18th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Budapest, August 4-9, 2005

From Color Glass Condensate to Quark-Gluon Plasma through the event horizon

D. Kharzeev

BROOKHAVEN NATIONAL LABORATORY

based on work with K. Tuchin, hep-ph/0501234; Nucl.Phys.A

With thanks to

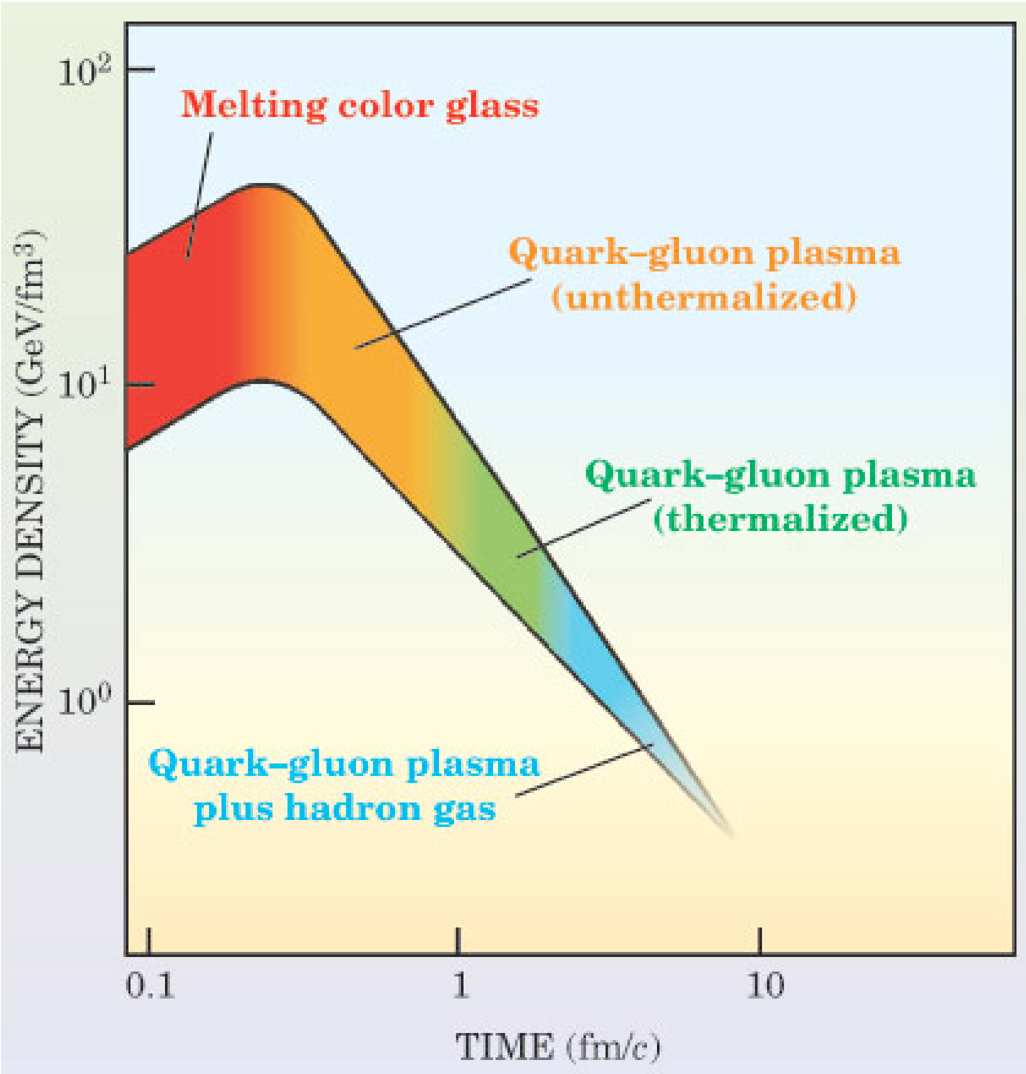
T. Csorgo (KFKI)
G. Dunne (UConn)
R. Glauber (Harvard)
E. Levin (Tel Aviv)
L. McLerran (BNL)
G. Nayak (Stony Brook)

for the ongoing discussions and collaborations

Outline

- Motivation
- Black holes and accelerating observers
- Event horizons and pair creation in strong fields
- Hawking phenomenon in the parton language
- Thermalization and phase transitions
  in relativistic nuclear collisions

The emerging picture



Big question:

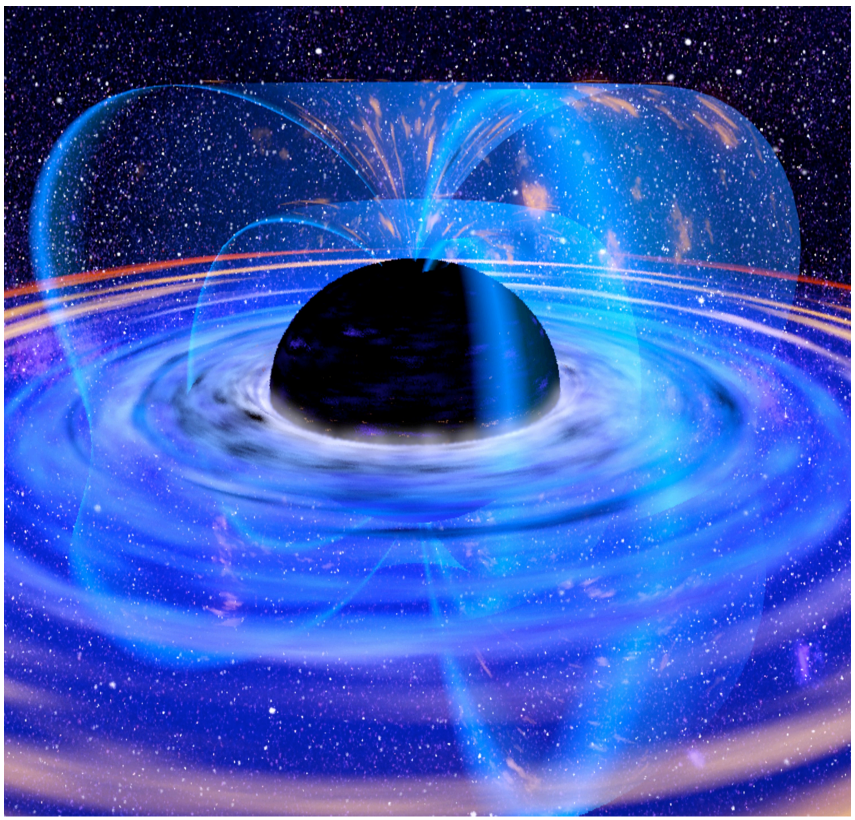
How does the produced matter thermalize so fast?

