COLD, DENSE,

(but not asymptotically dense)

QUARK MATTER:

PUZZLES AND HINTS

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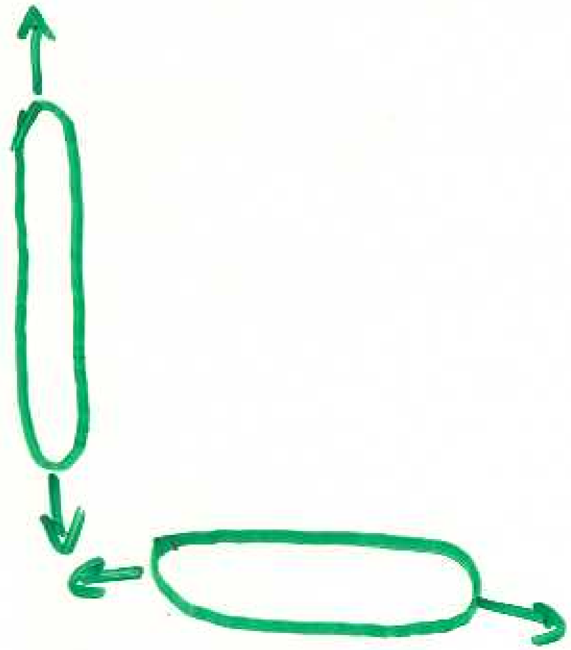
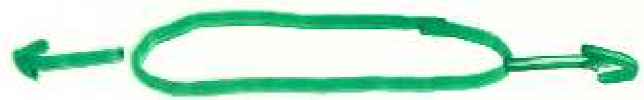
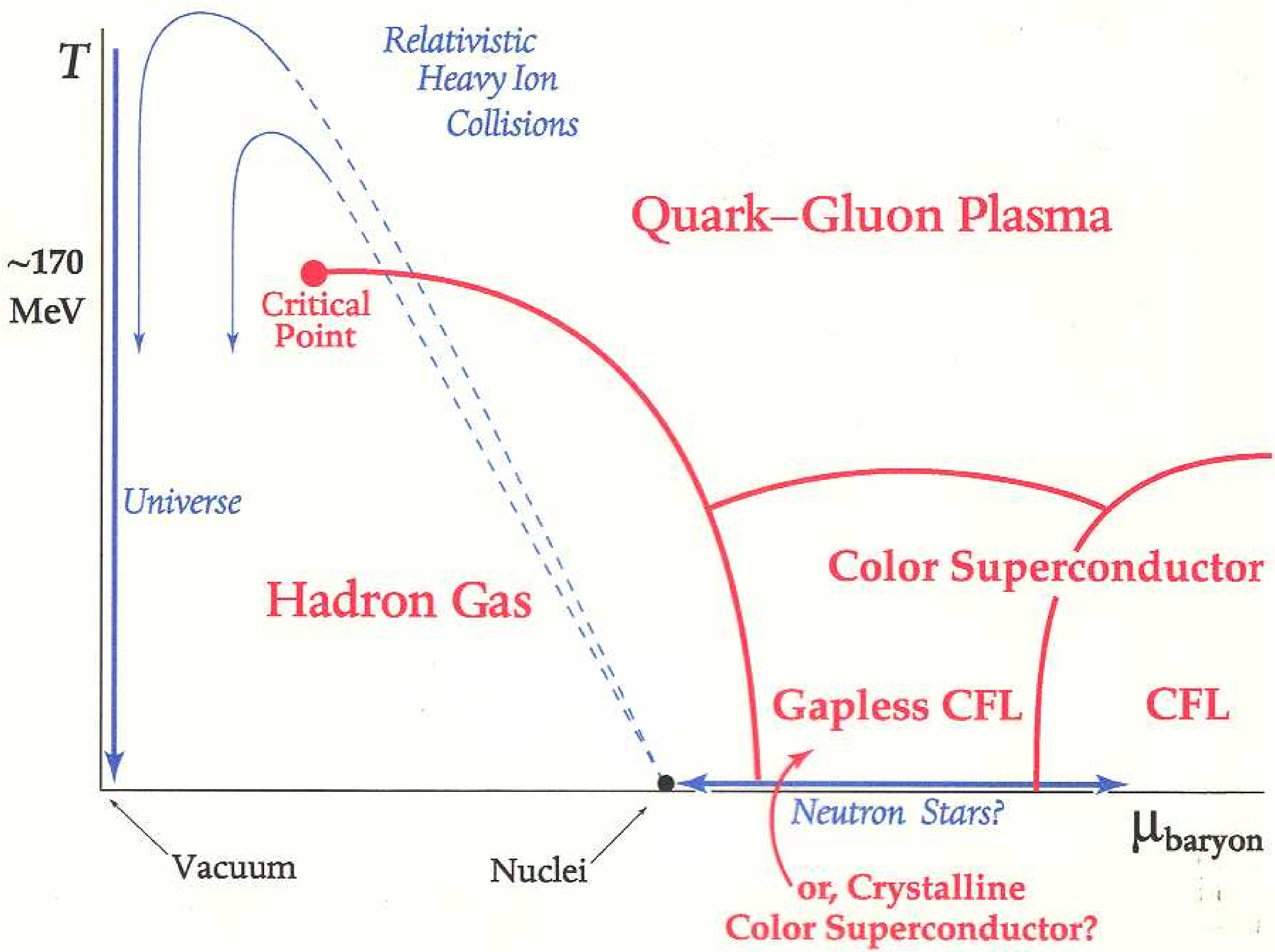
QUARK MATTER 2005, BUDAPEST, 7/8/2005



THE "IN-BETWEEN REGIONS":
PUZZLES & HINTS,
AND :: INTERESTING



EXPLORING *the* PHASES *of* QCD



HIGH DENSITY & LOW TEMPERATURE

Whereas at high T entropy wins
→ quark-gluon plasma with symmetries of QCD Lagrangian manifest

At large μ with small T we find quark matter with new patterns of order:

- Color superconductivity
- Color-Flavor Locking
- Crystalline Color Superconductivity

⋮

- At large enough μ (to be defined below) we have <u>answers</u>.
- At large but notso large μ, we have a <u>puzzle</u>, and <u>hints</u>.
- How can we use astrophysical observations of compact stars to provide answers?

WHY COLOR SUPERCONDUCTIVITY?

Large $\mu \to$ quarks filling Fermi sea up to a large Fermi energy. (E_F)

asymptotic freedom $\to$ weak interactions between quarks at Fermi surface.

BUT any attractive interaction, no matter how weak, $\to$ COOPER PAIRS ; $\langle qq \rangle$

One gluon exchange (& instanton interaction) attractive in color $\bar{3}$.
(no need to resort to phonons; $\therefore$ superconductivity more robust in QCD than in metals. Higher T_c/E_F.)

$\langle qq \rangle$, ie Cooper pairs of quarks,
$\Rightarrow$ - electric & color currents superconduct
- mass for photon & (some) gluons (?)
- Meissner effects. (Magnetic & color magnetic fields excluded.)

Barrois; Bailin & Love

GAP AND T_c

Much work (that I will not review) suggests that @ $\mu_q \sim 500$ MeV [$\sim 10\times$ nuclear density]

$$\Delta \lesssim 100 \text{ MeV}$$

$$T_c \lesssim 50 \text{ MeV}$$

Note: $T_c / E_F \sim 1/10 \rightarrow$ THIS is high T_c s.c.!

Two classes of methods $\sim$ agree:

i) models normalized to $\mu = 0$ physics

(Alford, K.R, Wilczek, Rapp, Schäfer, Shuryak, Velkovsky, Berges, Carter, Diakonov, Evans, Hsu, Schwetz, ...

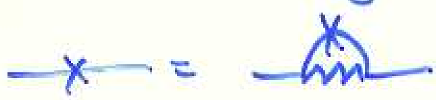
ii) weak-coupling QCD calculations, valid for $\mu \rightarrow \infty$; $g \rightarrow 0$. (Quantitatively, valid for $g \lesssim 1$ which means $\mu \gtrsim 10^8$ MeV K.R, Shuster)

$$\frac{\Delta}{\mu} \sim \underbrace{256\pi^4 e^{-\frac{\pi^2+4}{8}} \left(\frac{N_f}{2}\right)^{5/2}}_{} \underbrace{\frac{1}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g}\right)}_{}$$

Schaefer, Wilczek; Pisarski, Rischke; Hong, Miransky, Shovkovy, Wijewardhana; Evans, Hsu, Schwetz; Brown, Liu, Ren; Beane, Bedaque, Savage; K.R, Shuster; Rischke, Wang; ....

