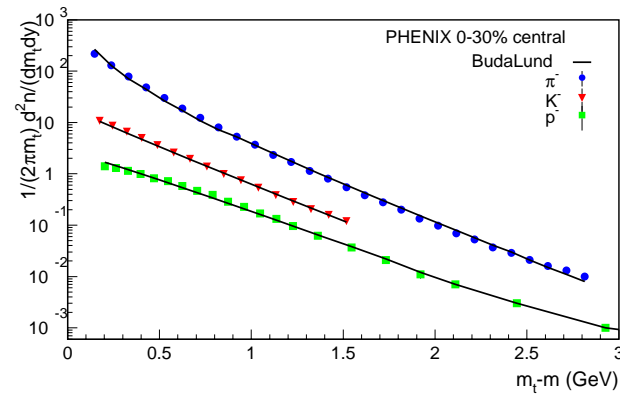
Buda-Lund parameters	Au+Au 200 GeV	Au+Au 130 GeV
$T_0$ [MeV]	196 ± 13	214 ± 7
$T_e$ [MeV]	117 ± 12	102 ± 11
$R_G$ [fm]	13.5 ± 1.7	28.0 ± 5.5
$R_s$ [fm]	12.4 ± 1.6	8.6 ± 0.4
$\langle u'_t \rangle$	1.6 ± 0.2	1.0 ± 0.1
$\tau_0$ [fm]	5.8 ± 0.3	6.0 ± 0.2
$\Delta\tau$ [fm]	0.9 ± 1.2	0.3 ± 1.2
$\Delta\eta$	3.1 ± 0.1	2.4 ± 0.1
$\chi^2/\text{NDF}$	114 / 208	158.2 / 180
$\mu_B [MeV] (\mu_p - \mu_{\bar{p}})$	61 ± 52	77 ± 38



BudaLund hydro fits to 130 AGeV Au+Au

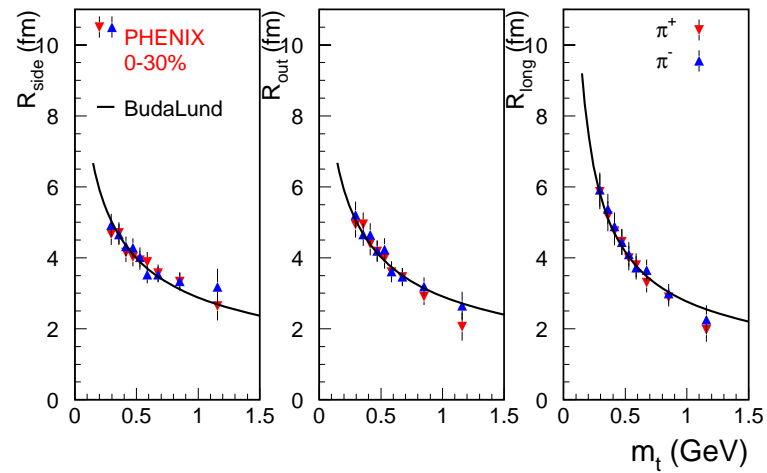
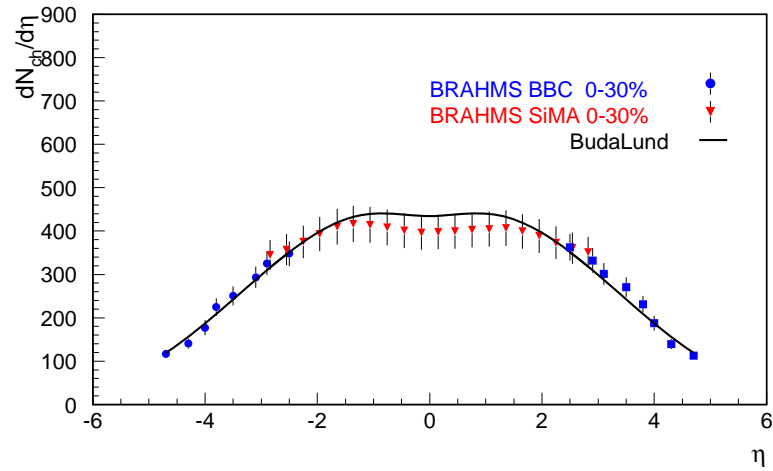


