DIFFRACTION THEORY, QUANTUM OPTICS, AND HEAVY IONS

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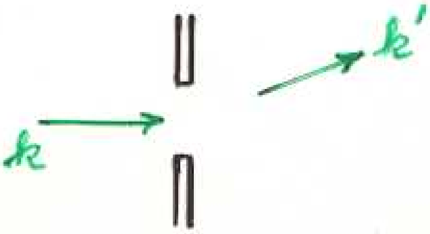
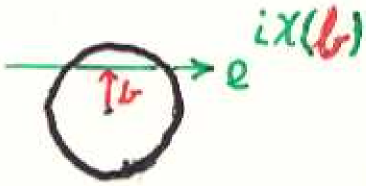
Nuclear Diffraction Theory

Optical (Fraunhofer) diffr.



$kR \gg 1$, $\Delta k = |k' - k| \ll k$

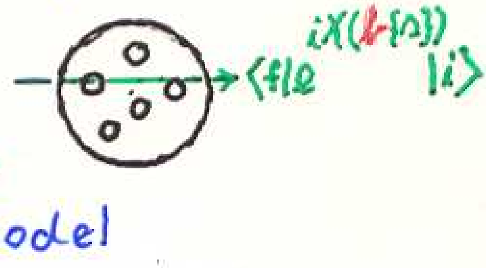
"Optical Model"



Ampl. , phase change at $\vec{b}$

Unitary approx.

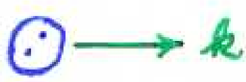
Insert intra-nuclear coordinates
Assum their motion slow,
inelasticities modest,
$\Delta E \ll E(k)$,



Treats nuclear transitions $|i\rangle \to |f\rangle$
For $|i\rangle \to |i\rangle$ derives optical model

Projectile can have internal coordinates too :