Son

[$\Delta \sim \exp(-1/g)$ comes from divergence in small angle scattering via exchange of unscreened magnetic gluons:



$$\rightarrow 1 = g^2 \underbrace{\ln \frac{\Delta}{\mu}}_{\text{BCS}} \underbrace{\ln \frac{\Delta}{\mu}}_{\text{collinear divergence}}$$

CFL

In cold quark matter, quarks near their Fermi surfaces pair
→ color superconductivity

Pattern of pairing:

$$\langle \psi^{\alpha}_{a} C\gamma_5 \psi^{\beta}_{b} \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}$$

(β ← color; a ← flavor; $C\gamma_5$: Lorentz scalar)

- antisymmetry in color + Dirac indices energetically favored; flavor antisym. forced by Pauli
- If density great enough that M_s can be neglected, $\Delta_1 = \Delta_2 \simeq \Delta_3$
- All 9 quarks pair, maximizing condensation energy; leaves largest symmetry unbroken
- Demonstrated rigorously at asymptotic density.
- unbroken symmetries all are color + flavor

	ru	gd	bs	rd	gu	bu	rs	gs	bd
ru		$-\Delta_3$	$-\Delta_2$						
gd	$-\Delta_3$		$-\Delta_1$						
bs	$-\Delta_2$	$-\Delta_1$							
rd					Δ_3				
gu				Δ_3					
bu							Δ_2		
rs						Δ_2			
gs									Δ_1
bd								Δ_1	

Define

$$\tilde{Q} = \begin{pmatrix} 2/3 \\ -1/3 \\ -1/3 \end{pmatrix} \text{for } \begin{matrix} u \\ d \\ s \end{matrix} + \begin{pmatrix} -2/3 \\ 1/3 \\ 1/3 \end{pmatrix} \text{for } \begin{matrix} r \\ g \\ b \end{matrix}$$

and check $\tilde{Q} = 0$ for every pair in the condensate.

$\Rightarrow$ One linear combination of photon + gluon does <u>not</u> get "Meissnered".

$$U(1)_{EM} \times SU(3)_{color} \rightarrow U(1)_{\tilde{Q}}$$

COLOR-FLAVOR LOCKED QUARK MATTER

- occurs for $\mu \to \infty$, and at any μ if $m_s = m_u = m_d$
- all 9 quarks pair and are gapped
- superfluid
- chiral symmetry spontaneously broken, by a new mechanism (CFL).
  - ⇒ "pions" and "kaons" lightest excitations
    - massless if $m_s = m_u = m_d = 0$
    - ~ few MeV mass ($\ll \Delta$) for real $m_{s,u,d}$.
    - "K_0" may condense
- Unbroken gauged $U(1) \to$ massless photon
- As long as T < meson masses ~ few MeV:
  - Transparent insulator (neutral without electrons)
    - index of refraction and reflection/refraction coefficients known
  - Very small specific heat, neutrino emissivity, viscosity. Good thermal conductor
- All these properties, and more, rigorously calculable in $\mu \to \infty$, $g \to 0$ limit. Chiral symmetry breaking and all its consequences understood at high density.
- Occurs in nature wherever $\mu > m_s^2 / 2\Delta$. (density depend.)
- What are the properties of quark matter at lower density??? ...

WHAT CAN BE CALCULATED?

from QCD from first principles?

- At asymptotic densities, answer is "everything"; more than in any other circumstance in QCD.
  - in the CFL phase, there are no unresolved nonperturbative ambiguities: no gapless fermions; no massless gluons. No IR difficulties.
  - calculation of Δ is nonperturbative, but controlled by smallness of g.
  - analogues of confinement and chiral symmetry breaking are calculable at weak coupling.

- At potentially accessible densities, g not small. Means Δ cannot be calculated precisely (barring a major lattice QCD breakthrough.)

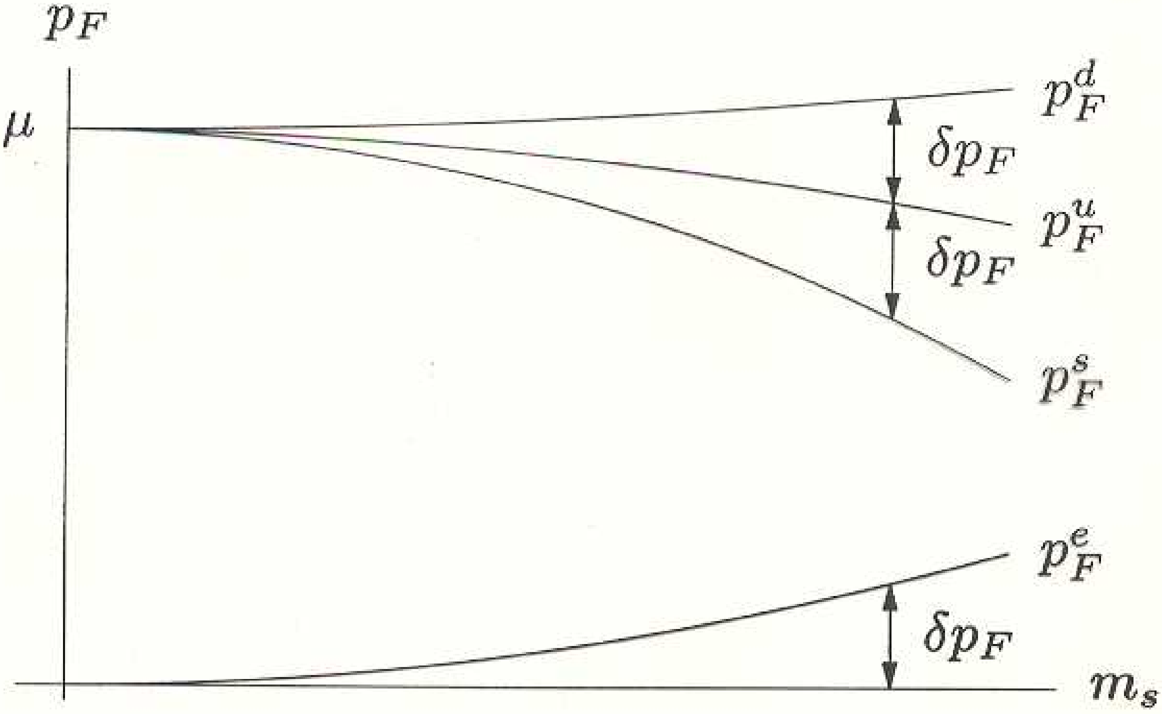
BUT: if you take Δ as given (ie treat as a parameter whose value known at order of magnitude level) then <u>many</u> physical properties calculable in terms of Δ.

Eg: specific heat, thermal conductivity, index of refraction, neutrino opacity, neutrino emissivity, shear viscosity, bulk viscosity, ...

[Many of these described within an effective field theory for the Goldstone bosons, whose parameters are determined by Δ.]

INTERMEDIATE DENSITY QUARK MATTER

- M_s important
- For orientation, consider noninteracting quarks, $M_u = M_d = 0$ $M_s \neq 0$, impose electrical neutrality and weak eqbm:



- In noninteracting quark matter, $\delta p_F \simeq \frac{M_s^2}{4\mu}$
- Motivates result that CFL pairing "breaks" when $\frac{M_s^2}{4\mu} > \Delta$??
- Also, when CFL "breaks", no residual <ud> pairing either. Alford, KR

WHAT REPLACES CFL, AT LOWER μ?

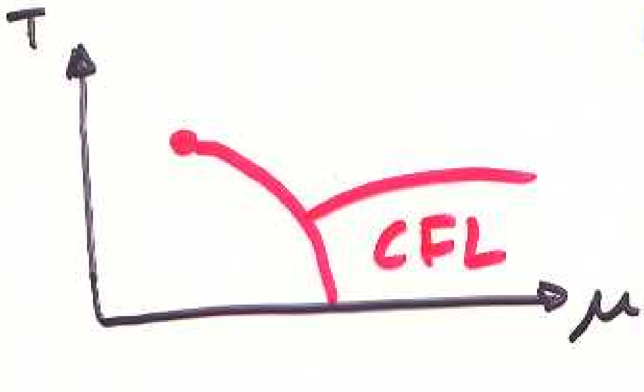
We don't yet know.....

We do know:

- CFL pairing is unstable once $\mu < \frac{M_s^2}{2\Delta}$

  (Alford Kouvaris KR)

  and stable for larger μ.
- $\therefore$ If Δ large enough & M_s not too large, CFL quark matter is stable all the way down to transition to nuclear matter.



eg: $M_s = 300$ MeV
$\Delta > 125$ MeV
or
$M_s = 200$ MeV
$\Delta > 55$ MeV

QUESTIONS:

What if less symmetrically paired quark matter intervenes? Ie, what are properties of quark matter with $\mu < M_s^2/2\Delta$?

What are astrophysical consequences if neutron stars have CFL cores?

LESS SYMMETRICALLY PAIRED Q.M.

Gapless CFL Phase?

