Equation of state from lattice QCD

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Outline

1. Introduction
2. Recent results
3. New results for the Equation of State
4. Summary

Introduction

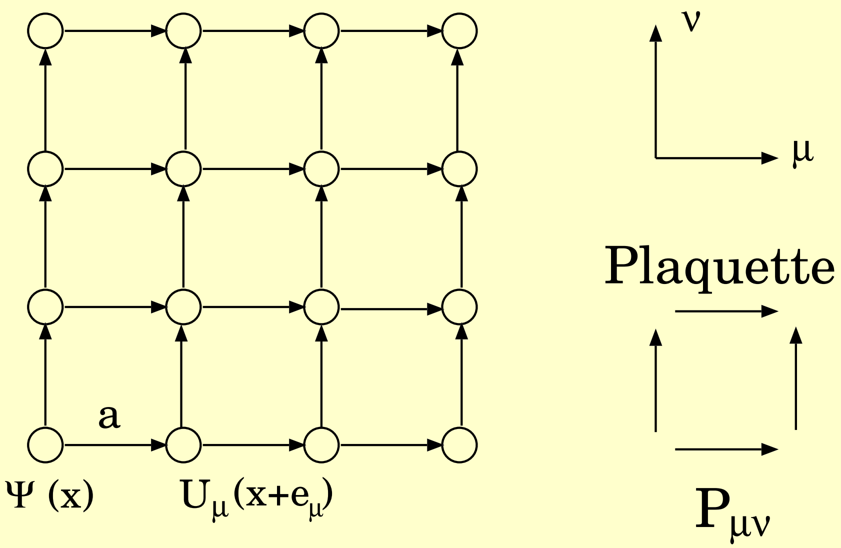
- The Equation of State (EoS); p, ϵ, s as a function of T is an unambiguous prediction of the QCD Lagrangian

- The EoS is an important input for hydrodynamical models of heavy-ion collisions

- Perturbation theory is only reliable at very large T

- Lattice QCD is an applicable non-perturbative tool to determine the EoS

Lattice QCD introduction



Fundamental Fields:

Gauge fields:

$U_\mu(x) \in SU(3)$ live on the links

Quarks:

$\Psi(x), \bar{\Psi}(x)$

anti-commuting Grassmann variables live on the sites

Wilson fermions: $\mathcal{O}(a)$ artefacts

Staggered fermions: $\mathrm{O}(a^2)$, BUT flavour symmetry violation

Partition function

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E}$$

S_E is the Euclidean action

Parameters:

gauge coupling g

quark masses m_i $(i = 1..N_f)$

(Chemical potentials μ_i)

Volume (V) and temperature (T)

Finite T $\leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a}$$

Continuum limit: $a \rightarrow 0$

Renormalization: keep the physical spectrum constant

at finite T:

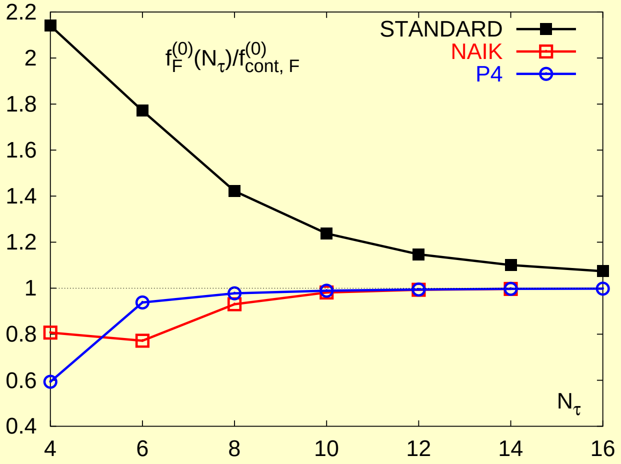
continuum limit $\Longleftrightarrow$ $N_t \rightarrow \infty$