Deuteron



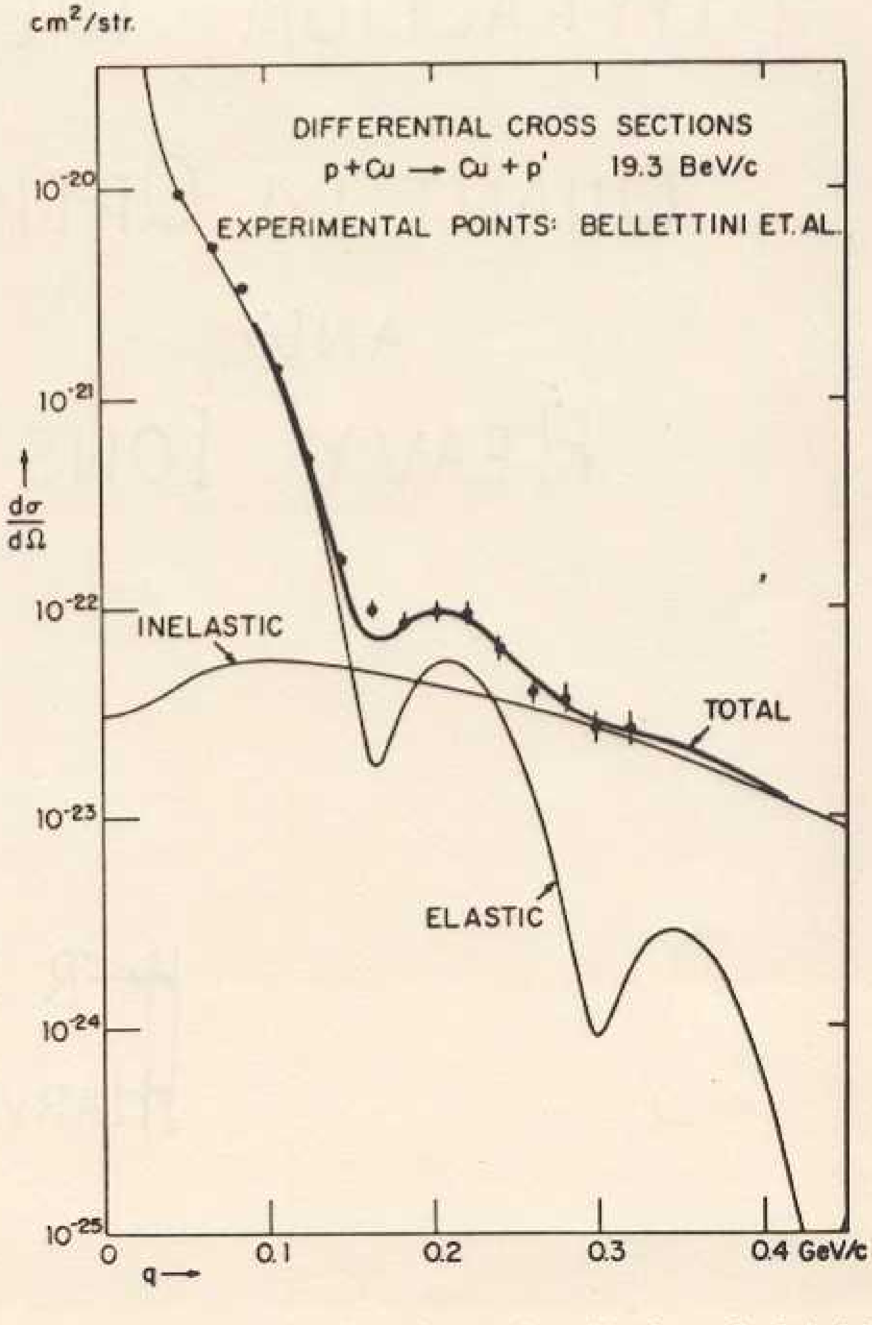


Fig. 20. Elastic, inelastic and summed scattering calculated by Kofoed-Hansen[52] for 19.3 GeV/c protons incident on Cu.