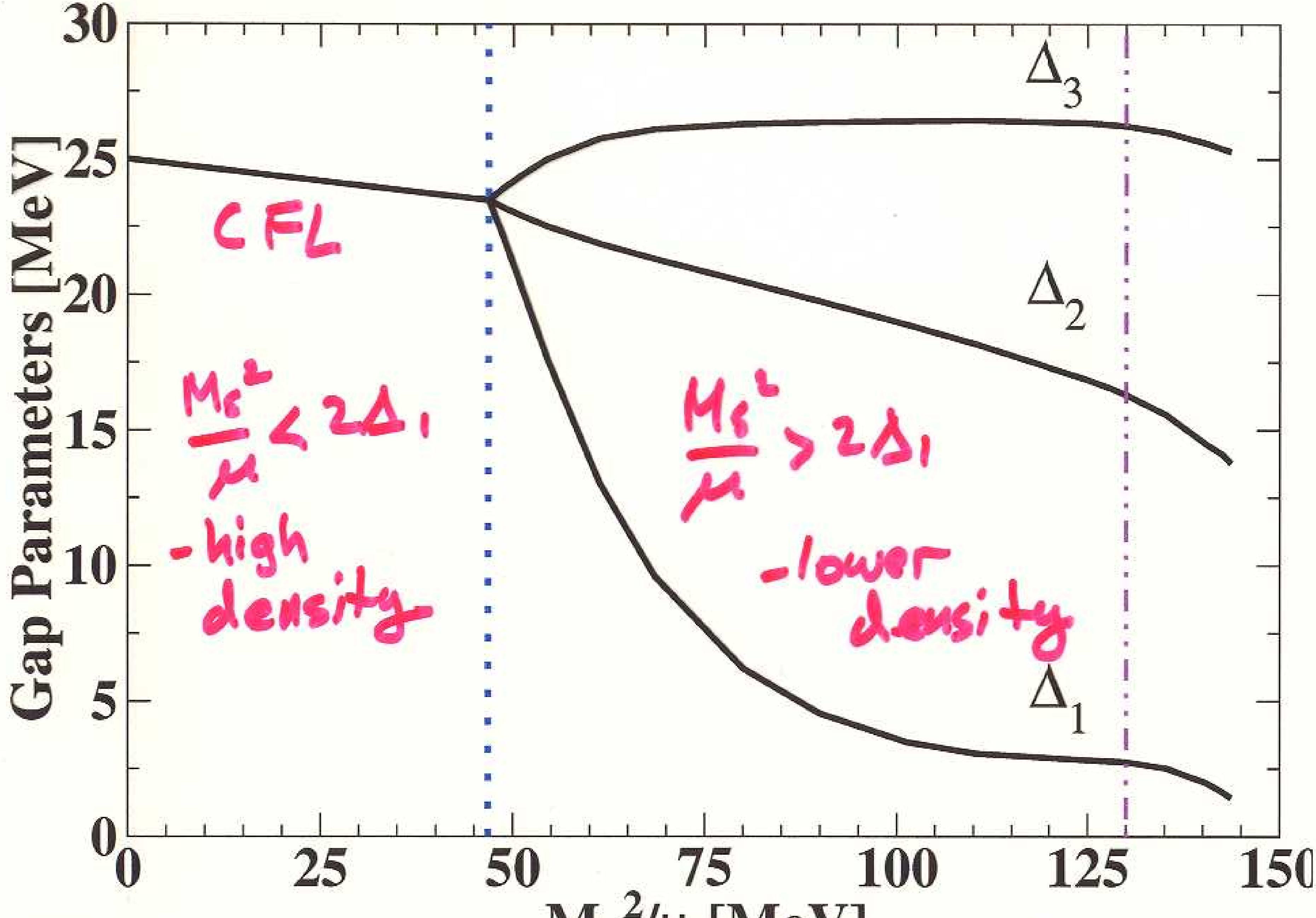
- 2nd highest density phase within a spatially uniform ansatz
- nice distinctive astrophysical signature (Alford Jotwani Kouvaris Kundu KR)
- unstable to currents → inhomogeneity (Huang Shovkovy; Casalbuoni et al; Giannakis Ren; Fukushima)

Crystalline Color Superconductivity?

- may be the answer, but:
- prior to last week, analyzed only in 2-flavor setting, without imposing neutrality
- potential for astrophysical signatures, (Alford, Bowers, KR) but not yet analyzed sufficiently to say how distinctive

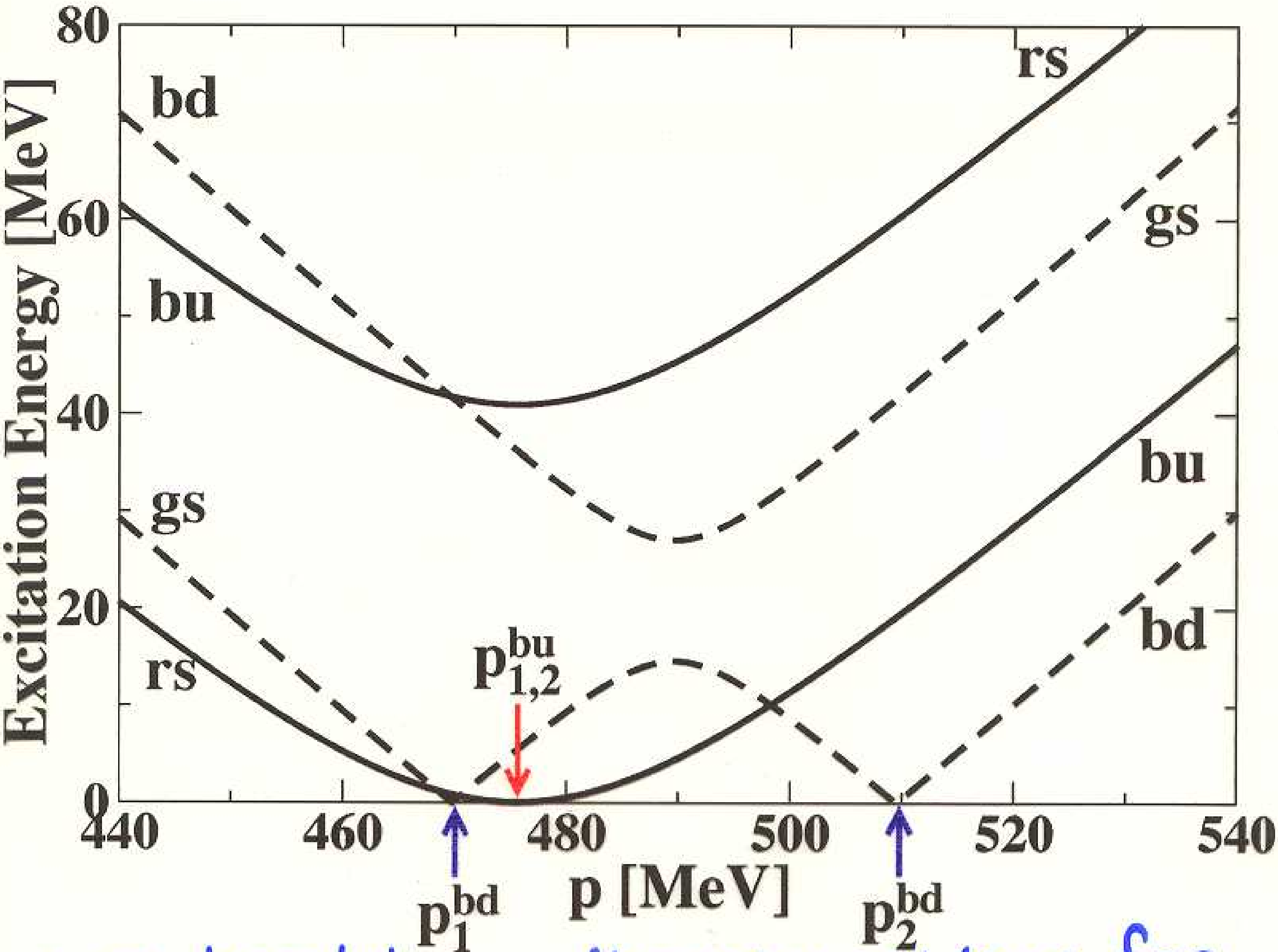
THE GAPLESS CFL PHASE

- All 9 quarks still pair, with same partners as before. BUT:
  - $\Delta_1 < \Delta_2 < \Delta_3$
  - there are shells in momentum space containing <u>unpaired</u> quarks. Eg: for some momenta, find bd quarks with no gs quarks with which to pair.
  - bd - gs pairing disrupted; Fermi surfaces split.
  - Nonzero density of electrons needed to maintain neutrality
  - CFL $\rightarrow$ gCFL transition is an insulator $\rightarrow$ conductor transition, 2nd order at $T=0$ and crossover at $T>0$.



We shall see that this phase is gapless. Same unbroken U(1) sym. as CFL.