Improved actions

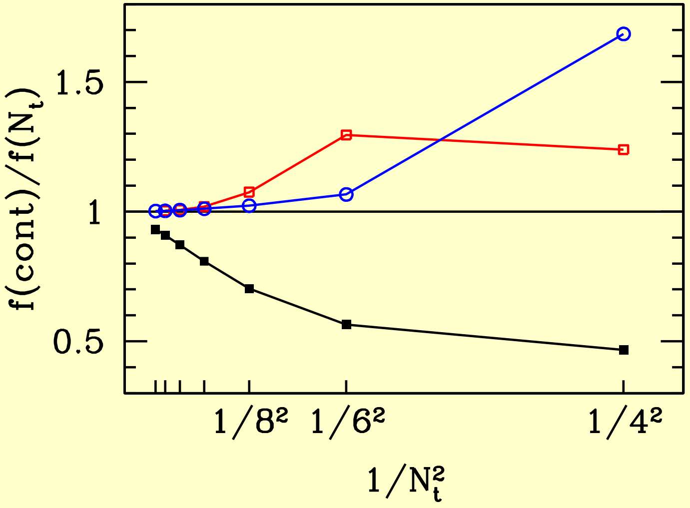
S_E is not unique; many possibilities

from flic to clover and tadpole, hyper-improved and even overimproved improvements

Continuum limit is always important!



[Heller, Karsch, Sturm '99]



Continuum extrapolation from N_t and $N_t + 2$ standard action may be better than using only N_t with improved action

How reliable is lattice QCD?

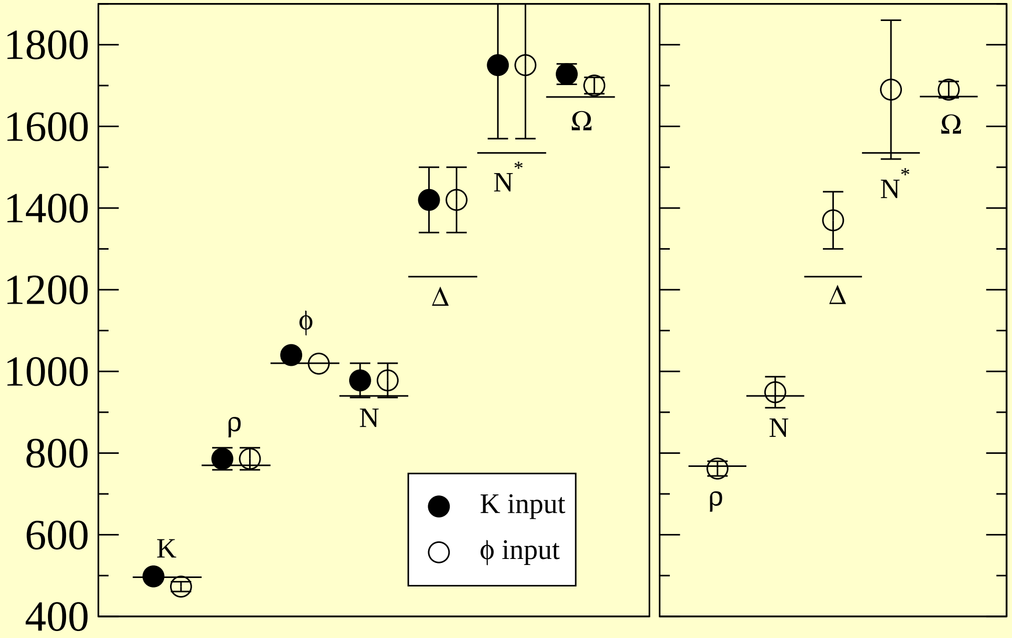
At $T = 0$: Hadron spectrum

based on the QCD Lagrangian (quarks+gluons)

no more – no less than the experimental spectrum

quantitative agreement on the percent level

already in the quenched approximation



[Hasenfratz, Juge, Niedermayer, 2004]

At $T > 0$: no clear connection between experiments and lattice (yet)

experiences from $T = 0$ are promising

Equation of state from lattice simulations

energy density (ϵ) and pressure (p) from partition function:

$$\epsilon(T) = \frac{T^2}{V}\frac{\partial(\log Z)}{\partial T} \qquad\qquad p(T) = T\frac{\partial(\log Z)}{\partial V}.$$

T, V are varied by a, take derivative with respect of a

$$\frac{\epsilon - 3p}{T^4} = -\frac{L_t^3}{L_s^3}a\frac{d(\log Z)}{da}$$

the pressure ($p \propto \log[Z]$) along the LCP by the integral method:

$$\frac{p}{T^4} = L_t^4 \int d(\beta, m \cdot a)\left(\frac{\partial(\log Z)}{\partial \beta}, \frac{\partial(\log Z)}{\partial(m \cdot a)}\right)$$