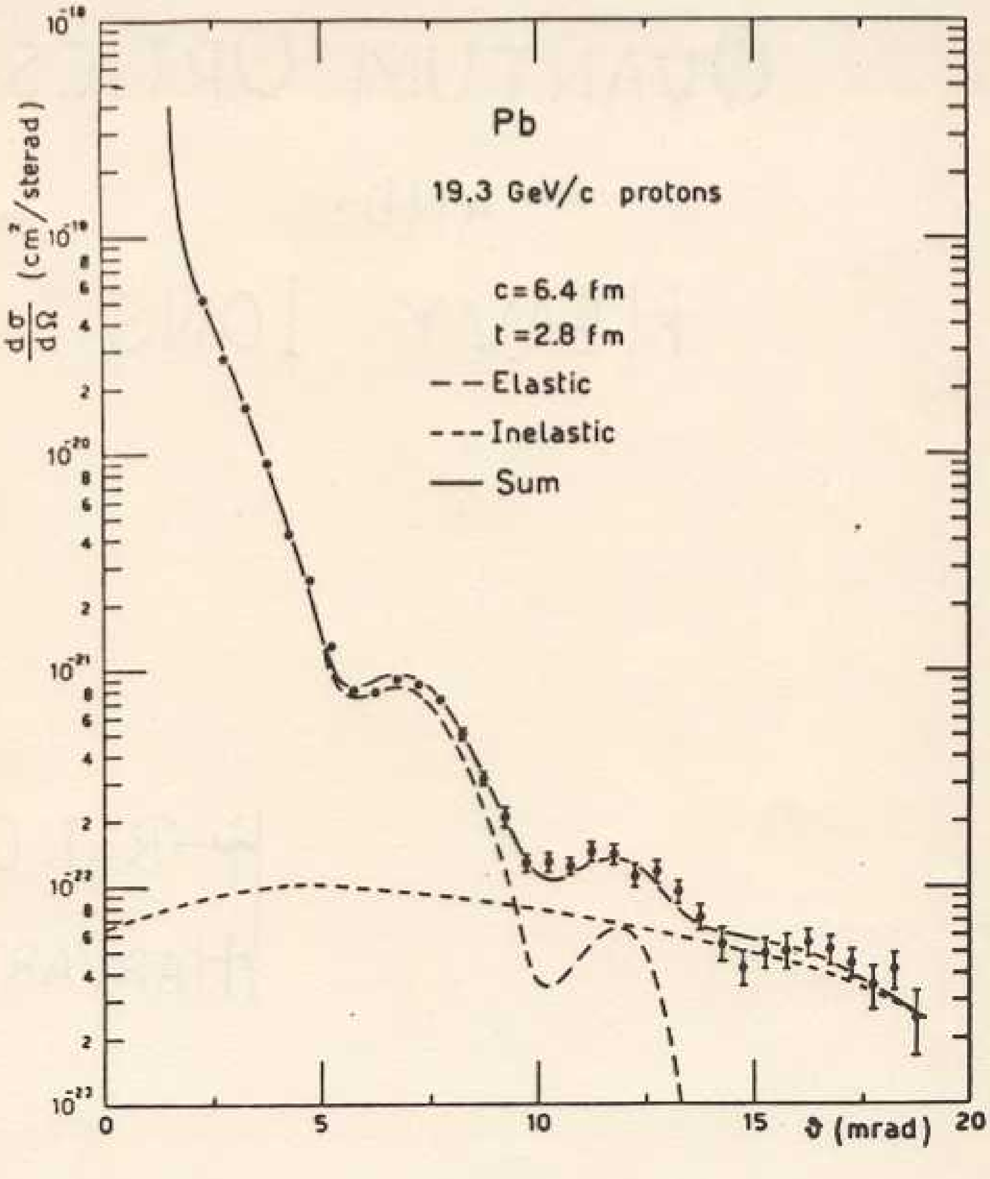
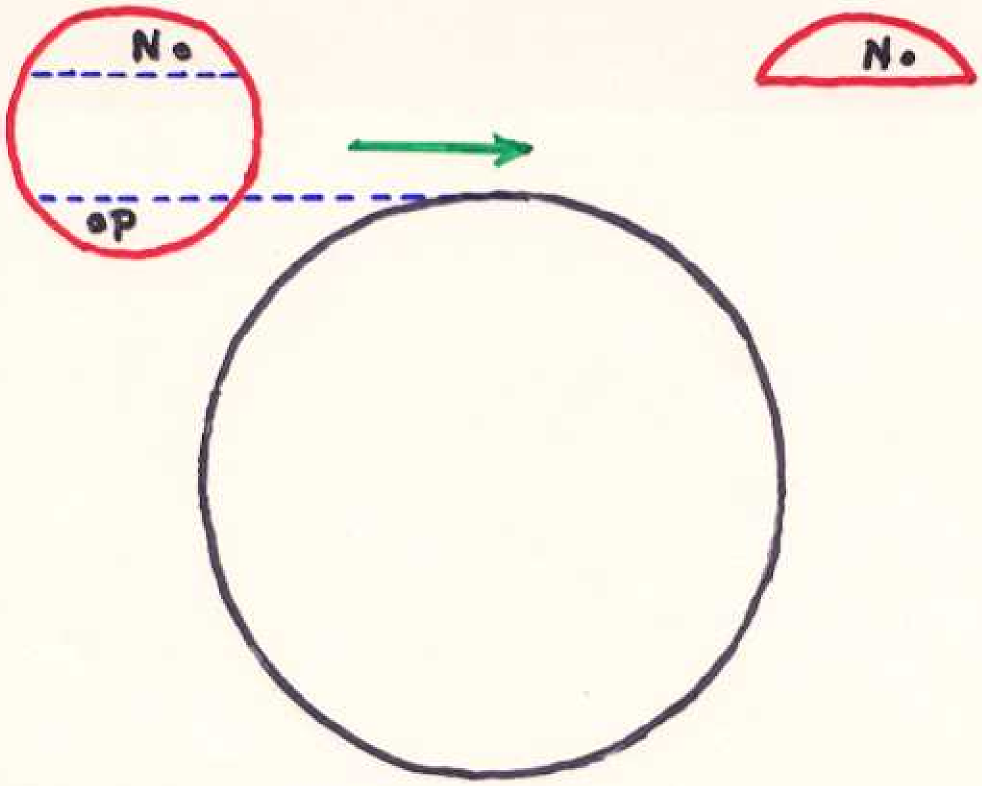
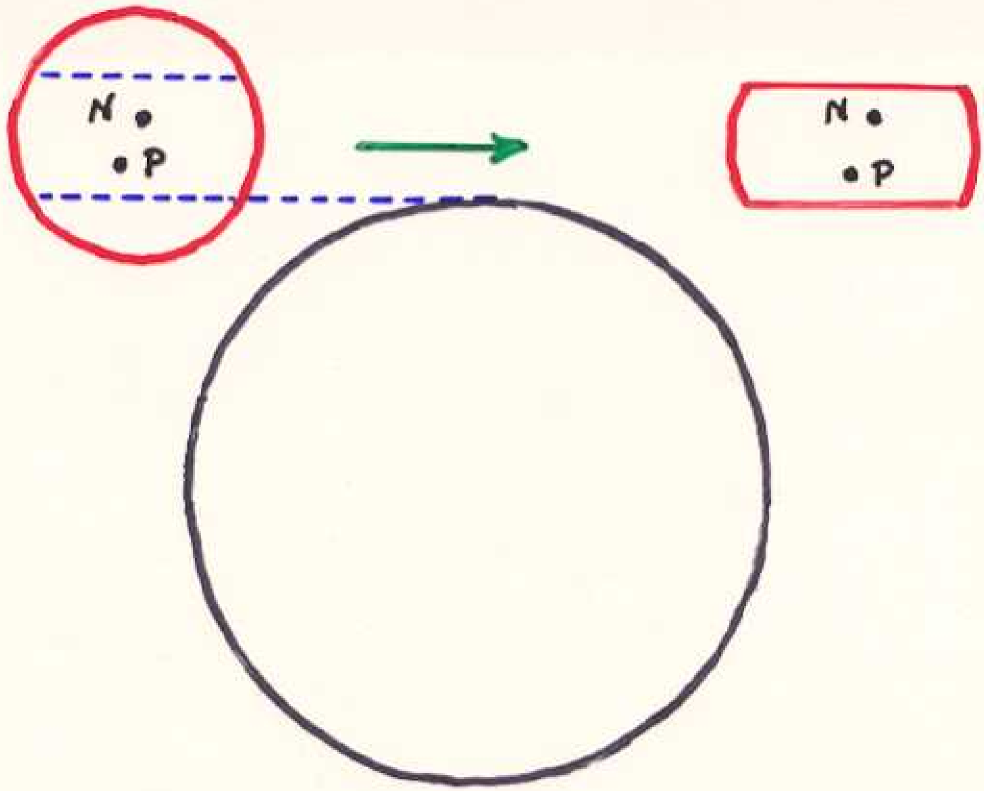