Non-perturbative phenomena in strong fields?

T. Ludlam,
L. McLerran,
Physics Today ’03

Black holes radiate

S.Hawking '74



Black holes emit thermal radiation with temperature

$$T = \frac{\kappa}{2\pi}$$

acceleration of gravity at the surface, $(4GM)^{-1}$

Similar things happen in non-inertial frames

Einstein's Equivalence Principle:

Gravity ⟷ Acceleration in a non-inertial frame

↓

An observer moving with an acceleration a detects a thermal radiation with temperature

$$T = \frac{a}{2\pi}$$

W.Unruh '76

In both cases the radiation is due to the presence of event horizon

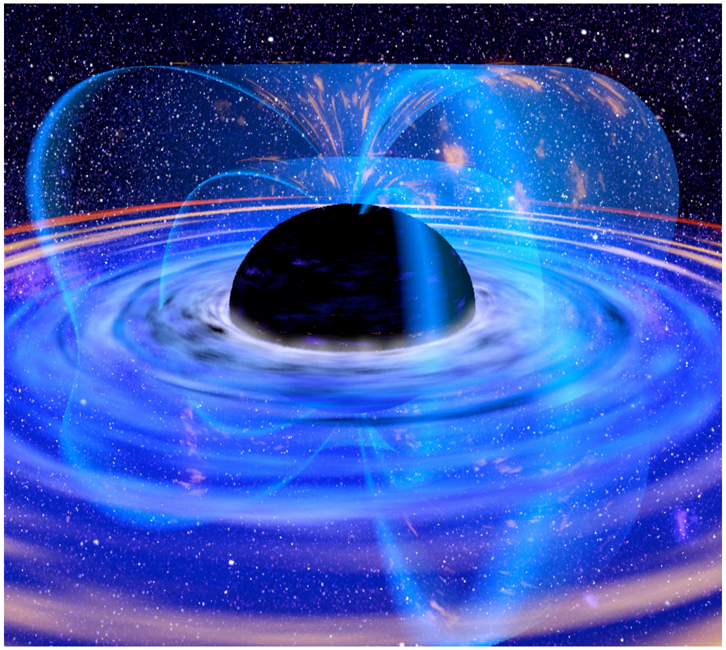
Black hole: the interior is hidden from
an outside observer;
Schwarzschild metric

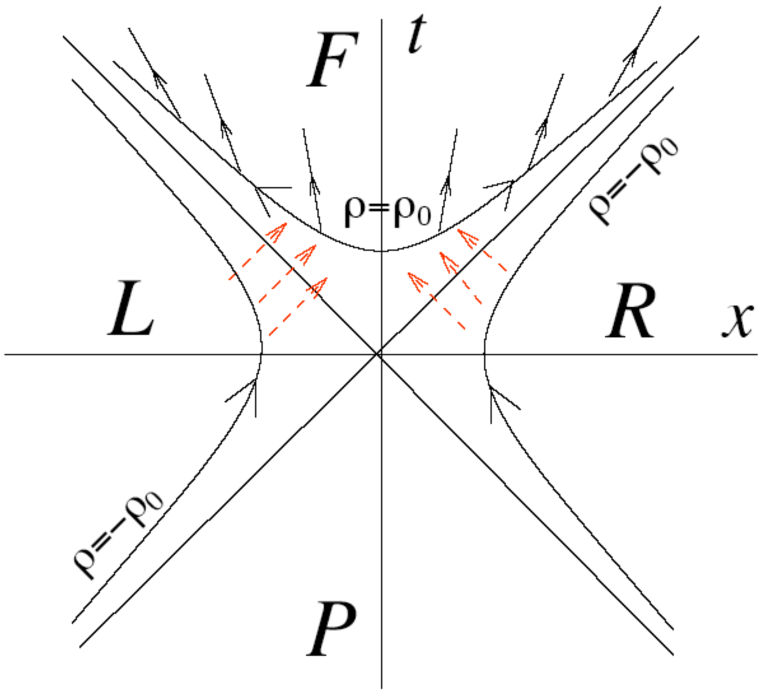
Accelerated frame: part of space-time is hidden
(causally disconnected) from
an accelerating observer;
Rindler metric

$$\rho^2 = x^2 - t^2, \qquad \eta = \frac{1}{2}\ln\left|\frac{t+x}{t-x}\right|$$

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - dx_\perp{}^2$$

Pure and mixed states: the event horizons





$$|O_{Out}\rangle\langle O_{Out}| = \sum_{Inside} |O_M\rangle\langle O_M| \qquad |O_L\rangle\langle O_L| = \sum_R |O_M\rangle\langle O_M|$$

mixed state — pure state — mixed state — pure state

Accelerating detector

- Positive frequency Green's function (m=0):

$$G^+(x,x') = \langle \phi(x)\phi(x') \rangle = -\frac{1}{4\pi^2\left[(t-t'-i\epsilon)^2 - |\vec{x}-\vec{x}|^2\right]}$$