Renormalization of the pressure

We want $p(T=0)=0$ and $\epsilon(T=0)=0 \rightarrow$
Simulations at both
$T>0 \ \ (N_t \ll N_s)$ and $T=0 \ \ (N_t \gtrsim N_s)$
are necessary and then subtraction:

$$\frac{p}{T^4}=\frac{p_T}{T^4}-\frac{p_0}{T^4}; \qquad \frac{\epsilon}{T^4}=\frac{\epsilon_T}{T^4}-\frac{\epsilon_0}{T^4}$$

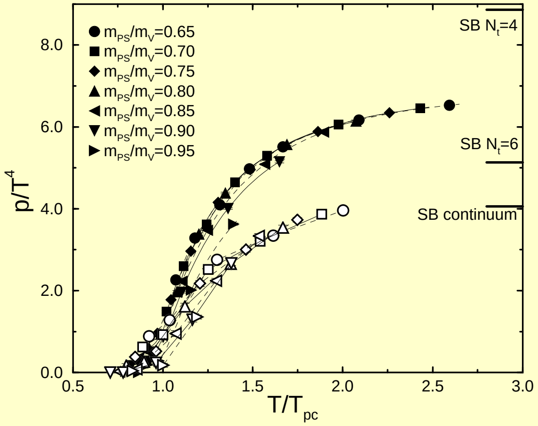
numerical precision needed for the subtraction increases with N_t^4
→ CPU costs grow faster ($\mathcal{O}(1/a^{13})$) than for $T=0$ simulations

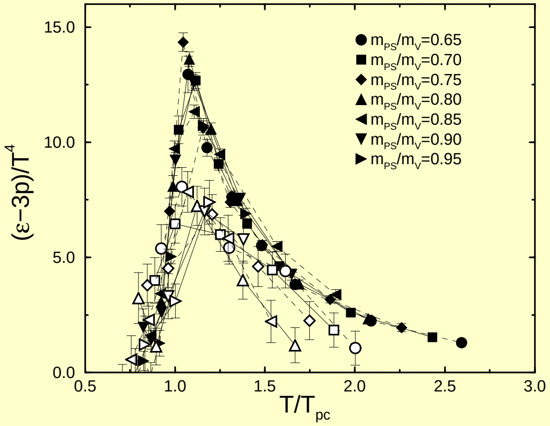
Today

$N_t=4$ is easy
$N_t=6$ is difficult
$N_t=8$ is a challenge

Recent lattice results

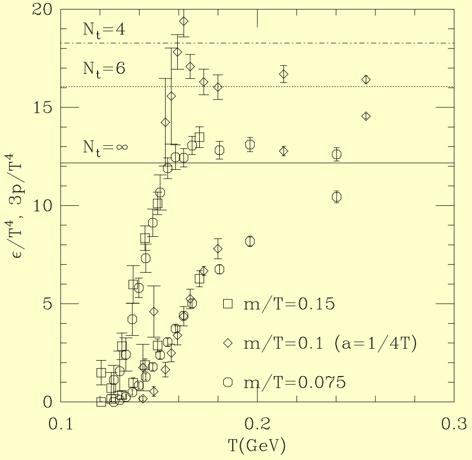
Wilson fermions: $\mathcal{O}(a)$, slower

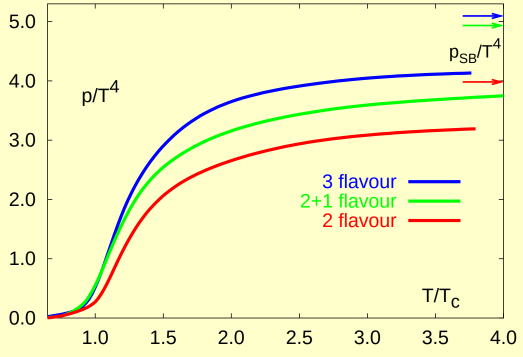




[Ali-Khan et al, '01]

Staggered fermions: $\mathcal{O}(a^2)$, faster





[Bernard et al, '96]

[Karsch, Laermann, Peikert, 2000]