Fig. 21. Elastic, inelastic and summed scattering calculated by Matthiae for 19.3 GeV/c protons incident on Pb. Experimental points from Bellettini et al.[56]



"Stripping"

R. Serber - 1947



Two-particle wave function

contains unbound components

→ Diffraction dissociation

R.G. Phys Rev. 99 1515 (1955)

1 Particle colliding with A

Inelastic collisions $\rightarrow$ particle prod.

$$\sigma_{\text{part. prod.}} = \int \{1 - (1 - \sigma_{in} t(b))^A\} d^2b$$

$t(b)$ = Thickness function for one nucleon

$$= \int \rho(b, z) dz$$

$$\int t(b) d^2b = 1$$

Prob. (m inelastic coll.) $= P_m = \binom{A}{m} (\sigma_{in} t)^m (1 - \sigma_{in} t)^{A-m}$

Particle production model:

1 part. prod. in **first** collision

μ " " " **second** "

μ^2 " " " **third** "

...

No. of particles prod. in n coll. $= 1 + \mu^2 + \cdots + \mu^{n-1}$

$$= \frac{1 - \mu^n}{1 - \mu}$$

Let $\sigma_{in} t(b) = x$

Av. multiplicity $= \sum_m P_m \frac{1-\mu^m}{1-\mu}$

$$= \frac{1}{1-\mu} \sum \binom{A}{m} x^m (1-x)^{A-m} (1-\mu^m)$$

$$= \frac{1}{1-\mu} \left\{ (x+1-x)^A - (\mu x + 1 - x)^A \right\}$$

$$= \frac{1}{1-\mu} \left\{ 1 - (1-(1-\mu)x)^A \right\}$$

Two limits:

$\mu = 0$ Av. mult. $= 1-(1-x)^A =$ Prob. of at least one collision

$\rightarrow$ "Wounded" nucleon model (B-B-C '76)

$\mu \rightarrow 1$ Av. mult. $\rightarrow Ax =$ Av. no. of collisions counting all equally

For A colliding with B at impact parameter $\vec{b}$

$$\langle \text{Mult} \rangle_{\mu} = \frac{A}{1-\mu} \int T_A(s) \{ 1 - [1-(1-\mu)\sigma_{in} T_B(b-s)]^B \} d^2s$$
$$+ \frac{B}{1-\mu} \int T_B(b-s) \{ 1 - [1-(1-\mu)\sigma_{in} T_A(s)]^A \} d^2s$$

where $\int T_A(s)\, d^2s = \int T_B(s)\, d^2s = 1$

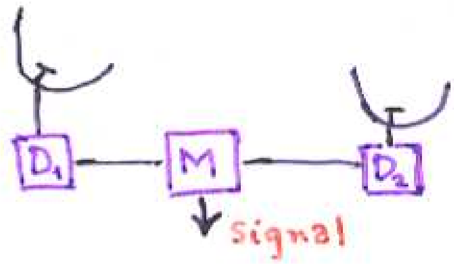
For $\mu = 0$ $\quad \langle \text{Mult} \rangle_0 = N_{\text{part.}}(b)$

$\mu \to 1 \quad \langle \text{Mult} \rangle_1 \to 2 N_{\text{coll.}}(b)$

$2\, N_{\text{coll.}}$ is the no. of collided nucleons

R. Hanbury Brown + R. Q. Twiss

Intensity interferometry



$$E(r t) = E^{(+)}(r t) + E^{(-)}(r t)$$

$$E^{(+)}(r t) \sim e^{-i\omega_k t}$$

$$E^{(-)}(r t) \sim \{E^{(+)}(r t)\}^*$$

Ordinary (Amplitude) interferometry

measures $\quad G^{(1)}(r t\, r' t') \equiv \left\langle E^{(-)}(r t)\, E^{(+)}(r' t') \right\rangle_{av.}$

Intensity interferometry measures

$$G^{(2)}(r t\, r' t'\, r' t'\, r t) = \left\langle E^{(-)}(r t)\, E^{(-)}(r' t')\, E^{(+)}(r' t')\, E^{(+)}(r t) \right\rangle$$

Two-photon dilemma

Roc.

THE PRINCIPLES OF QUANTUM MECHANICS

BY

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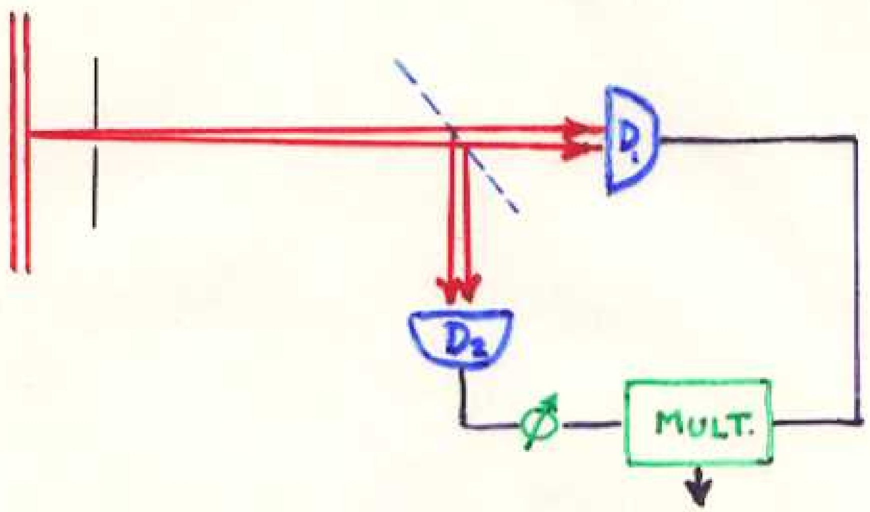
THIRD EDITION

§3, P.9 . . .