- Along an inertial trajectory

$$G^+(\Delta\tau) = -\frac{1}{4\pi^2(\Delta\tau - i\epsilon)^2}$$

$$\vec{x} = \vec{x}_0 + \vec{v}t$$

- Along a uniformly accelerated trajectory

$$x = y = 0, z = (t^2 + a^{-2})^{1/2}$$

$$G^+(\Delta\tau) = -\frac{1}{4\pi^2} \sum_{n=-\infty}^{\infty} (\Delta\tau - 2\pi i\epsilon + in2\pi/a)^{-2}$$

Accelerated detector is effectively immersed into a heat bath at temperature $T_U = a/2\pi$

Unruh,76

An example: electric field

The force:

$$F = ma = eE$$

The acceleration:

$$a = \frac{eE}{m}$$

The rate:

$$R \sim \exp\left(-\frac{2\pi m}{a}\right) = \exp\left(-\frac{2\pi m^2}{eE}\right)$$

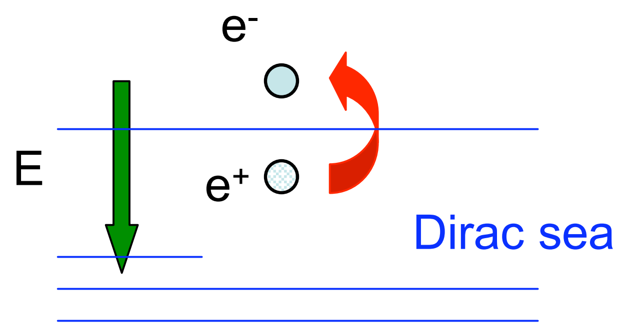
What is this?

Schwinger formula for the rate of pair production;

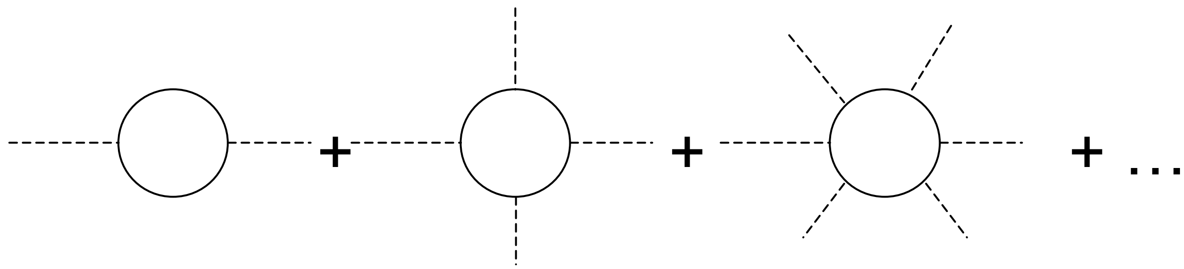
an exact non-perturbative QED result

factor of 2: contribution from the field

The Schwinger formula



$$R \sim \exp\left(-\frac{\pi m^2}{eE}\right)$$



The Schwinger formula

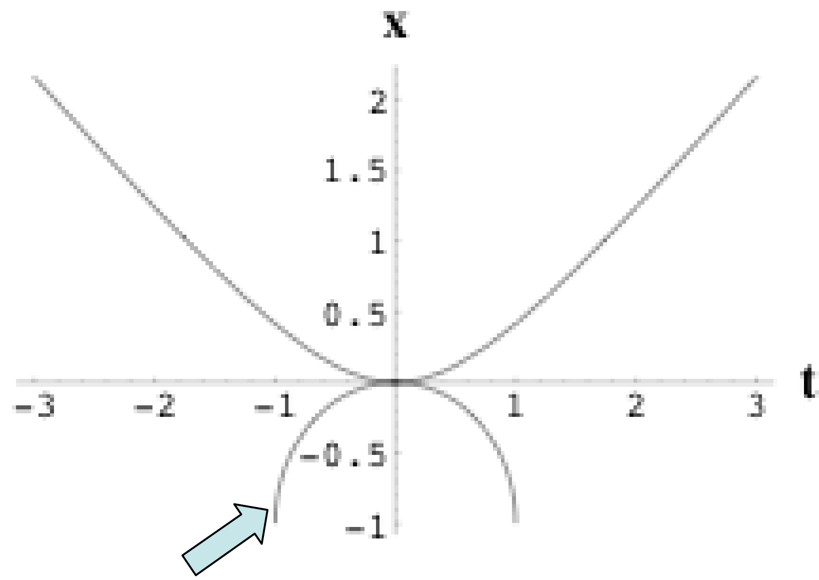
- Consider motion of a charged particle in a *constant electric field E*. Action is given by

$$S = \int (-m\, ds - e\, \phi\, dt)$$

Equations of motion yield the trajectory

$$x(t) = a^{-1}(\sqrt{1 + a^2 t^2} - 1)$$

where `a=eE/m` is the acceleration



Classically forbidden trajectory $t \rightarrow -it_E$

- Action along the classical trajectory:

$$S(t) = -\frac{m}{a} arcsh(at) + \frac{eE}{2a^2}\left(at(\sqrt{1+a^2t^2} - 2) + arcsh(at)\right)$$

- In Quantum Mechanics S(t) is an analytical function of t
- Classically forbidden paths contribute to $\operatorname{Im} S(t) = \frac{m\pi}{a} - \frac{eE\pi}{2a^2} = \frac{\pi m^2}{2eE}$
- Vacuum decays with probability

$$\Gamma_{V\to m} = 1 - \exp\left(-e^{-2\operatorname{Im} S}\right) \approx e^{-2\operatorname{Im} S} = e^{-\pi m^2/eE}$$