BudaLund v1.5 fits to 200 AGeV Au+Au



fit to the PHENIX data on Au+Au collisions at  $\sqrt{s_{NN}} = 130$  GeV and  $\sqrt{s_{NN}} = 200$  GeV,  
 Csanad, Csörgő, Lörstad, and Ster, J. Phys. G30, S1079 (2004)

BudaLund v1.5 fits to 200 AGeV Au+Au



fit to the PHENIX data on Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV, Csanad, Csörgő, Lörstad, and Ster, J. Phys. G30, S1079 (2004)

## summary on the Buda-Lund Model

- i. good description of  $p_T$  spectra,  $v_2$  and HBT in the full rapidity range
- ii. correct dependence of  $v_2$  on pseudorapidity (talk by Ster, parallel session 2b)
- iii. small values of  $\tau_0$  ( $\sim 6$  fm) and  $\Delta\tau$  ( $\sim 1$  fm)
- iv. explicitly no resonances are included (long living resonances in the halo, short living resonances in the discussed here core)
- v. BL parameterization is related to simple hydrodynamic solutions (with boundary conditions different from those used in the advanced hydro)
- vi. # of parameters for the standard version = 8 (and 6 normalization constants)

decoupling happens at different values of the temperature, mostly at  $T \sim 100 - 120$  MeV, the very hot center, with  $T \sim 200$  MeV, is considered as a signal/remnant of the deconfined phase

## V. Seattle Model

J.G. Cramer, G.A. Miller, J.M.S Wu, J.-H. Yoon PRL 94, 102302 (2005)

cylindrical symmetry, no boost-invariance

effects of the final state interactions of outgoing pions are included by the use of **distorted wave functions**

the generalized emission function used to get one-particle and two-particle distributions in momentum space has the form

$$S(x, p, q) = \int d^4 K' S_0(x, K') \int \frac{d^4 x'}{(2\pi)^4} e^{-iK' \cdot x'} \Psi_{\mathbf{p}_1}^{(-)}(x + x'/2) \Psi_{\mathbf{p}_2}^{(-)*}(x - x'/2)$$

$$p = (p_1 + p_2)/2, \quad q = p_1 - p_2$$

in the special case

$$\Psi_{\mathbf{p}}^{(-)}(x) \rightarrow e^{ip \cdot x}, \quad S(x, p, q) \rightarrow S_0(x, p) e^{iq \cdot x}$$

$$S(x, p, 0) \rightarrow S_0(x, p)$$

the standard formalism is recovered

the emission function  $S_0$  resembles the blast-wave and Buda-Lund parameterizations

$$S_0(x, p) = \frac{m_T \cosh \eta}{\sqrt{2\pi(\Delta\tau)^2}} \exp \left[ -\frac{(\tau - \tau_0)^2}{2\Delta\tau^2} - \frac{\eta^2}{2\Delta\eta^2} \right] \frac{\Omega(r)}{(2\pi)^3} \frac{1}{\exp(\frac{p \cdot u - \mu_\pi}{T}) - 1}$$

$$\Omega(r) = \frac{1}{\left[ \exp \left( \frac{r - R_{WS}}{a_{WS}} \right) + 1 \right]^2}, \quad r = \sqrt{r_x^2 + r_y^2}$$

$$p \cdot u = m_T \cosh \eta \cosh \rho(r) - p_T \sinh \rho(r) \cos \phi, \quad \rho(r) = \eta_f \frac{r}{R_{WS}}$$