Ongoing projects: MILC, Bielefeld-Brookhaven-Columbia

Weaknesses of these results

1. Unrealistic quark masses
   might be important, since $T_c \gtrsim m_\pi$

2. No Line of constant physics (LCP) used
   $T = 1/(N_t a)$ is increased with decreasing a
   physical spectrum $(m_\pi, m_K, m_\rho, \dots)$ should not change

3. flavour symmetry violation (staggered)
   unphysical, large pion non-degeneracy

4. Approximate algorithms were used
   R algorithm: systematic error due to finite stepsize
   high precision subtraction can be sensitive to it

5. Lattice artefacts
   improved action with $N_t = 4$ only

6 Scale determination
   no string-tension in dynamical QCD

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   $q\bar{q}$ force at 0.5fm

New lattice results for the EOS

[Y. Aoki, Z. Fodor, SDK, K.K. Szabo]

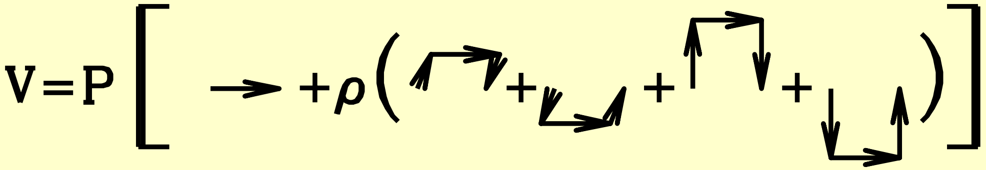
Main features:

- Physical mass spectrum is used for $T > 0$ simulations
- Use of LCP:
  physical spectrum unchanged while a changes
- Exact algorithm (RHMC) is used
  to get rid of stepsize errors
- Supressed flavour symmetry violation
  1-loop improved Symanzik gauge action +
  stout improved fermionic action
- Two sets of lattice spacings
  $N_t = 4$ and 6 simulations
- Unambiguous scale setting

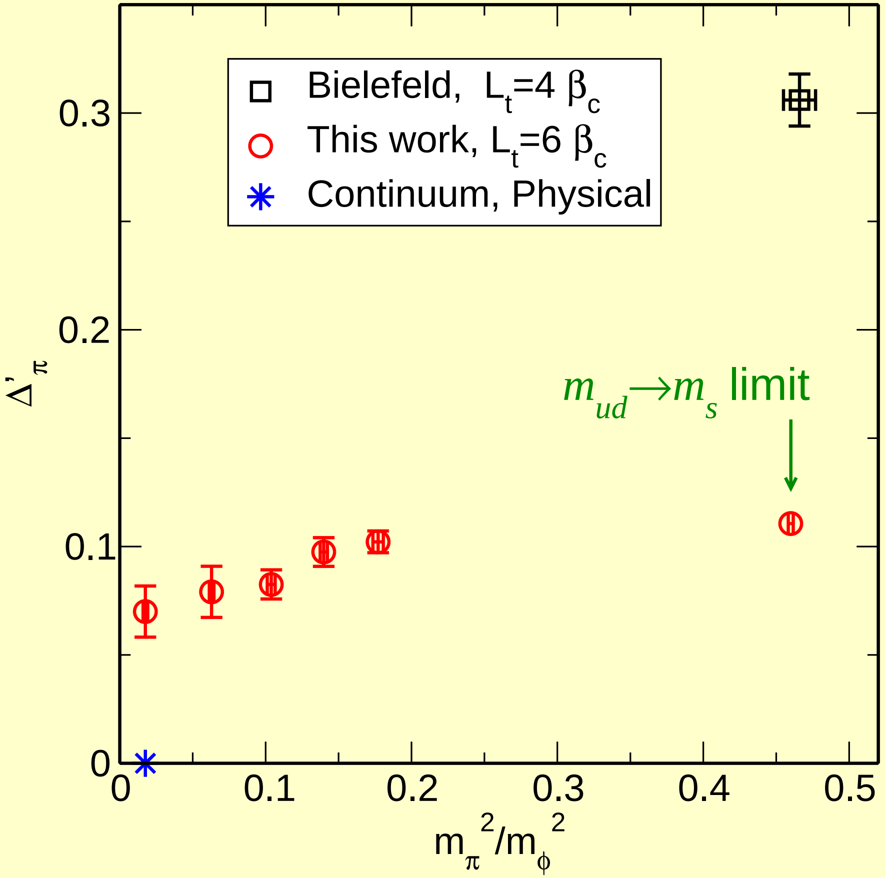
Stout improvement

Stout smearing:

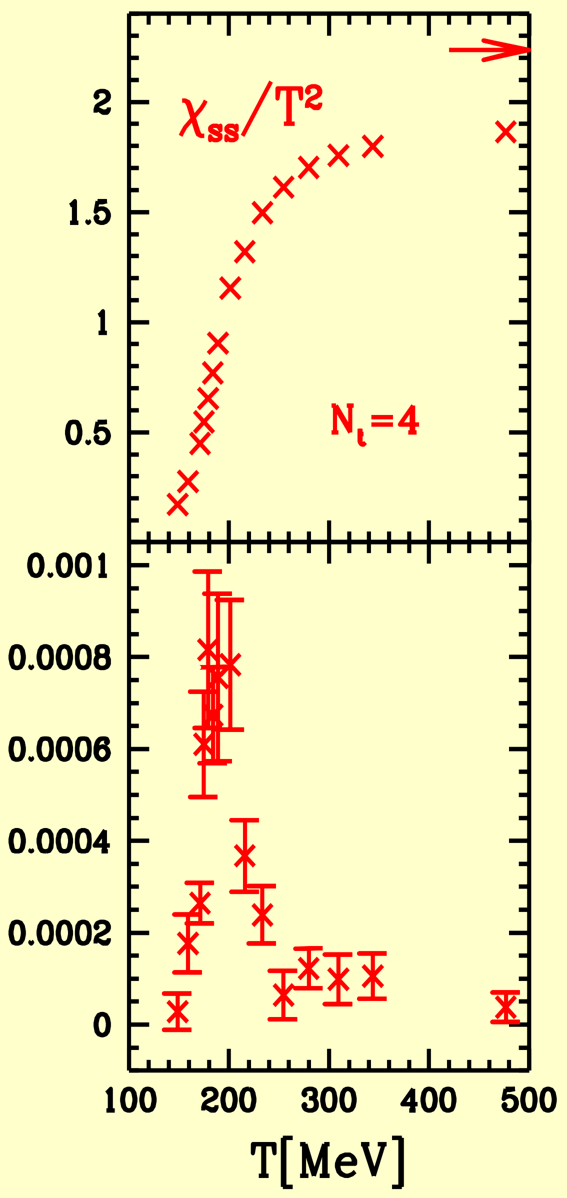
replace the $U(x)_\mu$ gauge links with V stout links:

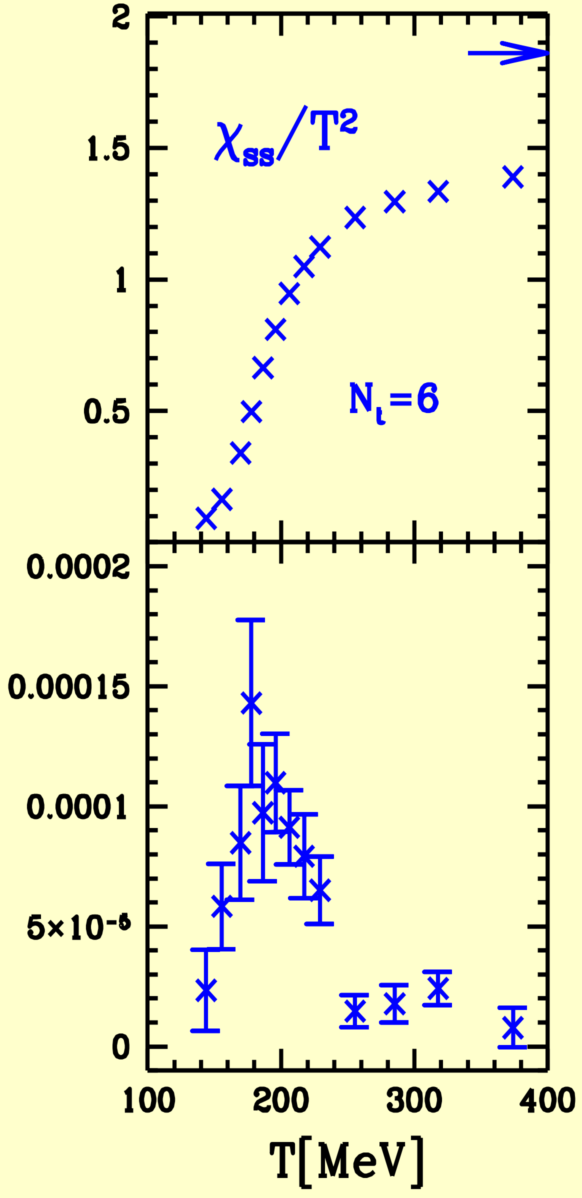


unphysical non-degeneracy of pions largely reduced:



Quark number susceptibilities transition temperature





$$\chi_{ff'} = \frac{T}{V}\frac{\partial^2 \log Z}{\partial\mu_f \partial\mu_{f'}}$$

- experimentally relevant
- nice peak in $\partial\chi_{ss}/\partial\beta$

(pseudo)critical coupling for physical quark masses:

$N_t = 4$:

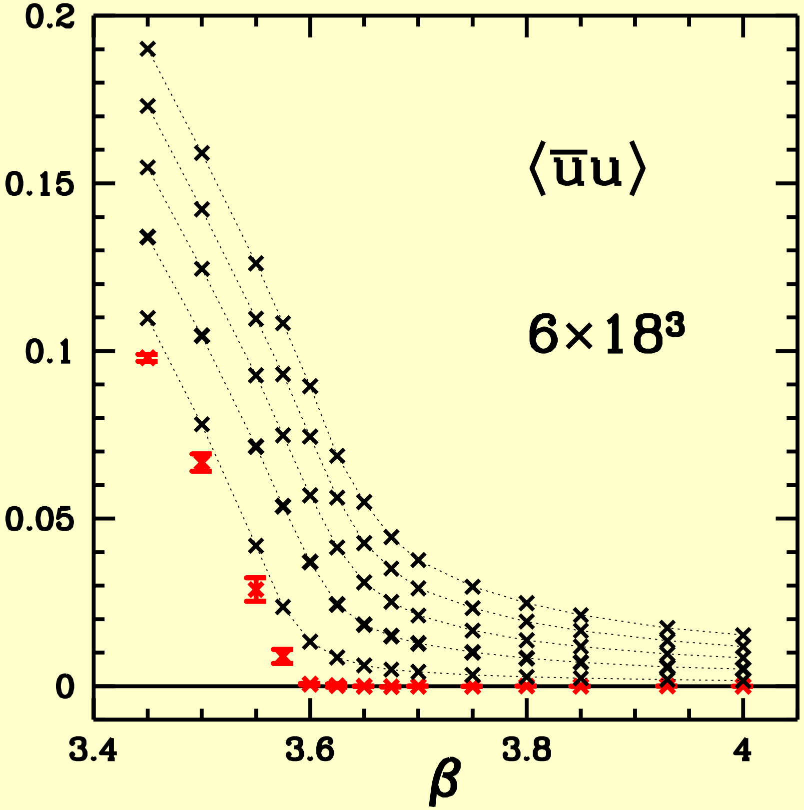
$T_c = 186(3)(3)$ MeV

$N_t = 6$:

$T_c = 193(6)(3)$ MeV

Chiral condensate

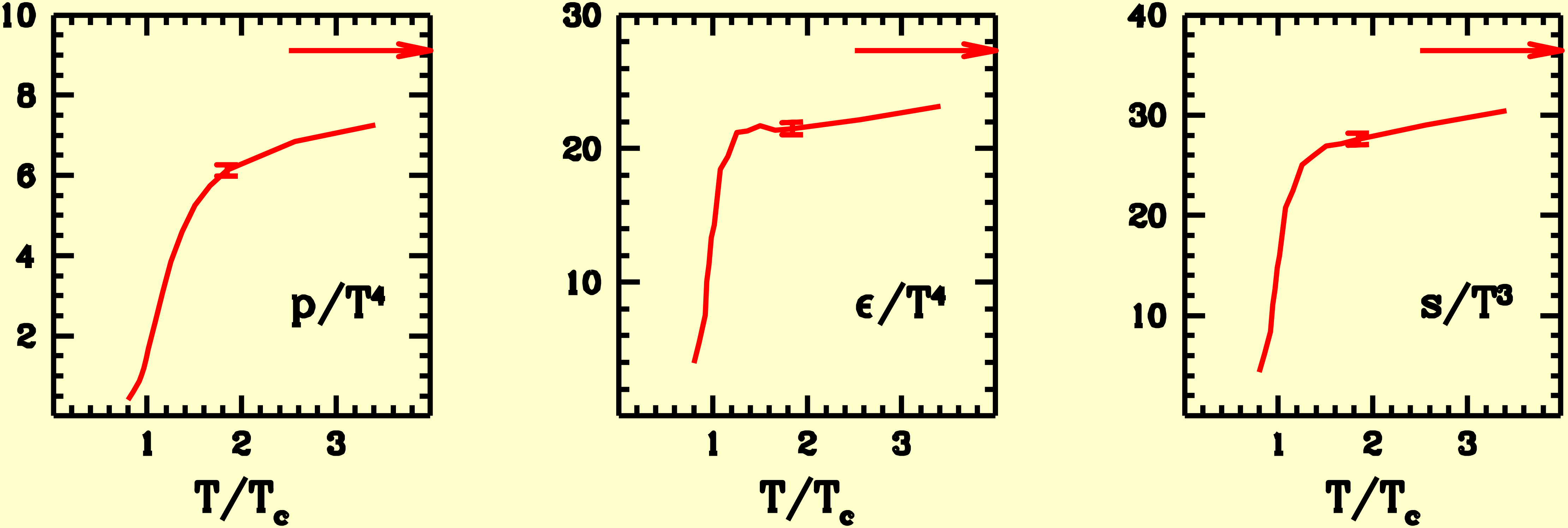
Simulations for $m_{ud} = \{1, 3, 5, 7, 9\} m_{\text{phys}}$ at finite T



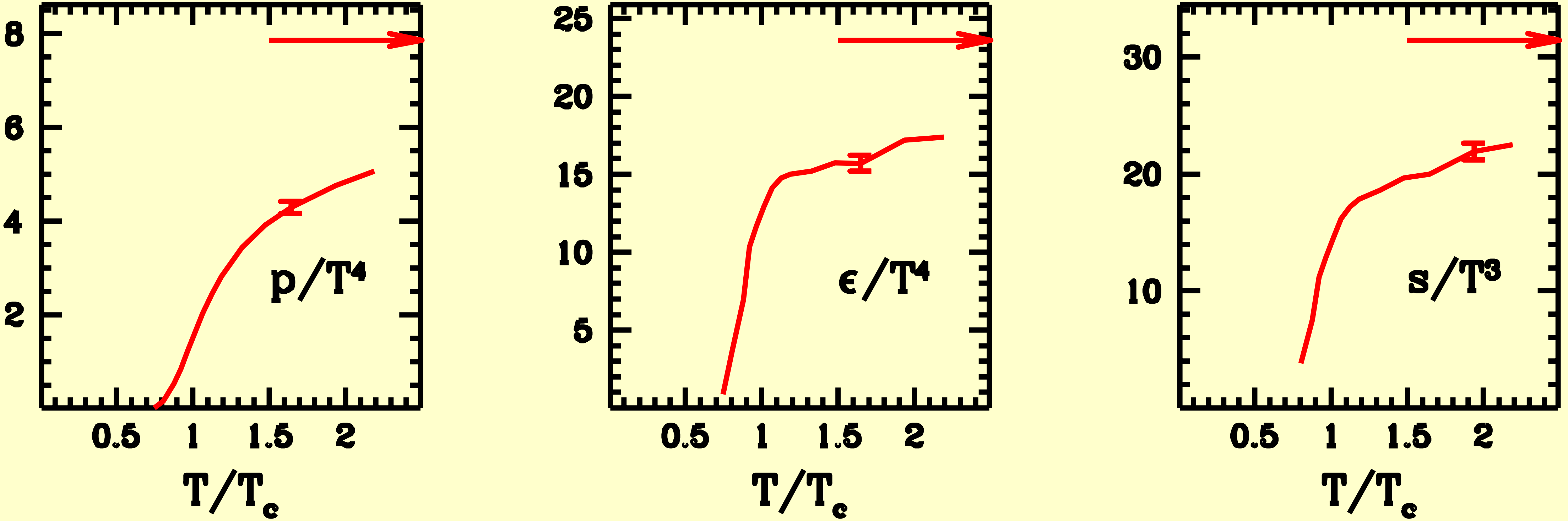
extrapolate to $m = 0$ → 2^{nd} order phase transition expected