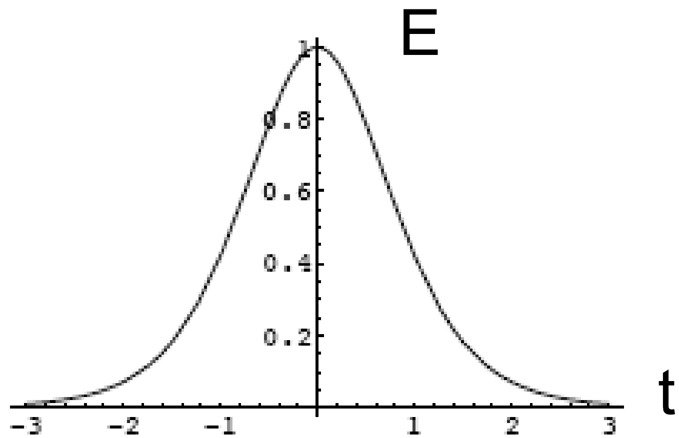
Sauter,31
Weisskopf,36
Schwinger,51

- Note: this expression can not be expanded in powers of the coupling - non-perturbative QED!

Pair production by a pulse

Consider a time dependent field

$$A^{\mu} = \left(0,0,0,-\frac{E}{k_0}\tanh(k_0 t)\right)$$



- Constant field limit $k_0 \to 0$
- Short pulse limit $k_0 \to \infty$

$$\Gamma \sim \exp\left[-\frac{2\pi|\vec{p}|}{k_0}\right]$$

a thermal spectrum with $T = \frac{k_0}{2\pi}$

Chromo-electric field: Wong equations

- Classical motion of a particle in the external non-Abelian field:

$$m\ddot{x}^{\mu} = gF^{q\mu\nu}\dot{x}_{\nu}I_{a}$$

The constant chromo-electric field is described by

$$A^{0}{}_{a} = -Ez\delta^{a3}, A^{i}{}_{a} = 0$$

Solution: vector I_3 precesses about 3-axis with I_3=const

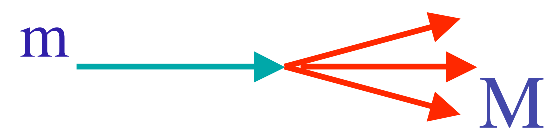
$$\ddot{x} = \ddot{y} = 0, m\ddot{z} = gE\dot{x}^{0}I_{3}$$

Effective Lagrangian: Brown, Duff, 75; Batalin, Matinian,Savvidy,77; Nayak, Nieuwenhuizen, 05

Strong interactions?

Consider a dissociation of a high energy hadron of mass m into a final hadronic state of mass M;

The probability of transition:



$$P(m \to M) = 2\pi |\mathcal{T}(m \to M)|^2 \rho(M)$$

Transition amplitude: $|\mathcal{T}(m \to M)|^2 \sim \exp(-2\pi M/a)$

In dual resonance model: $\rho(M) \sim \exp(4\pi\sqrt{b}M/\sqrt{6})$

$b = 1/2\pi\sigma$ – universal slope

Unitarity: $\Sigma_M P(m \to M) = \text{const}$,

$$\frac{a}{2\pi} \equiv T \leq \frac{\sqrt{6}}{4\pi\sqrt{b}}$$

limiting acceleration

Hagedorn temperature!

Fermi - Landau statistical model of multi-particle production

Enrico Fermi
1901-1954

Lev D. Landau
1908-1968

Hadron production
at high energies
is driven by
statistical mechanics;
universal temperature

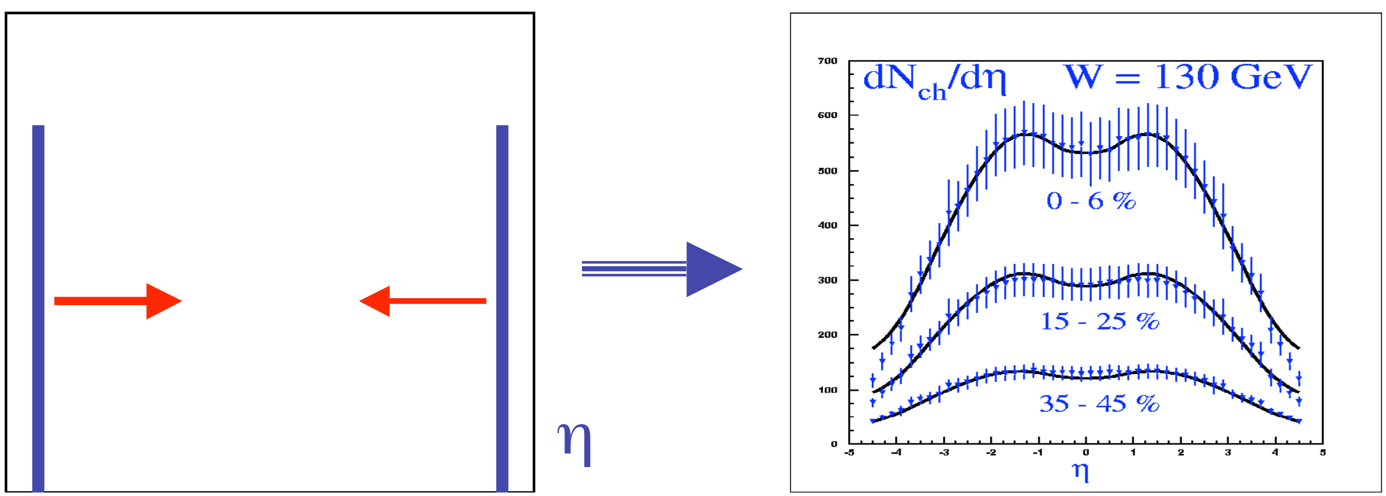
Where on Earth can one achieve the largest acceleration (deceleration) ?