$\psi_{\mathbf{p}}^{(-)}(\mathbf{r})$  obtained by solving a two-dimensional Klein-Gordon equation,  $\mathbf{r} = (r_x, r_y)$

$$\left[ -\nabla_{\perp}^2 + U(r) \right] \psi_{\mathbf{p}}^{(-)*}(\mathbf{r}) = p^2 \psi_{\mathbf{p}}^{(-)*}(\mathbf{r})$$

the optical potential  $U(r)$  is a complex, azimuthally-symmetric function depending on pion momentum and local density, it represents the strength of the interaction between a pion and the medium

$$U_p(r) = -(w_0 + w_2 p^2) \Omega(r)$$

values of the parameters from a more recent paper: [Miller and Cramer, nucl-th/0507004](#)

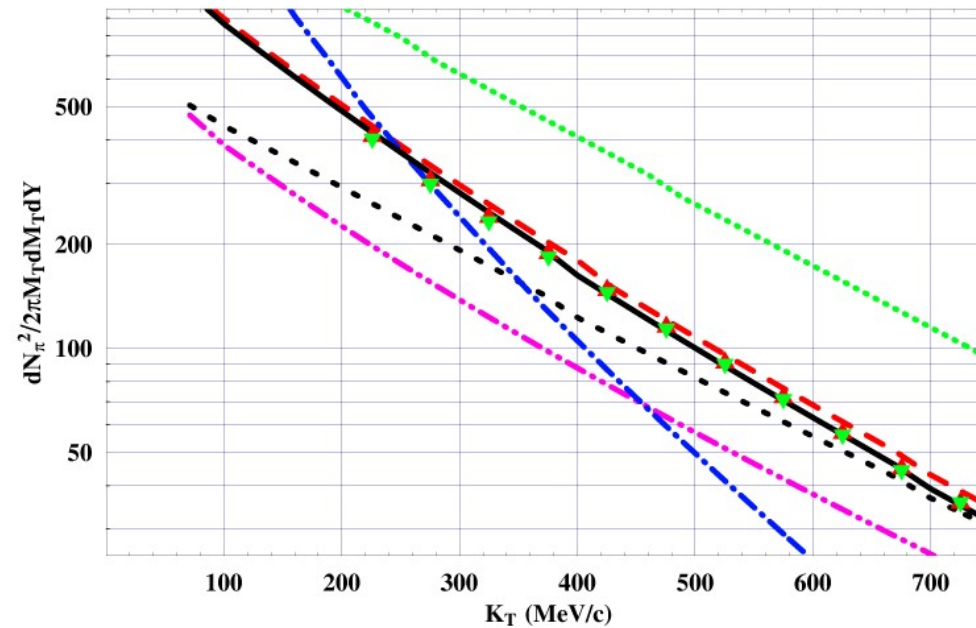
$$T = 214.8 \pm 1.6 \text{ MeV}, \quad \mu_\pi = 122.7 \pm 1.1 \text{ MeV}$$

$$\tau_0 = 8.14 \pm 0.10 \text{ fm}, \quad \Delta\tau = 2.650 \pm 0.07 \text{ fm}$$

$$R_{WS} = 12.070 \pm 0.06 \text{ fm}, \quad a_{WS} = 0.786 \pm 0.015 \text{ fm}$$

$$w_0 = 0.142 \pm 0.046 \text{ fm}^{-2}, \quad w_2 = 0.582 \pm 0.014 + i(0.123 \pm 0.002)$$

$$\Delta\eta = 1.040 \pm 0.032 \quad \eta_f = 1.539 \pm 0.025 \quad 11 \text{ (real) parameters}$$



fit to the STAR data, PRL 92, 112301 (2004), describing the pion  $p_T$ -spectrum  
the model describes well the HBT radii