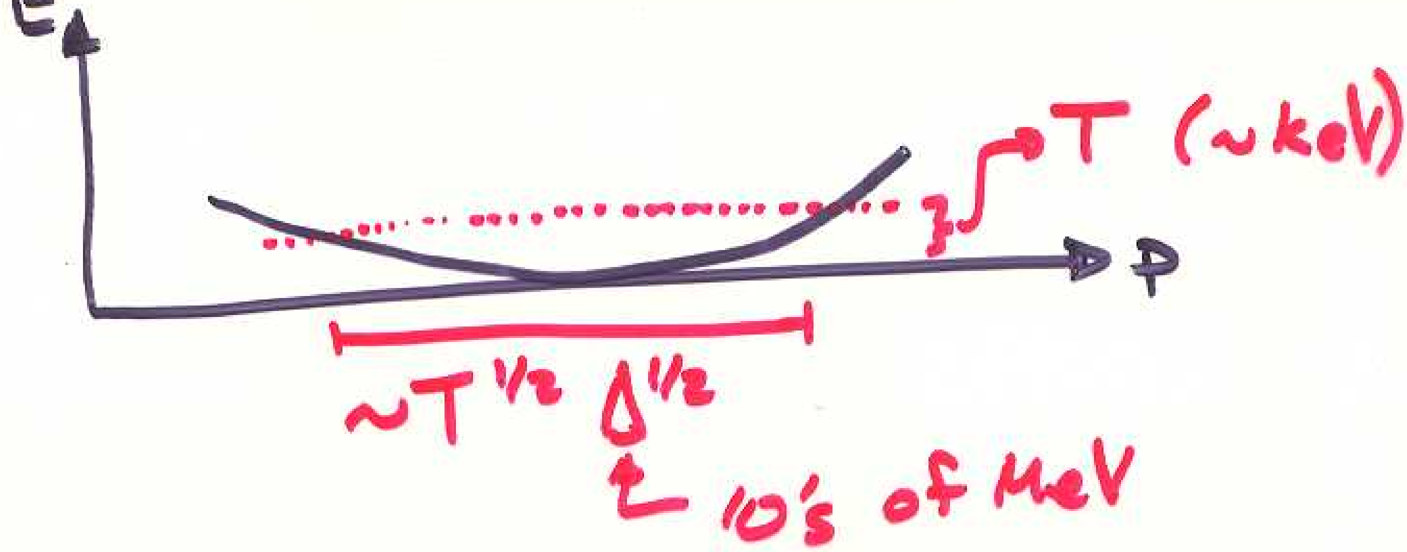
gCFL DISPERSION RELATIONS



- conventional linear dispersion relations for gapless fermions at two momenta (with unpaired quarks between these momenta)
- unconventional "grazing" dispersion relation, practically quadratic, at a third momentum
  - characteristic of and unique to gCFL
  - due to the way electric neutrality is maintained, not due to any fine tuning
  - and, has consequences....

CONSEQUENCES OF A "GRAZING" DISPERSION RELATION

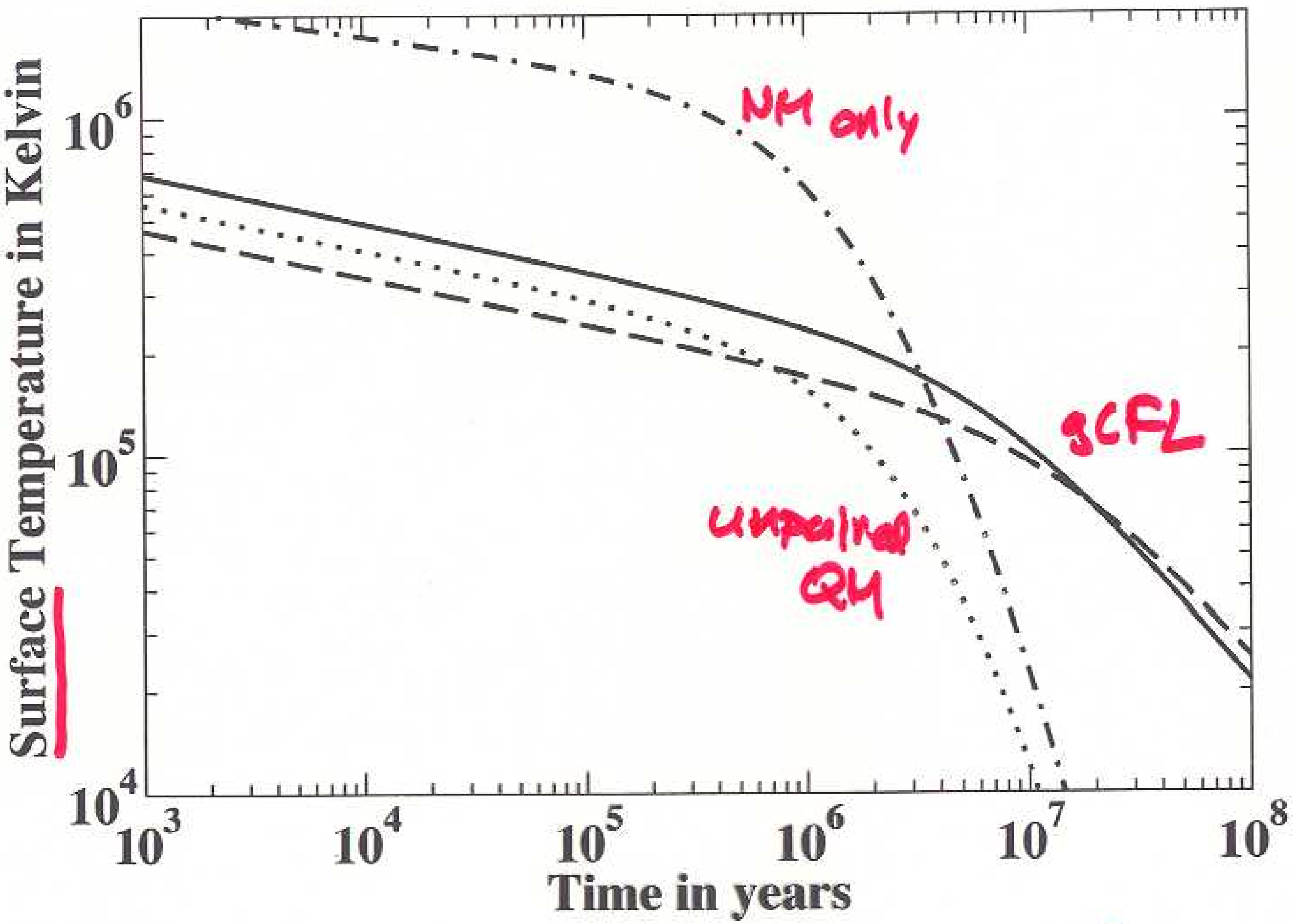
Alford Jotwani Kouvaris Kundu KR



$\Rightarrow C_V \sim P_F^2 T^{1/2} \Delta^{1/2} \gg P_F^2 T$

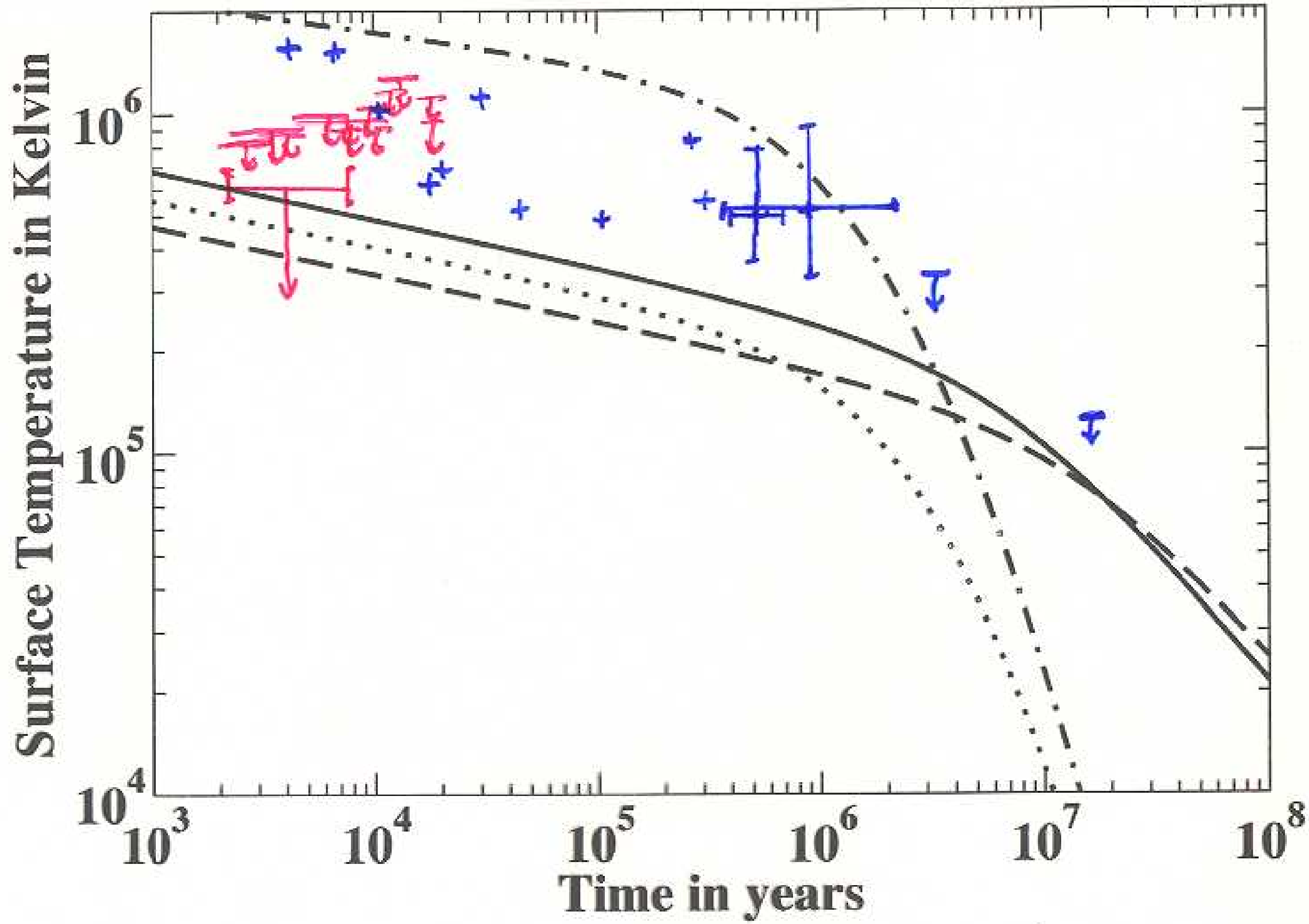
$\Rightarrow$ If present, a gCFL layer acts like a "hot water bottle", keeping aging neutron stars anomalously warm late in their life, because of its large heat capacity.