Relativistic heavy ion collisions! -

stronger color fields: $$E \sim \sigma \quad \rightarrow \quad E \sim \frac{Q_s^2}{g}$$

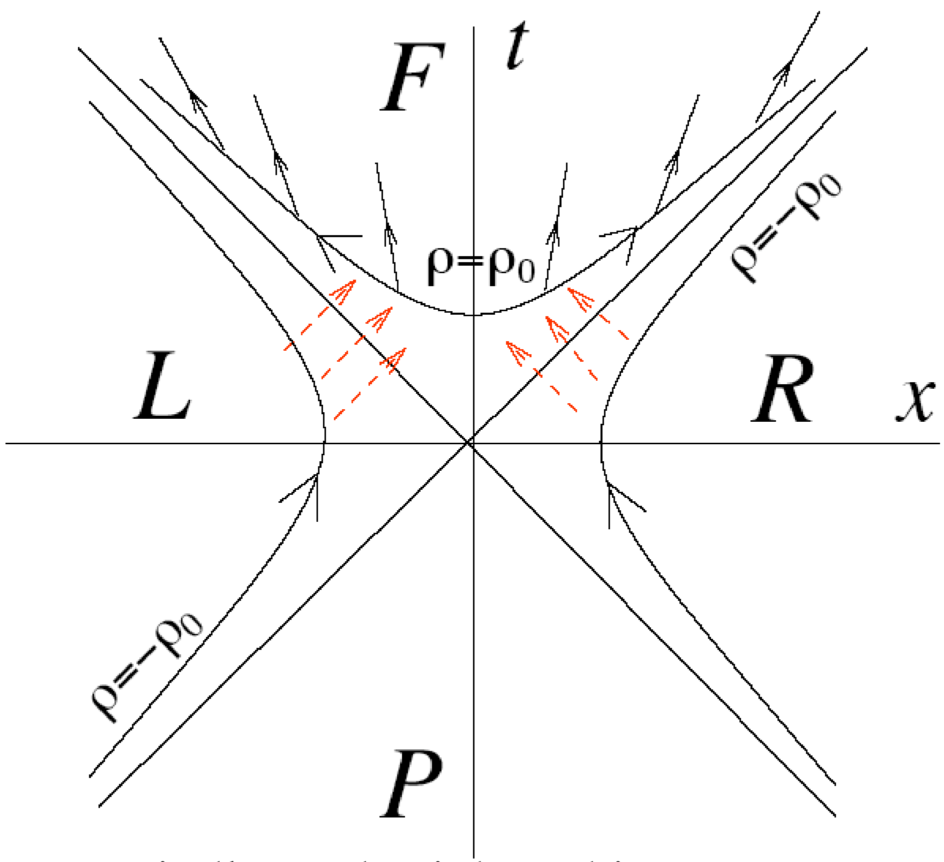
$$v_{initial} \simeq c; \quad v_{final} \simeq 0; \quad \Delta t \simeq 1/Q_s$$

$$a \simeq Q_s \sim 1 \text{ GeV}; \quad T = \frac{a}{2\pi} \sim 160 \text{ MeV}$$



Quantum thermal radiation at RHIC

$$T = \frac{a}{2\pi} \simeq \frac{Q_s}{2\pi}$$

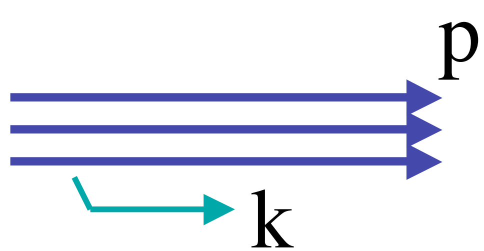


Rindler and Minkowski spaces

The event horizon emerges due to the fast decceleration $a \simeq Q_s$ of the colliding nuclei in strong color fields;

Tunneling through the event horizon leads to the thermal spectrum

Hawking phenomenon in the parton language



The longitudinal frequency of gluon fields in the initial wave functions is typically very small:

$$\omega \sim \frac{1}{t_{form}} \simeq \frac{k_\perp^2}{k_+} \ll k_\perp$$

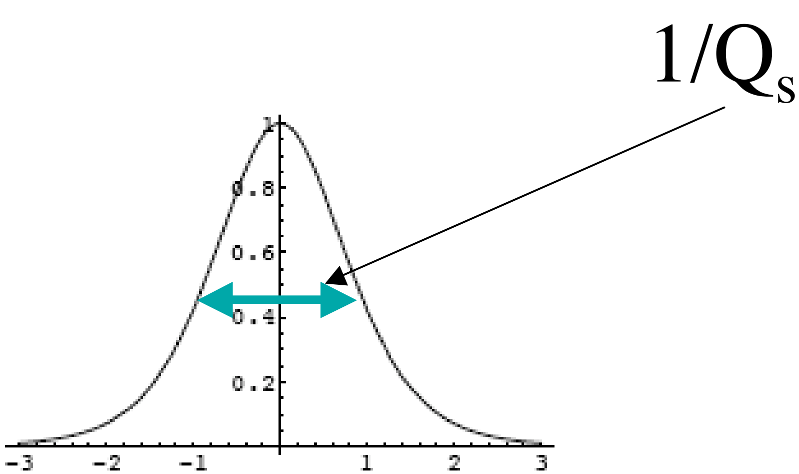
Parton configurations are frozen,
Gauge fields are flat in the longitudinal direction $G_{+-} = 0$

$$\vec{E} \perp \vec{H} \perp \vec{p}$$

But: quantum fluctuations (gluon radiation in the collision process) necessarily induce $G_{+-} \neq 0, E_z \neq 0$

Gluons produced at mid-rapidity have large frequency (c.m.s.) $\omega \simeq Q_s$

=> a pulse of strong chromo-electric field



Production of gluons and quark pairs with 3D thermal spectrum

$$\sim \exp\left[-\frac{2\pi|\vec{k}|}{Q_s}\right]$$

1) Classical Yang-Mills equations for the Weizsacker-Williams fields are unstable in the longitudinal direction => the thermal seed will grow

L.McLerran

(a link to the instability-driven thermalization?)