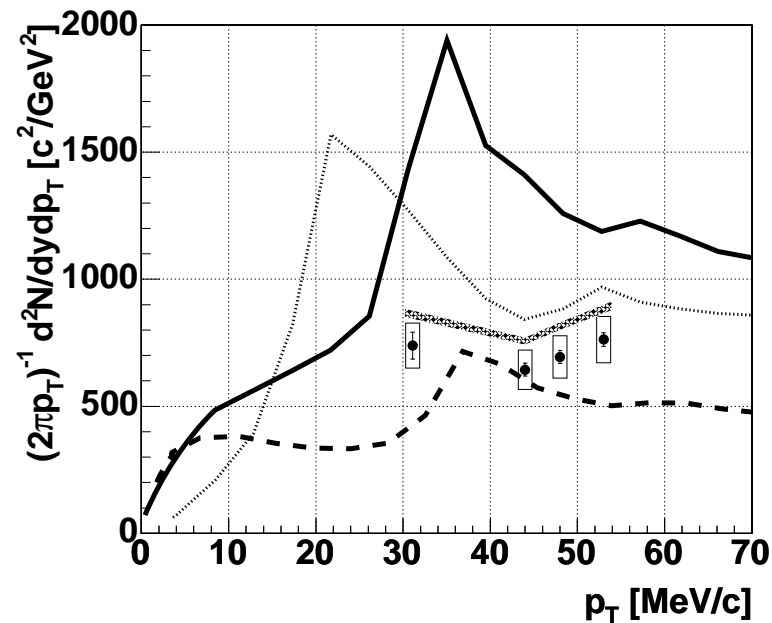
the strength of the attraction inside the medium is greater than  $m_\pi^2$   
a pion behaves as a massless particle

Son, Stephanov: connection with the restoration of chiral symmetry

talk by J. G. Cramer, parallel session 10 b

results on low- $p_T$  production may be confronted with the PHOBOS results

talk by A. Trzupek, parallel session 1 a



curves represent model results for different values of the parameters, four points represent the PHOBOS data

## VI. Durham Model

Renk, PRC 70, 021903(R) (2004)

parameterization of the hydrodynamic expansion in the proper-time interval  $\tau_0 < \tau < \tau_f$

transverse expansion (radius):  $R_c(\tau) = R_0 + \frac{1}{2}a_{\perp}\tau^2$

longitudinal expansion (spacetime rapidity range):  $\eta_c(\tau) = \eta_0 + a_{\eta}\tau \longrightarrow H_c(\tau)$

distribution of entropy determined by two Woods-Saxon functions

$$R(r, \tau) = \left( 1 + \exp \left[ \frac{r - R_c(\tau)}{d_{ws}} \right] \right)^{-1} \quad H(\eta, \tau) = \left( 1 + \exp \left[ \frac{\eta - H_c(\tau)}{\eta_{ws}} \right] \right)^{-1}$$

$$s = NR(r, \tau)H(\eta, \tau)$$

two parameters may be replaced by:  $v_{\perp}^f = a_{\perp}\tau_f$  and  $\eta_f = \eta_0 + a_{\eta}\tau_f$

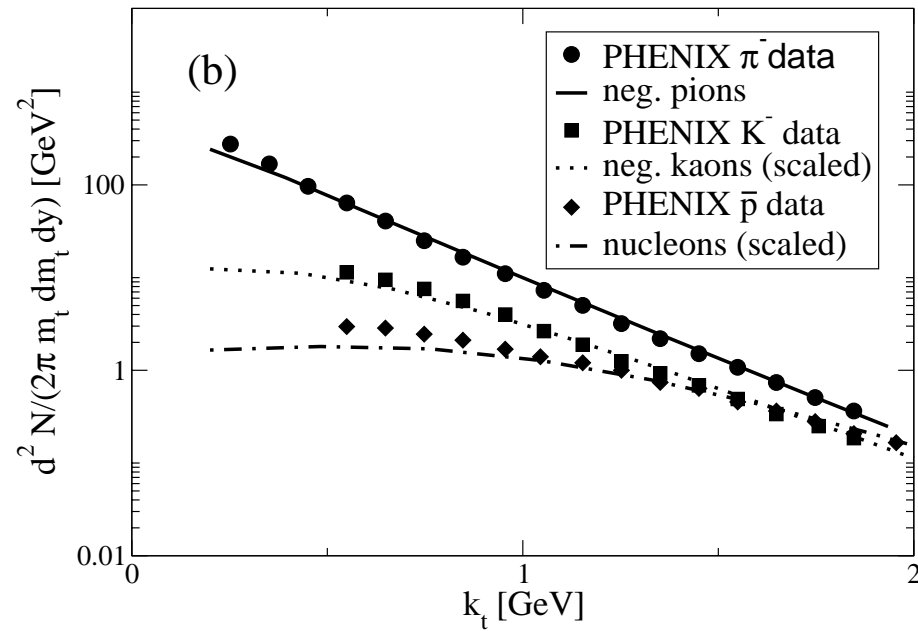
entropy per baryon is calculated from the data, temperature and pressure from the realistic EOS