A HOT WATER BOTTLE FOR AGING NEUTRON STARS



- $10^3 - 10^5$ years : gCFL → colder stars, as for many scenarios where direct Urca allowed
- 10^7 → years : unique signature of gCFL quark matter, due to its characteristic dispersion rel.

COMPARISON OF COOLING OF TOY STARS TO DATA ON REAL STARS



⊥ : upper bounds on T from nonobservation of neutron stars in 14 supernova remnants. Kaplan et al.

+ : measurements or upper bounds on T of various observed neutron stars. (compilation of Page et al) All + have error bars comparable to the two shown.

DETECTING WARM OLD STARS?

- First cut analysis by Kaplan (not worth doing second cut until more realistic density profile, atmosphere, added to calculation)
- It will be a challenge to find these stars, without knowing where they are. → Fig.
- Limits on thermal emission from pulsars with age $\sim 10^7$ yrs?
  - one example within a factor of two
- The gCFL hot water bottle keeps 10^7–10^8 yr. old stars warmer by orders of magnitude than if core has any other composition
- Not ruled out by current data
- An observational challenge.

gCFL INSTABILITY

Before astrophysical observations have had a chance to rise to the challenge of ruling out the presence of gCFL quark matter, theorists have done so first:

The gCFL phase has a "magnetic instability." (Huang Shovkovy; Casalbuoni et al; Giannakis Ren; Fukushima)

⇒ It can lower its energy by turning on <u>currents</u>.

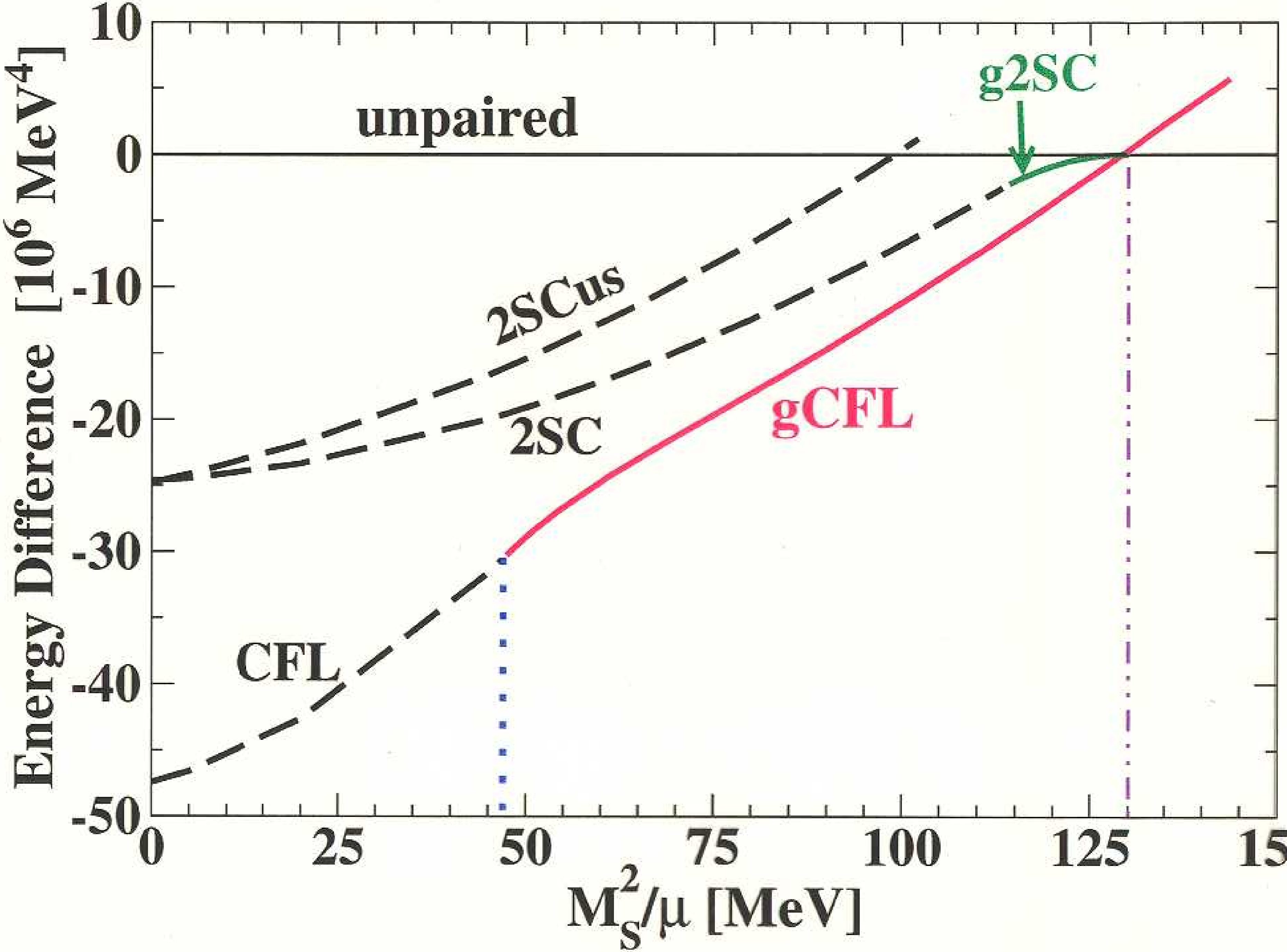
BUT: ground state of a system has no net current. (Bloch's theorem)

SO: the instability tells us there must be some lower energy phase, but does not tell us what.

- Not 2SC or variants. Not a mixed phase
- Currents hint we should revisit....

↓ ↓ INSTABILITY ↓ ↓ ↓ ↓ ↓ ↓

Challenge: Find a phase with lower energy than gCFL.



unpaired
g2SC
2SCus
2SC
gCFL
CFL
Energy Difference [10^6 MeV^4]
M_S^2/μ [MeV]
10
0
-10
-20
-30
-40
-50
0
25
50
75
100
125
150

CRYSTALLINE COLOR SUPERCONDUCTIVITY

Alford Bowers K.R.; Bowers Kundu K.R. Shuster; Leibovich K.R. Shuster; Casalbuoni Gatto Mannarelli Nardulli; Giannakis Liu Ren; Bowers K.R.;

As $\mu \downarrow$, if CFL "breaks" before you get to hadronic matter, quark matter at intermediate density may have:

Pairing between quarks with <u>different p_F</u>

<u>GOAL</u>: both quarks in a pair on respective Fermi surfaces

<u>IDEA</u>: Cooper pairs with momentum!

$(\vec{p}+\vec{q}, -\vec{p}+\vec{q})$ for any $\vec{p}$.