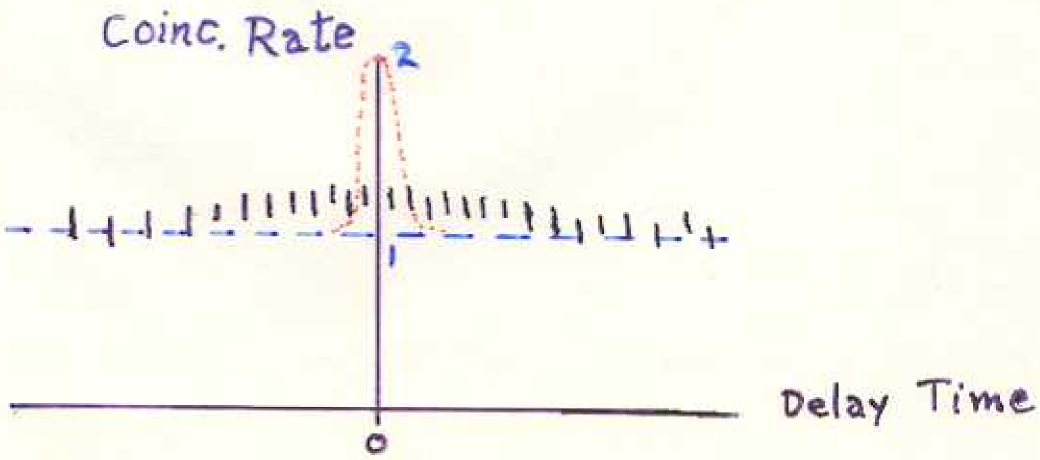
Some time before the discovery of quantum mechanics people realized that the connexion between light waves and photons must be of a statistical character. What they did not clearly realize, however, was that the wave function gives information about the probability of *one* photon being in a particular place and not the probable number of photons in that place. The importance of the distinction can be made clear in the following way. Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs.

The association of particles with waves discussed above is not restricted to the case of light, but is, according to modern theory, of universal applicability. All kinds of particles are associated with waves in this way and conversely all wave motion is associated with

Hanbury Brown + Twiss '56



Pound + Rebka '57



S.C.A.B.

Laser: very narrow linewidth

HB-T effect for laser ?? '61

Argued **No, none!** R.G. – Phys. Rev. 130 '63

→ What is coherence?

Optical (quadratic) coherence: $E = E^{(+)} + E^{(-)}$

$G^{(1)} = \langle E^{(-)}(rt) E^{(+)}(r't') \rangle$ factorizes into $\mathcal{E}^*(rt)\mathcal{E}(r't')$

~ First order coherence

Higher order $G^{(m)}$ can also factorize,

e.g. $G^{(2)} = \langle E^{(-)}(rt) E^{(-)}(r't') E^{(+)}(r't') E^{(+)}(rt) \rangle$ into $|\mathcal{E}(rt)|^2 |\mathcal{E}(r't')|^2$

~ Second order coherence

— Wipes out photon bunching + HB-T effect

All $G^{(n)}$ can factorize → Full coherence

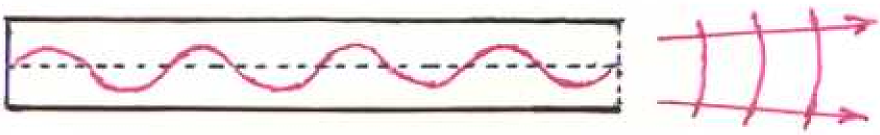
States that do it: Coherent states $|\alpha\rangle$

~ because $a|\alpha\rangle = \alpha|\alpha\rangle$

Any classical (i.e. predetermined) current $\vec{j}$

radiates coherent states —R.G.-Phys. Rev. 84, '51

What is current $\vec{j}$ for a laser?