longitudinal acceleration is self consistently related to the pressure

$$p(\tau) \rightarrow a_{\eta}(\tau) \rightarrow s(\tau) \rightarrow p(\tau)$$

freeze-out happens at fixed  $T_f$ , Cooper-Frye formula is used to calculate the spectra





transverse-momentum spectra, fit to the PHENIX data at  $\sqrt{s_{NN}} = 200$  GeV  
 $\tau_0 = 1$  fm,  $\tau_f = 19$  fm,  $T_f = 110$  MeV,  $\eta_0 = 1.8$ ,  $\eta_f = 3.5$ ,  $\eta_{ws} = 0.6$

the model describes also nicely the HBT radii

no resonance decays are included in the calculation of the spectra

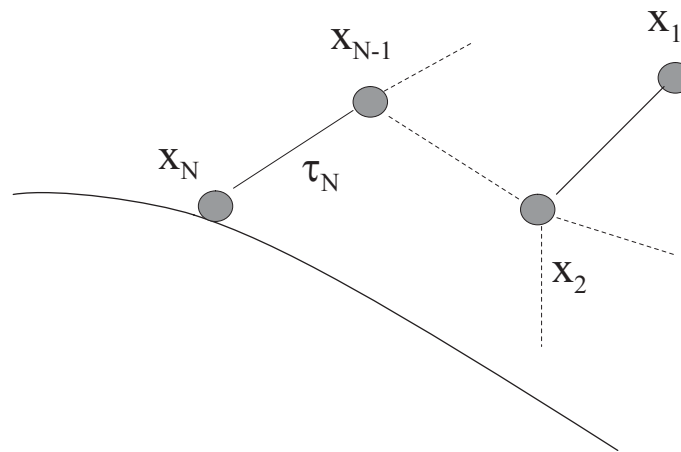
comparison with a real hydro would strengthen the physical significance of the DURHAM model

## VII. Cracow Single-Freeze-Out Model

standard version formulated for boost-invariant and cylindrically symmetric systems,  
Broniowski and WF, PRL 87, 272302 (2001)

radical simplification: **chemical and kinetic freeze-out coincide**

direct connection to all statistical/thermal models which reproduce the ratios of hadronic abundances: Rafelski + Letessier + Torrieri, Gorenstein + Gaździcki, Becattini, Sinyukov + Akkelin + Kostyuk, Braun-Munzinger + Stachel + Redlich + Magestro + Humanic, Xu + Kaneta, Cleymans + Kämpfer + Wheaton, Wilk + Włodarczyk



all resonances included, the emission function includes (all) cascading decays  
**absolute yields predicted**

the emission function is the Cooper-Frye formula convoluted with the momentum splitting functions  $B$  and the space-time displacement functions  $\delta$

$$S(x_1, p_1) = E_1 \frac{dN_1}{d^3p_1 d^4x_1} = \int \frac{d^3p_2}{E_{p_2}} B(p_2, p_1) \int d\tau_2 \Gamma_2 e^{-\Gamma_2 \tau_2} \int d^4x_2 \delta^{(4)}\left(x_2 + \frac{p_2 \tau_2}{m_2} - x_1\right) \dots \times \int d\Sigma_\mu(x_N) p_N^\mu \delta^{(4)}\left(x_N + \frac{p_N \tau_N}{m_N} - x_{N-1}\right) f_N[p_N \cdot u(x_N)]$$

the freeze-out hypersurface defined by the conditions:

$$\tau_{\text{inv}} = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2} = \text{const}, \quad r = \sqrt{r_x^2 + r_y^2} < r_{\text{max}}$$