Each pair has total momentum $2\vec{q}$

- $|\vec{q}| \simeq 1.2\, \delta p_F$ determined energetically
- "pattern" of $\{\hat{q}_i\}$ " " Bowers K.R

$$\langle \psi\psi \rangle \sim \Delta \sum_i e^{i\vec{q}_i\cdot\vec{x}}$$

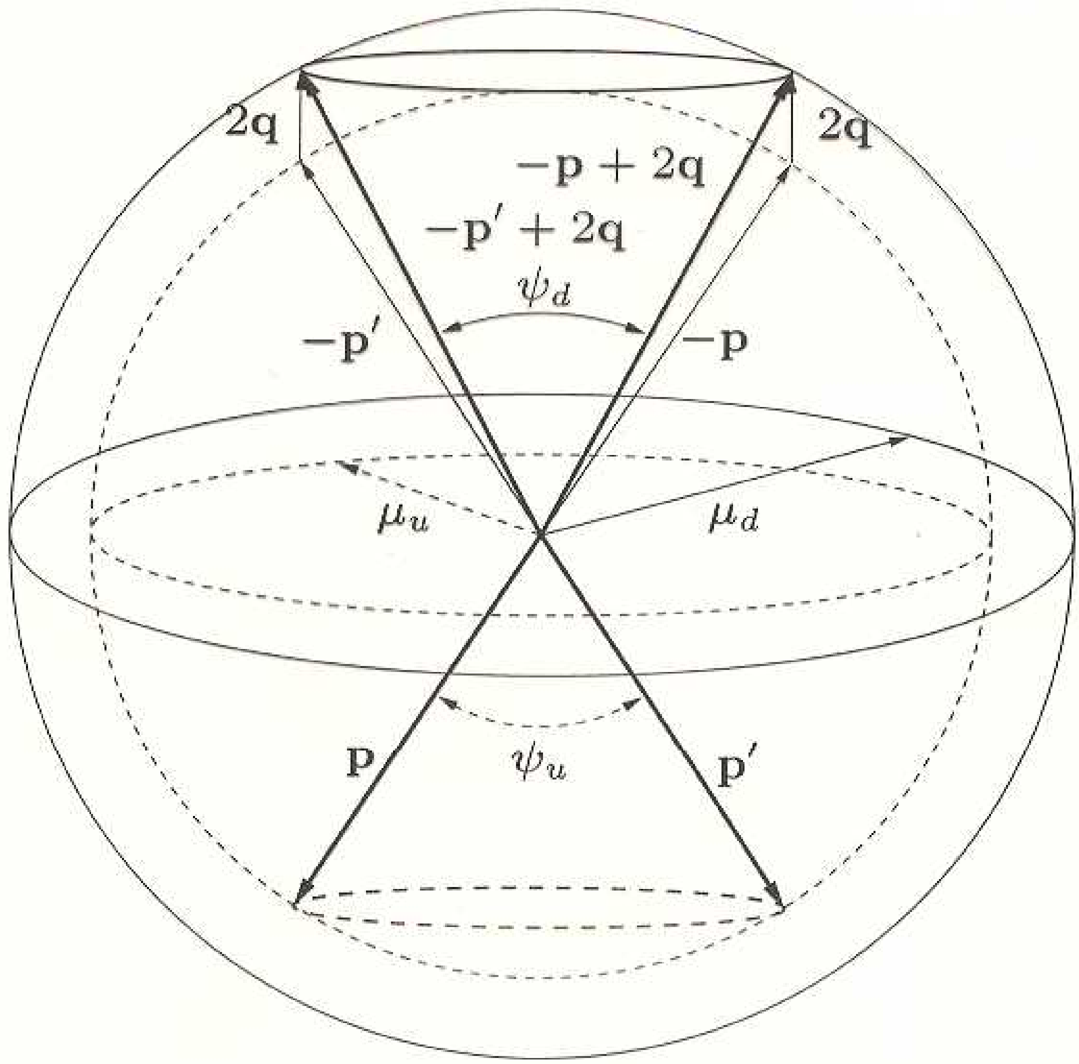
- spontaneous breaking of rotational and translational symmetry.

<u>LOFF</u>: Larkin Ovchinnikov Fulde Ferrell (1964) considered this state for $\langle e_\uparrow e_\downarrow \rangle$ pairing with Zeeman splitting. State not seen in condensed matter. Problem is that $\vec{B} \to$ orbital effects, not just Zeeman. QCD, with its "flavor Zeeman splitting" turns out to be the natural context for LOFF's idea!

Basic LOFF idea

Try Cooper pairs $(\mathbf{p}, -\mathbf{p}+2\mathbf{q})$

- total momentum $2\mathbf{q}$ for each and every pair
- each quark at its Fermi surface, even with $p_F^u \neq p_F^d$
- $\hat{q}$ chosen spontaneously, $|\mathbf{q}|$ determined variationally (result is $|\mathbf{q}| = q_0 \approx 1.20\delta\mu$)
- condensate forms a ring on each Fermi surface, with opening angle $\psi_u \approx \psi_d \approx 2\cos^{-1}(\delta\mu/q_0) \approx 67.1^\circ$



MULTIPLE PLANE WAVES

If system unstable to formation of 1 plane wave, this allows quarks lying on one ring on each F.S. to pair. Much of F.S. remains unpaired....

Why not multiple $\vec{q}$'s? i.e. multiple rings?

Want to compare many different possible $\{\vec{q}_i\}$;

$$\langle \psi(x)\,\psi(x)\rangle = \sum_{\{\vec{q}_i\}} \Delta\, e^{i\,2\vec{q}_i\cdot\vec{x}}$$

and for each $\{\vec{q}_i\}$ calculate Δ and Ω

$\{q_i\}$, ie crystal structure, with lowest Ω wins.

FCC Crystal

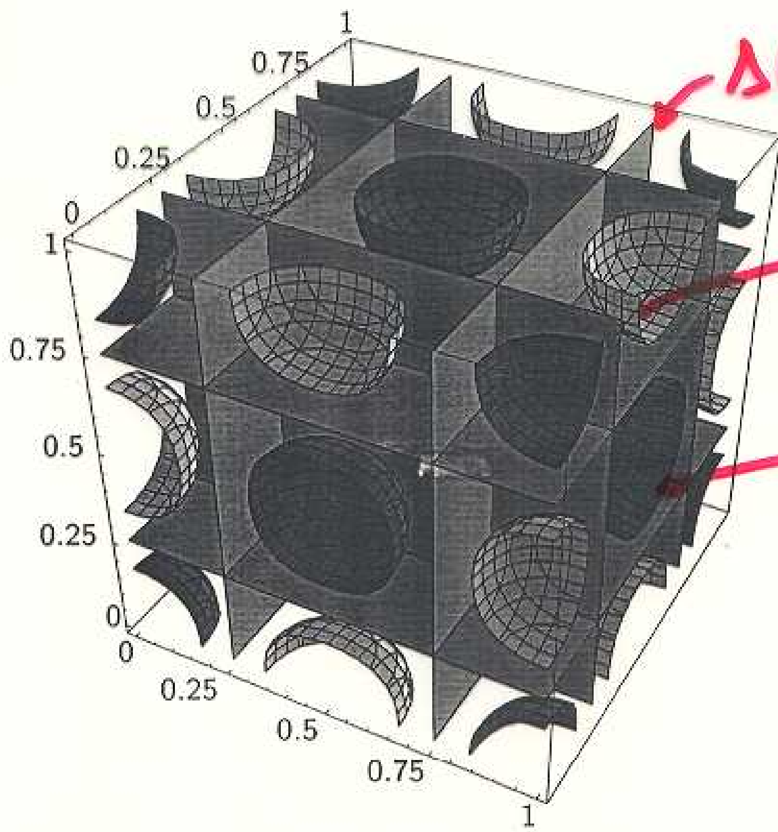
Favored according to Ginzburg-Landau analysis, that is not yet quantitatively reliable. Bowers KR

- The cube structure is the favored ground state: eight wave vectors pointing towards the corners of a cube, forming the eight shortest vectors in the reciprocal lattice of a face-centered-cubic crystal. The gap function is

$$\Delta(\mathbf{x}) = 2\Delta\left[\cos\frac{2\pi}{a}(x+y+z)+\cos\frac{2\pi}{a}(x-y+z) + \cos\frac{2\pi}{a}(x+y-z)+\cos\frac{2\pi}{a}(-x+y+z)\right]$$

$\Delta \sim \Delta_{CFL}$

A unit cell:



$\Delta(x)=0$

$\Delta(x) = -4\Delta$

$\Delta(x) = +4\Delta$

with contours $\Delta(\mathbf{x}) = +4\Delta$ (black), 0 (gray), -4Δ (white). Lattice constant is $a = \sqrt{3}\pi/|\mathbf{q}| \simeq 6.012/\Delta_0$.

Sum of 8 currents; zero net current.