→ Talk by S.Mrowczynski

2) Non-perturbative effect, despite the weak coupling (non-analytical dependence on g)

→ Talk by Yu.Kovchegov

3) Thermalization time

c~1 (no powers of 1/g)

$$\tau = c \, \frac{\pi}{Q_s}$$

Deceleration-induced phase transitions?

- Consider Nambu-Jona-Lasinio model in Rindler space

$$L = \overline{\psi} i \gamma^{\nu}(x) \nabla_{\nu} \psi(x) + \frac{\lambda}{2N}\left[(\overline{\psi}(x)\psi(x))^2 + (\overline{\psi}(x) i\gamma_5 \psi(x))^2 \right]$$

- Commutation relations:

e.g. Ohsaku,04

$$\left\{ \gamma_{\mu}(x), \gamma_{\nu}(x) \right\} = 2 g_{\mu\nu}(x)$$

- Rindler space:

$$\rho^2 = x^2 - t^2, \qquad \eta = \frac{1}{2}\ln\left|\frac{t+x}{t-x}\right|$$

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - dx_{\perp}{}^2$$

Gap equation in an accelerated frame

- Introduce the scalar and pseudo-scalar fields

$$\sigma(x) = -\frac{\lambda}{N}\overline{\psi}(x)\psi(x) \qquad \pi(x) = -\frac{\lambda}{N}\overline{\psi}(x)i\gamma_5\psi(x)$$

- Effective action (at large N):

$$S_{eff} = \int d^4x\sqrt{-g}\left(-\frac{\sigma^2+\pi^2}{2\lambda}\right) - i\ln\det(i\gamma^\nu\nabla_\nu - \sigma - i\gamma_5\pi)$$

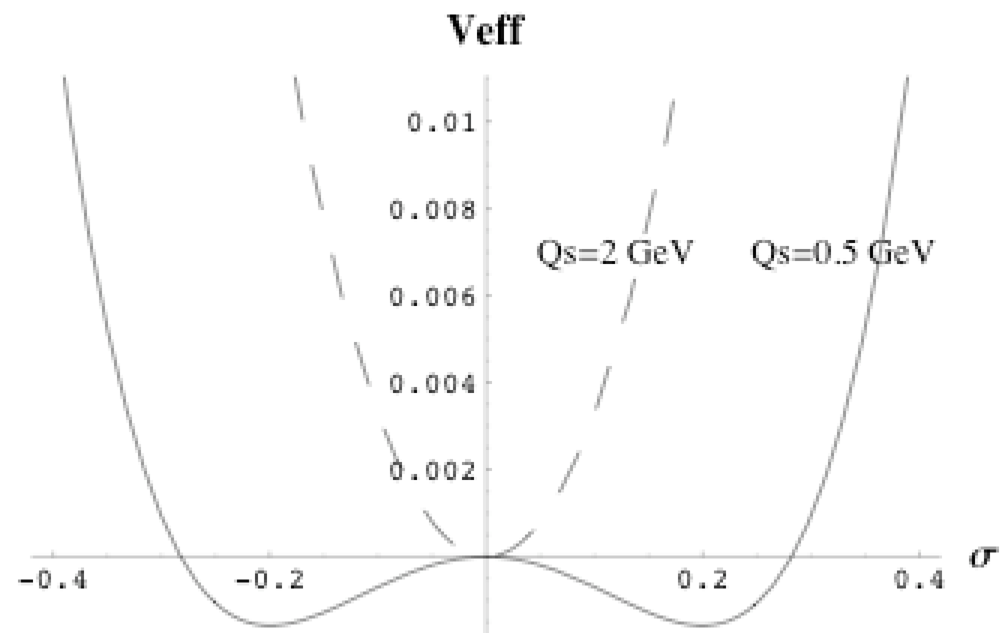
- Gap equation:

$$\sigma = -\frac{2i\lambda\sigma}{a}\int\frac{d^2k}{(2\pi)^2}\int_{-\infty}^{\infty}d\omega\,\frac{\sinh(\pi\omega/a)}{\pi^2}\left\{(K_{i\omega/a+1/2}(\alpha/a))^2 - (K_{i\omega/a-1/2}(\alpha/a))^2)\right\}$$