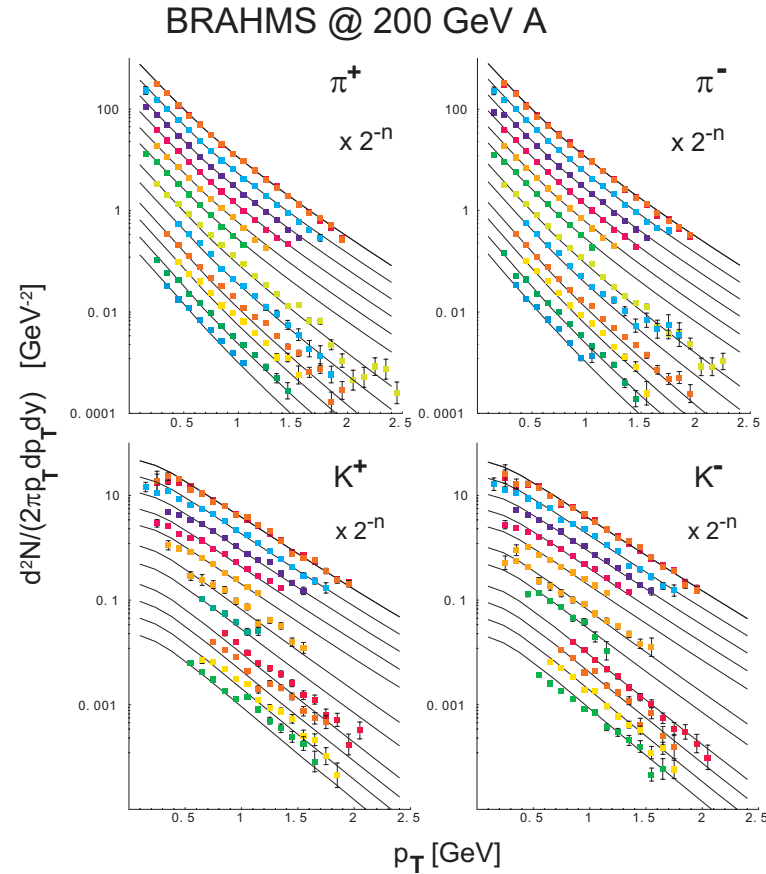
$$u^\mu = \frac{x^\mu}{\tau_{\text{inv}}} = \frac{t}{\tau_{\text{inv}}} \left(1, \frac{r_x}{t}, \frac{r_y}{t}, \frac{r_z}{t}\right), \quad d\Sigma^\mu = u^\mu \tau_{\text{inv}}^3 \sinh(\rho) \cosh(\rho) d\rho d\eta d\phi$$

only four parameters in the standard version:  $T = 165.5$  MeV,  $\mu_B = 28.5$  MeV (independent of centrality) and  $\tau_{\text{inv}}$  and  $r_{\text{max}}$  (dependent on centrality)

OTHER CHOICES ARE POSSIBLE AND USED, affect HBT, Kisiel

very good description of the ratios of hadronic observables,  $p_T$ -spectra,  $R_{\text{side}}$  and  $R_{\text{out}}$ , satisfactory description of the balance functions (with Bożek) and invariant masses of pion pairs (with Hiller)

non boost-invariant version: extra parameter  $\Delta\eta$ ,  $0 \leq \rho \leq \rho_{\max} \exp[-\eta^2/(2\Delta\eta^2)]$   
 ( $\rho_{\max}$  - transverse space-time rapidity,  $r_{\max} = \tau_{\text{inv}} \sinh \rho_{\max}$ )



$\tau_{\text{inv}} = 8.33$  fm,  $\rho_{\max} = 0.825$ ,  $\Delta\eta = 3.33$  - similar to the value in the Buda-Lund model  
 ( $n$  numbers rapidity bins,  $r_{\max} = \tau_{\text{inv}} \sinh \rho_{\max}$ )

# of parameters: 2 (ratios of abundances) + 2 ( $p_T$  spectra) + 2 ( $v_2(p_T, y = 0)$ )  
 + 1 (rapidity distributions of pions and kaons) = 7  $\rightarrow \langle v_T \rangle \sim 0.5$