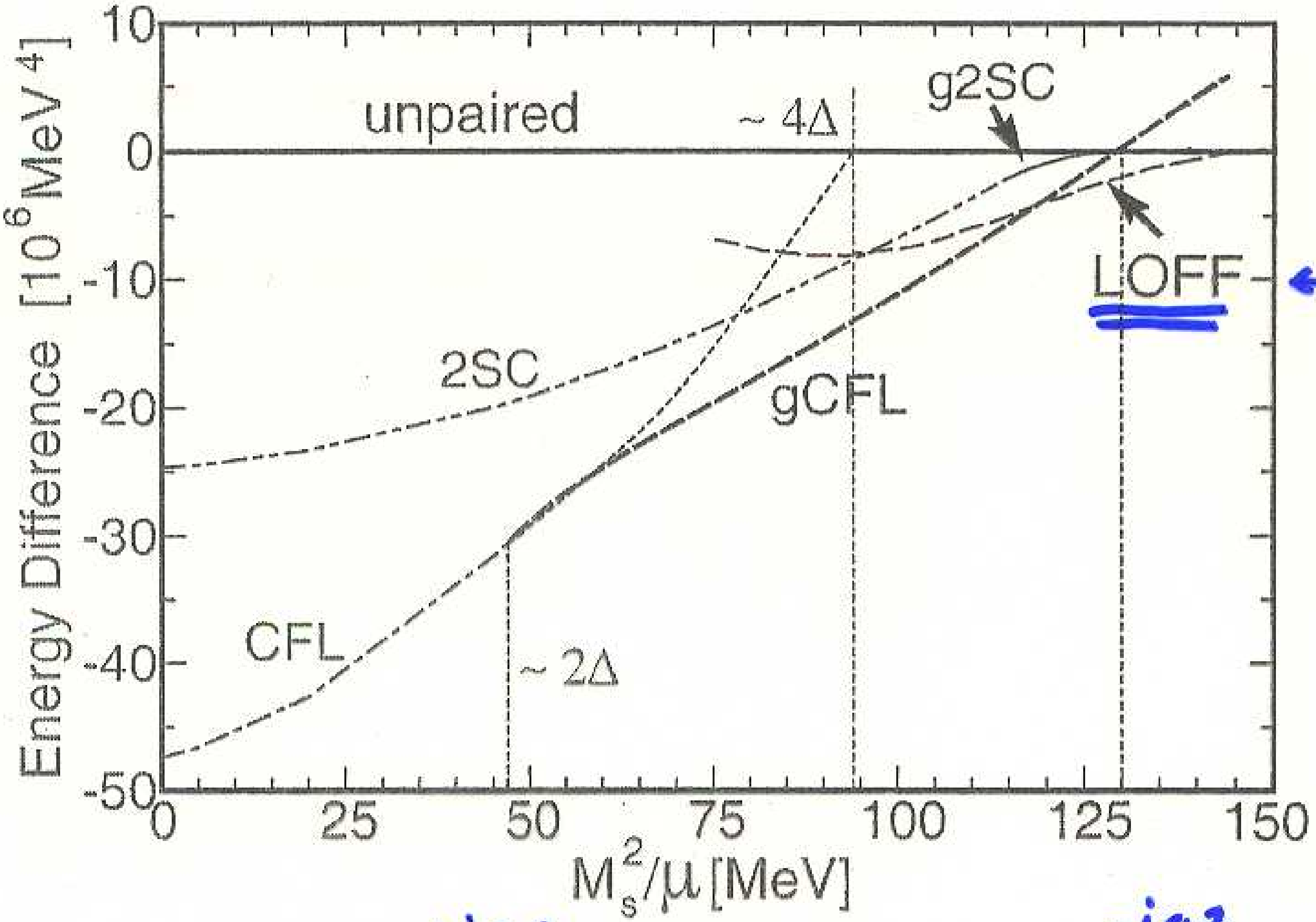
OUTLOOK AND IMPLICATIONS

QUESTION: Is there a crystalline phase with lower energy than gCFL?

- Ginzburg-Landau analysis yields qualitative answers - eg what is favored crystal structure - but is not quantitatively reliable. Must go beyond G.L.
- Need three-flavor analysis.
  → News from last week....
- Current status: nobody has shown that a crystalline phase can have lower energy than gCFL throughout the gCFL window, but the gCFL instability is a strong hint pointing in this direction.

THREE-FLAVOR "CRYSTALLINE" PHASE

Casalbuoni Gatto Ippolito Nardulli Ruggieri



$\langle ud \rangle \sim \Delta_3 e^{+iqz}$; $\langle us \rangle \sim \Delta_2 e^{-iqz}$

Greatly simplified ansatz for "crystal" structure, and yet we see this phase does beat gCFL over a part of the gCFL window.

A strong hint, since whatever the favored three-flavor crystal structure turns out to be, it must have lower energy still.

OUTLOOK AND IMPLICATIONS

CRYSTALLINE SUPERFLUIDITY

- A two species version of crystalline superfluid may be created in gases of ultracold fermionic atoms
  Combescot; Son Stephanov
  - trap 2 hyperfine states of atom;
  - arrange strong attractive interaction between 2 "species". (Done via a Feshbach resonance.)
  - load the trap with different number densities for 2 "species".

VORTEX PINNING & PULSAR GLITCHES

- Rotate the crystal; what happens? Alford Bowers KR
  Vortices? Vortices pinned at intersections of crystal's nodal planes?
- If there are pinned vortices, the presence of a layer of crystalline color superconducting quark matter within neutron stars could make this layer a locus for Pulsar Glitches.

IMPLICATIONS FOR COMPACT STARS

Or, flipping that around,... How can we use observations of compact stars to determine the high density region of the phase diagram?

- If core of "neutron" star is quark matter, then it IS a color superconductor.

$$T_{star} \sim \text{keV} \lll T_c$$

- Not known whether neutron stars have quark matter cores. Goal: understand observational consequences, so we can find out.

FIRST: Can we discover whether there is a crystalline color superconductivity window?

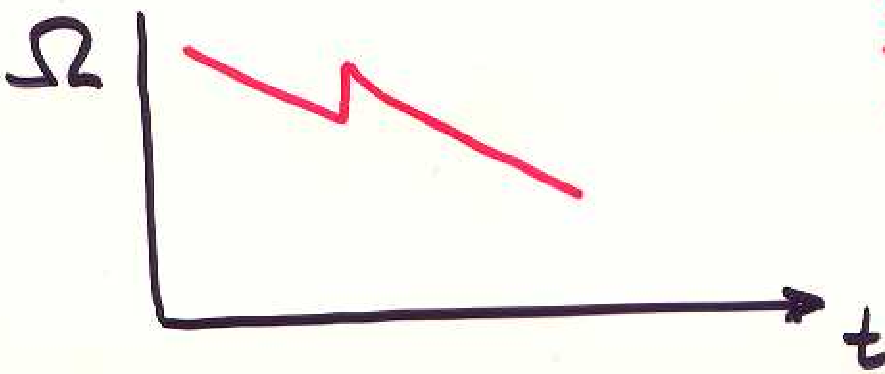
- As a function of increasing depth, m_s^2/μ Δ decreases.

$\therefore$ LOFF WINDOW → LOFF SHELL

THEN: List other examples of ways to answer this question.