where $\alpha^2 = k_\perp{}^2 + \sigma^2$

Ohsaku'04

Rapid deceleration induces phase transitions



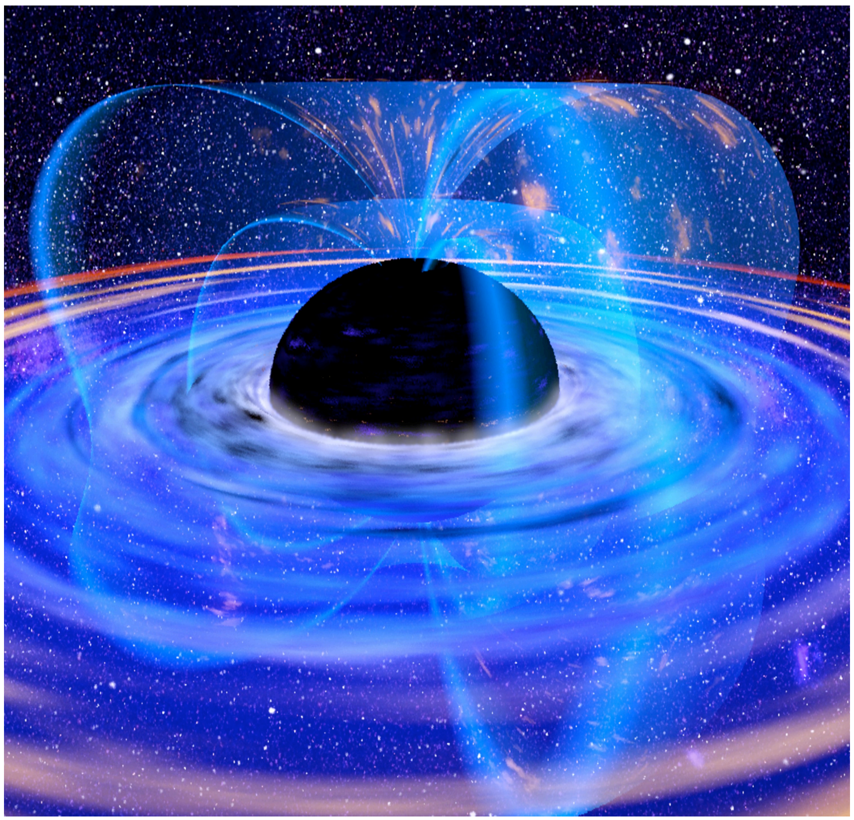
Nambu-
Jona-Lasinio model
(BCS - type)

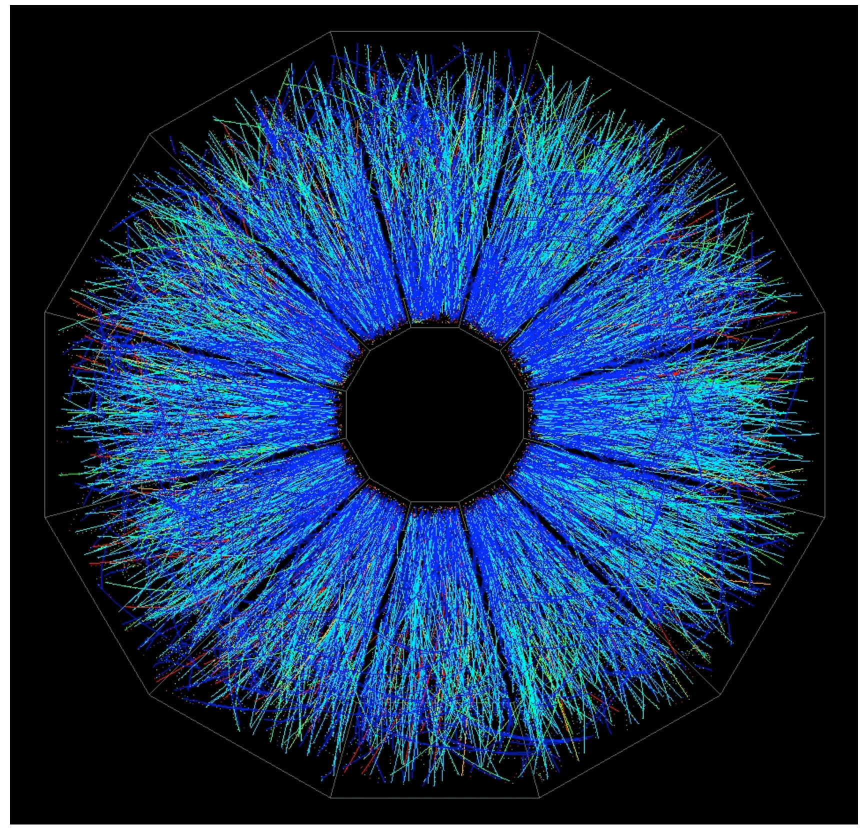
$$V_{eff}(Q_s,\sigma) = -\frac{1}{96\pi^2}\left(Q_{s,c}{}^2 - Q_s{}^2\right)\sigma^2 + \frac{1}{32\pi}\sigma^4 + \ldots$$

Similar to phenomena in the vicinity of a large black hole:
Rindler space ≈ Schwarzschild metric

A link between General Relativity and QCD?
solution to some of the RHIC puzzles?

Black holes ⟷ RHIC collisions





Additional slides

Thermal radiation can be understood as a consequence of tunneling through the event horizon

Let us start with relativistic classical mechanics:

$$v(t) = \frac{a\,t}{\sqrt{1 + a^2\,t^2}}$$

velocity of a particle moving with an acceleration a

classical action:

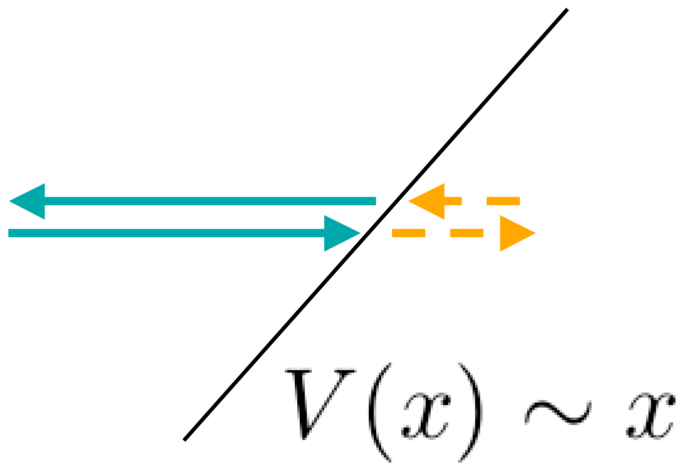
$$S(\tau) = -m \int^{\tau} dt\,\sqrt{1 - v^2(t)}$$