$N_t = 6$: $T_c(m = 0) = 191(5)(2)$ MeV

The pressure,energy density and entropy density for $N_t = 4$

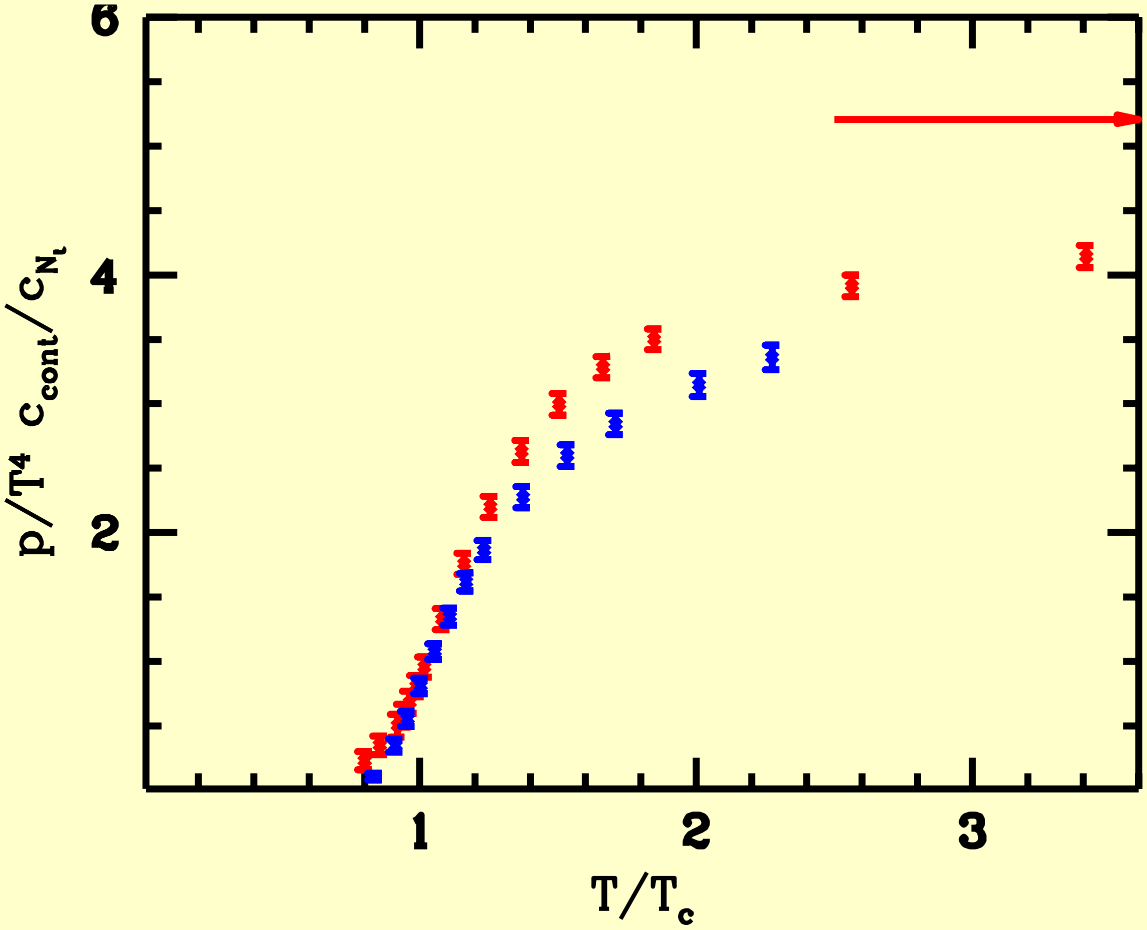


The pressure,energy density and entropy density for $N_t = 6$



Scaling of the pressure

Comparison of $N_t = 4$ and $N_t = 6$



- No good scaling yet. Most probably $N_t = 4$ is too coarse

$\rightarrow$ $N_t = 8$ might be needed for final continuum-extrapolated result

Summary, Conclusions

- Previous results on EoS suffer from several weaknesses
- New results improve on these points
- Transition temperature using different methods: $T_c \approx 189(8)$ MeV
- EoS is presented for two sets of lattice spacings
- Continuum-extrapolation already possible, but better to wait for even finer lattices