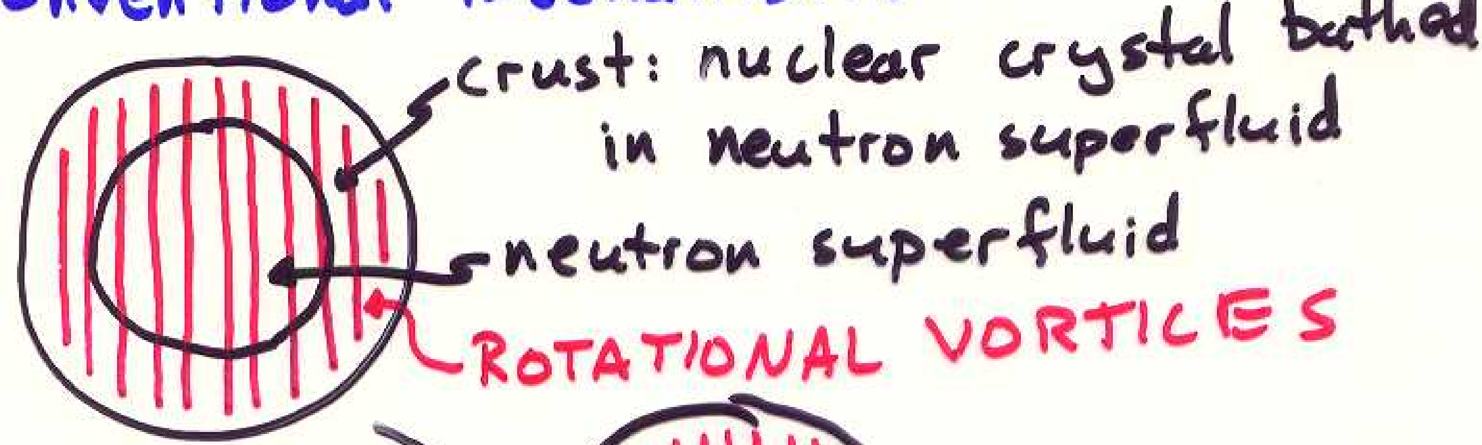
GLITCHES

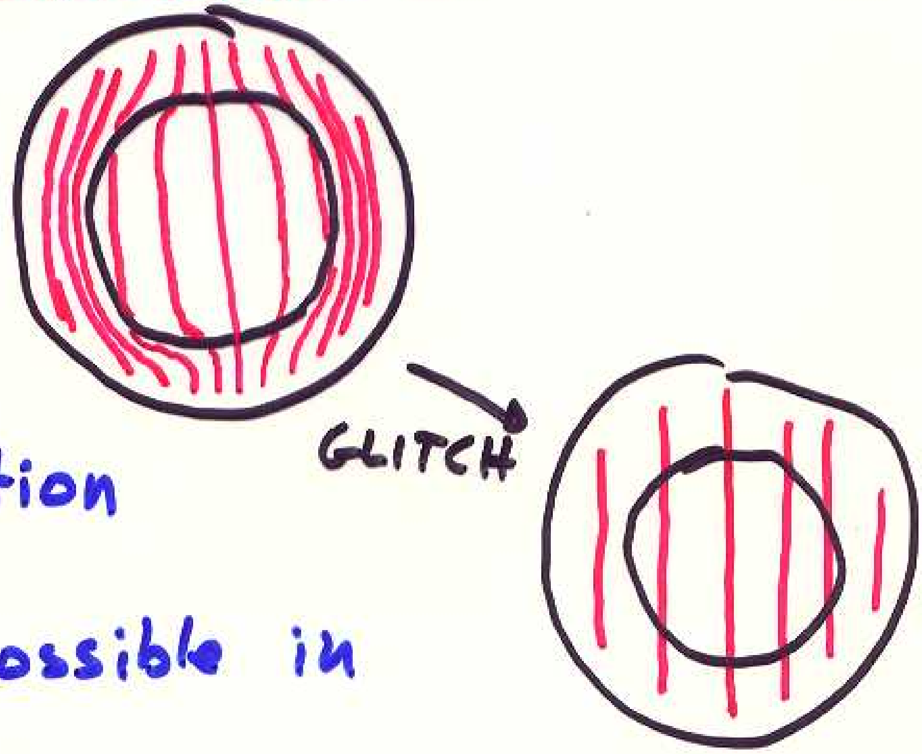
Pulsars glitch:



$$\frac{\delta \Omega}{\Omega} \sim 10^{-9} \rightarrow 10^{-6}$$

Conventional mechanism:



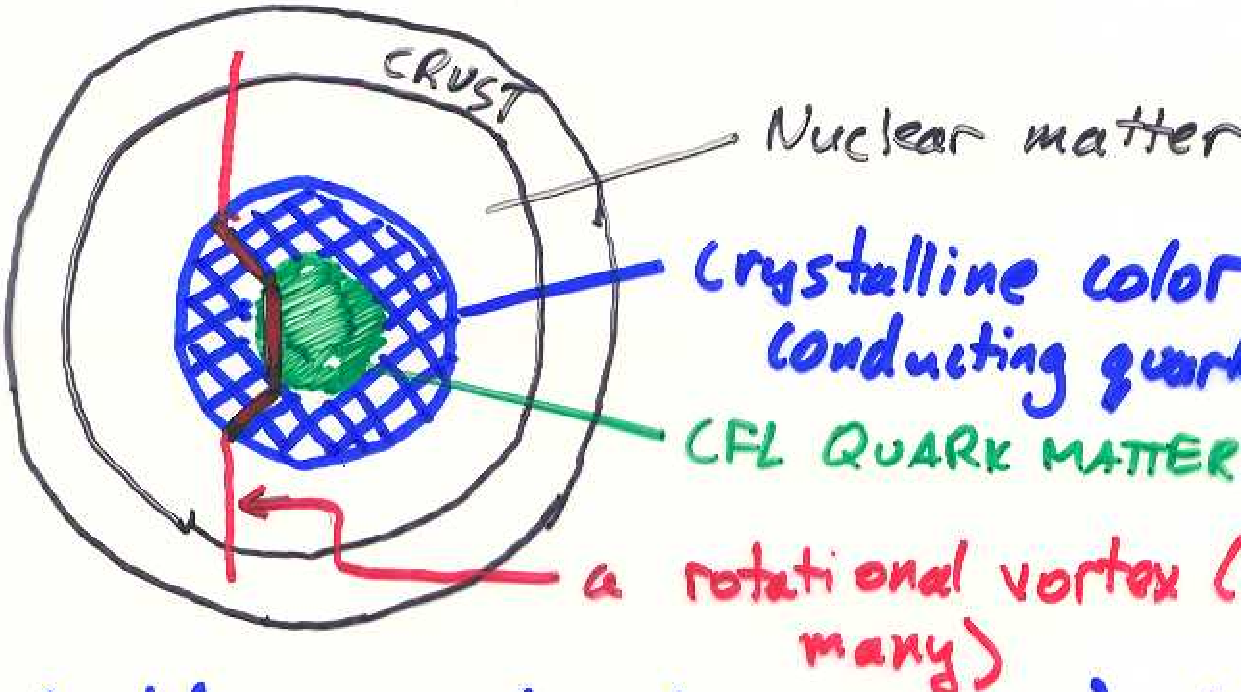


Glitches require non-uniformity (ie crystal) to impede (pin) motion of vortices.

∴ thought impossible in QM.

GLITCHES IN QUARK MATTER?

- crystalline condensate may pin vortices, ∵ they prefer to follow intersections of nodal planes



- could some (eg the smaller?) glitches originate in a crystalline layer in core?
- would features of observed glitches rule out existence of crystalline layer?
- serious glitch phenomenology requires calculation of pinning force and shear modulus. Both can be calculated, once neutral, 3-flavor, crystalline phase with real crystal structure is in hand.

FROM PHYSICAL PROPERTIES TO ASTROPHYSICAL CONSEQUENCES

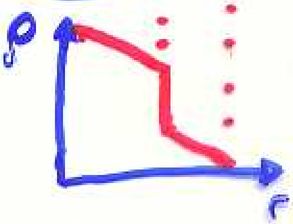
How can we learn whether neutron stars have CFL cores?

nb: Unless surface tension is unexpectedly small, this is a case where a single sharp interface (with charged boundary layers) is favored over a mixed phase.

Alford KR Reddy Wilczek

ASTROPHYSICAL CONSEQUENCES IF NEUTRON STARS HAVE CFL CORES

- For given M, R a little smaller. But, uncertainty in R still dominated by nuclear outer layer. *Alford Reddy*
- At a sharp interface, big density step. $\xrightarrow{?}$ LIGO signal *Alford KR Reddy Cokad*



- If spherical stars have CFL cores but oblate stars do not, → unusual spin-up history. *Glendenning, Weber; Blaschke, Grigorian, Poghosyan*
- Transparent insulator. → $\vec{B}$ in core not in flux tubes; not frozen. → $\vec{B}$ evolution governed by outer layer.
- For T < few MeV: *Page Prakash Lattimer Steiner*
  - very small specific heat, neutrino emissivity, neutrino opacity. *Jaikumar Prakash Schaefer*
  - superfluidity → very large thermal conductivity *Shovkovy Ellis*

  ⇒ cooling of star controlled by nuclear outer layer
- During supernova, T ~ tens of MeV > meson mass
  → mesons emit and scatter neutrinos *Reddy Sadzikowski Tachibana; Carter Reddy*
  - and, also, may be phase transitions

  $\xrightarrow{?}$ signals in time distribution of supernova ν
- Bare quark star would be nice. **NOT** seen...

NEWS

Nice, Splaver, Stairs, Löhmer, Jessner, Kramer, Cordes
astro-ph/0508050 (appeared this week)

A pulsar (named PSR J0751+1807) with mass:

$M = 2.1 \pm 0.2$ solar masses

$1.6 < M < 2.4$ at 95% confidence

- A 3.5 ms pulsar in a 6.3 hr orbit around a 0.19 solar mass white dwarf.
- Over 10 years of observation, the 6.3 hr orbit has slowed by $19.6 \pm 2.5\ \mu s$! due to gravitational wave emission.
- Shapiro delay measured. No accretion, mass transfer, or X-ray emission.
- ie this is gold-plated, as clean as the best previous mass measurements. And, the error bar will shrink like $(1/\text{duration of observation})^{2.5}$
- cf: $M_{NS} < 2.3\ M_\odot$ (stiff nuclear E.O.S.)
- cf: M_{NS} with quark core $< (1.9-2.0)\ M_\odot$ Alford, Braby, Paris, Reddy

GOALS

PUZZLE: If non-CFL quark matter intervenes between CFL & nuclear, what are its properties?

HINTS: gCFL instability $\overset{?}{\Rightarrow}$ crystalline condensate

COMING: neutral, 3-flavor crystalline color superconductor, with realistic crystal structure: does it have lower energy than gCFL?

LONGER TERM: Improve calculations of properties and consequences of these phases, allowing observations to rule their presence within neutron stars out or in. Eg:

- pinning force & shear modulus of X-tal $\overset{?}{\to}$ glitches
- almost no limit to possible improvement in calculation of CFL properties
- new data coming on M, R, ν-cooling, SN-ν, LIGO, ...