$$= -\frac{m}{a}\,\mathrm{arcsinh}(a\,\tau)$$

it has an imaginary part…

well, now we need some quantum mechanics, too:

$$\mathrm{Im}\, S(\tau) \;=\; \frac{m\,\pi}{a}$$



The rate of tunneling under the potential barrier:

$$R \sim \exp(-2\ \mathrm{Im} S) = \exp\left(-\frac{2\pi\ m}{a}\right)$$

This is a Boltzmann factor with $T = \dfrac{a}{2\pi}$

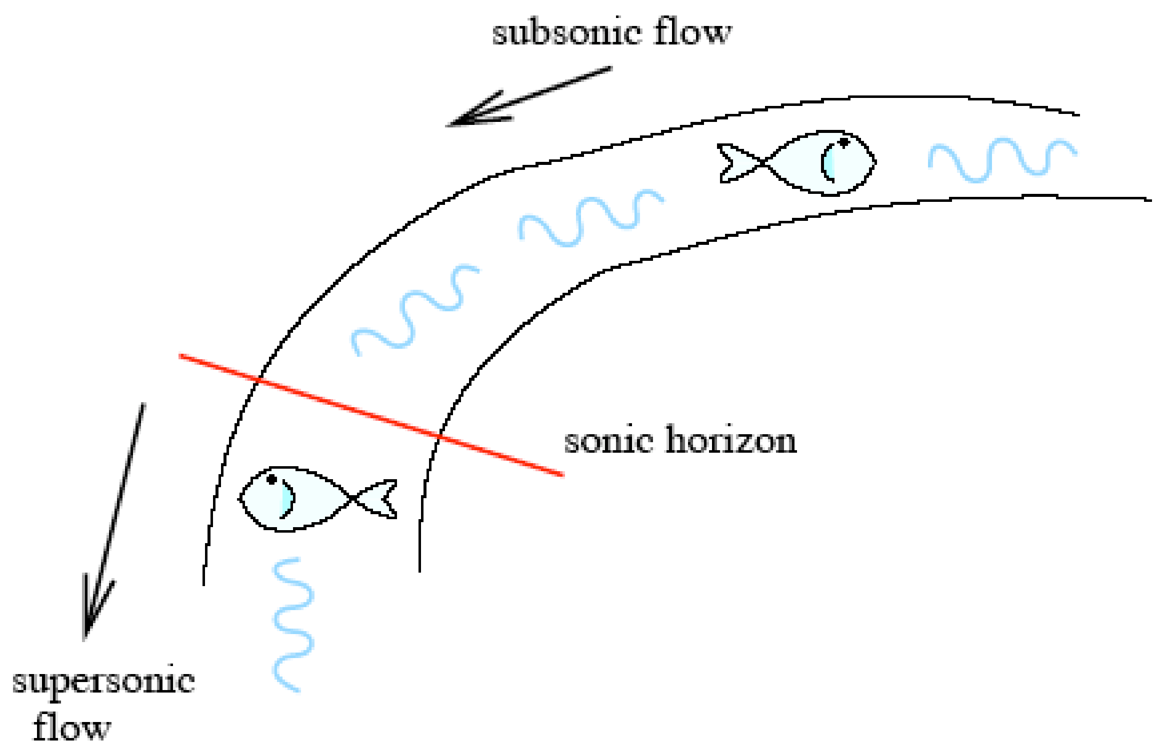
Electric fields? Lasers?

Laser Parameters						
		Optical [43]	X-ray FEL			
		Focus: Diffraction limit	Design [33]	Focus: Available [44]	Focus: Goal [37]
Wavelength	λ	1 μm	0.4 nm	0.4 nm	0.15 nm
Photon energy	$\hbar\omega = \frac{hc}{\lambda}$	1.2 eV	3.1 keV	3.1 keV	8.3 keV
Peak power	P	1 PW	110 GW	1.1 GW	5 TW
Spot radius (rms)	σ	1 μm	26 μm	21 nm	0.15 nm
Coherent spike length (rms)	$\triangle t$	500 fs $\div$ 20 ps	0.04 fs	0.04 fs	0.08 ps
Derived Quantities					
Peak power density	$S = \frac{P}{\pi\sigma^2}$	$3 \times 10^{26}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$5 \times 10^{19}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$8 \times 10^{23}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$7 \times 10^{31}\ \frac{\mathrm{W}}{\mathrm{m}^2}$
Peak electric field	$\mathcal{E} = \sqrt{\mu_0\, c\, S}$	$4 \times 10^{14}\ \frac{\mathrm{V}}{\mathrm{m}}$	$1 \times 10^{11}\ \frac{\mathrm{V}}{\mathrm{m}}$	$2 \times 10^{13}\ \frac{\mathrm{V}}{\mathrm{m}}$	$2 \times 10^{17}\ \frac{\mathrm{V}}{\mathrm{m}}$
Peak electric field/critical field	$\mathcal{E}/\mathcal{E}_c$	3×10^{-4}	1×10^{-7}	1×10^{-5}	0.1
Photon energy/e rest energy	$\frac{\hbar\omega}{m_e c^2}$	2×10^{-6}	0.006	0.006	0.02
Adiabaticity parameter	$\gamma = \frac{\hbar\omega}{e\,\mathcal{E}\,\lambda\!\!\!^{-}_e}$	9×10^{-3}	6×10^{4}	5×10^{2}	0.1

Table 1: Laser parameters and derived quantities relevant for estimates of the rate of spontaneous e^+e^- pair production. The column labeled "Optical" lists parameters which are typical for a

compilation by A.Ringwald

Condensed matter black holes?



Unruh '81;
e.g. T. Vachaspati,
cond-mat/0404480

"slow light"?