## VIII. SHARE & THERMUS

publicly available codes for the statistical model:

SHARE - Statistical Hadronization with Resonances,  
Cracow - Arizona Collaboration: Torrieri, Steinke, Broniowski, Letessier, Rafelski, and WF  
Computer Physics Communications 167 (2005) 229

set of programs designed to analyse the ratios of hadronic abundances and  
transverse-momentum spectra

Fortran – analysis of the ratios (chemical non-equilibrium possible)

Mathematica – analysis of the  $p_{\perp}$  spectra (in chemical equilibrium)

web-page calculator

[www.physics.arizona.edu/~torrieri/share/share.html](http://www.physics.arizona.edu/~torrieri/share/share.html)

THERMUS - a thermal model package for ROOT

Wheaton and Cleymans, hep-ph/0407174

## IX. THERMINATOR



### THERMal heavy-IoN generATOR

Monte-Carlo version of the Cracow and Blast-Wave models (Blast-Wave model extended to include complete set of the resonance decays) [Kisiel, Tałuć, Broniowski, WF, nucl-th/0504047](#)

two poster presentations in session 4: [Tałuć](#) (characteristics of the program) and [Kisiel](#) (interesting new physical results on correlations)

uses the same input from the Particle Data Tables as SHARE

poster presentation on SHARE in session 1 by [Torrieri](#)

# X. Humanic's rescattering model

simple transport models may play a similar role as hydro-inspired models

Humanic, Nucl. Phys. A715, 641 (2003)

Monte-Carlo simulation of the evolution of hadronic system

(i.) initialization = hadronization transverse geometry determined by the overlapping region

$$z_{\text{had}} = \tau_{\text{had}} \sinh y, \quad t_{\text{had}} = \tau_{\text{had}} \cosh y$$

$$\frac{1}{m_T} \frac{dN}{dm_T} = C \frac{m_T}{\exp(m_T/T) \pm 1}, \quad \frac{dN}{dy} = D e^{-\frac{(y-y_0)^2}{2\sigma_y^2}}$$

$$T = 300 \text{ MeV}, \quad \sigma_y = 2.4, \quad \tau_{\text{had}} = 1 \text{ fm}$$

(ii.) rescattering, binary collisions,  $\pi, K, N, \Delta, \Lambda, \rho, \omega, \eta\eta', \phi, K^*$  initial abundances fixed

(iii.) freeze-out happens at about 30 fm (Au+Au, 130 GeV), calculation of the observables, successful description of the slope parameters,  $v_2(m, p_T, \eta)$ , HBT( $p_T$ , centrality)

results sensitiv to  $\tau_{\text{had}}$ , larger  $\tau_{\text{had}}$  fewer collisions, rescattering-generated flows are reduced

is strong transverse flow generated in the preequilibrium phase? effect on the hydrodynamic evolution shown by Chojnacki, poster session 10

# CONCLUSIONS

- a common feature of the hydro-inspired models is the **short evolution time**  $\tau_0 < 10 \text{ fm}$ , and even shorter emission time  $\Delta\tau \ll \tau_0$   
this is in contrast with microscopic transport calculations
  - another common feature is **large transverse flow**,  $\langle v_t \rangle \sim 0.5 c$
- 
- **models not including resonances** or pion interactions, yield rather low values of the freeze-out temperature,  $T \sim 100 \text{ MeV}$ , but the effects of the resonance decays are large and should be included, resonance spectra are mostly known!
  - **models including full set of resonances** (Cracow) or **pion interactions** (Seattle) **give higher temperature**, compatible with the temperature of the chemical freeze-out,
- 
- is there any physical reason to neglect the emission from the space-like parts, not yet considered? – surface tension, pressure from the non-perturbative vacuum, ...
  - if not, include it in the blast-wave parameterizations, **Kiev-Nantes model (new)**

hydro-inspired models take into account the effect of the transverse flow, resonance decays are important and should be included in a complete way