$\vec{j}$ ↕ 

Strong oscillating polarization current $\vec{j} = \frac{\partial P}{\partial t}$

Quantum Optics = Photon Statistics

Heavy Ion Collisions

Analysis of boson statistics ($\pi, K \cdots$) parallels Q.O. in several ways. (Q.O. now has a large literature of solved problems.)

e.g. Statistics of Signal + Noise:

Signal - pure coh. state $|\beta\rangle$, $p(n) = \frac{|\beta|^{2n}}{n!} e^{-|\beta|^2}$

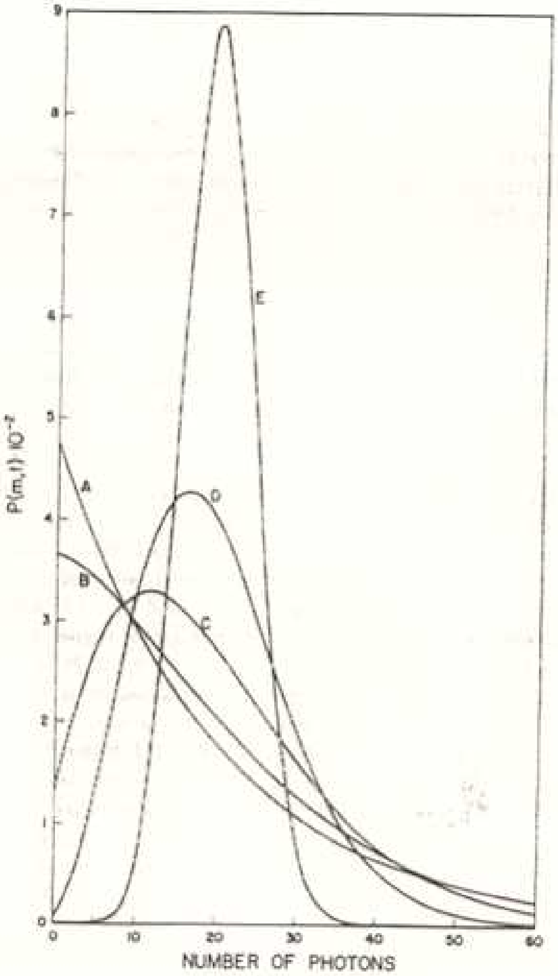
Noise - Mixture $P(\alpha) = \frac{1}{\pi\langle n\rangle} e^{-\frac{|\alpha|^2}{\langle n\rangle}}$, $p(n) = \frac{1}{1+\langle n\rangle}\left(\frac{\langle n\rangle}{1+\langle n\rangle}\right)^n$

Superposition: $P(\alpha) = \frac{1}{\pi\langle n\rangle} e^{-\frac{|\alpha-\beta|^2}{\langle n\rangle}}$

Result:

$$p(n) = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{n+1}} L_n\left(-\frac{|\beta|^2}{\langle n\rangle(1+\langle n\rangle)}\right) e^{-\frac{|\beta|^2}{1+\langle n\rangle}}$$

e.g. for $|\beta|^2 + \langle n\rangle = 20$ $\cdots$



ig. 1. Photon count distributions which would be measured in various superpositions of oherent (signal) and chaotic (noise) fields in a single mode. The total intensity of the eld is fixed for these cases so that the average number of photons counted is 20 for ich superposition. Curve A represents the distribution for a pure noise field, Eq. (35). ie other curves represent the distributions for fields in which the noise and signal com-nents would separately contribute the following average photon numbers: Curve B, 10 m noise and 10 from the signal; Curve C, 5 and 15; Curve D, 2 and 18; Curve E, 0 d 20, respectively. All of these distributions may be expressed in terms of Laguerre lynomials multiplying the distribution of Eq. (35). Curve E, in particular, is a Poisson tribution.

R.

Can there be coherence in heavy-ion output?

Simplest meson theory: $H_{int.} = \int \rho(r)\,\varphi(rt)\,d\vec{r}$

Ground state: bound coherent state,

$$\prod_k |\alpha_k\rangle \quad \text{with} \quad \alpha_k \sim \frac{\int \rho(r) e^{-ik\cdot r}\,dr}{E(k)}$$

If $\rho(r)$ suddenly vanishes, a coherent field excitation $\prod_k |\alpha_k\rangle$ is set free.

If $\rho(r)$ is suddenly replaced by a random source producing mode excitations γ_k the field density operator becomes the superposition:

$$\int \mathcal{P}(\{\gamma_k\}) \prod_k |\alpha_k+\gamma_k\rangle\langle\alpha_k+\gamma_k|\, d^2\alpha_k$$

The coherent excitation remains — as a relic — though it may be swamped.