

Critical end point and its consequences

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Recently a lot of evidence that there exists a critical end point (CEP) in the QCD phase diagram has been accumulating. However, so far, no reliable equation of state with the CEP has been employed in hydrodynamical calculations. In this article, we construct the equations of state with the CEP on the basis of the universality hypothesis and show that the CEP acts as an attractor of isentropic trajectories. We also consider the time evolution in the case with the CEP and discuss how the CEP affects the final state observables.

1. UNIVERSALITY AND EQUATION OF STATE

The existence of the CEP in the QCD phase diagram has been predicted by several effective theory analyses [1,2]. Recent lattice calculation has also shown it [3], although the lattice size is still rather small and recently a question on the procedure in Reference [3] was raised [4]. In this article, we construct equations of state with the CEP on the basis of the universality, which is a general principle in phase transition theory and discuss the consequences of the CEP in final state observables in high energy heavy ion collisions. See Reference [5] for further details of the formulation and calculation.

The equation of state with the CEP consists of two parts, the singular part and non-singular part. We assume that the CEP in QCD belongs to the same universality class as that in the three dimensional Ising model on the basis of the universality hypothesis. After mapping the variables and the equation of state near the CEP in the three dimensional Ising model onto those in QCD, we match the singular entropy density near the CEP with the non-singular QGP and hadron phase entropy densities which are known away from the CEP. From this procedure we determine the behavior of the entropy density which includes both the singular part and non-singular part in a large region in the $T-\mu_B$ plane.

In the three dimensional Ising model, the magnetization M (the order parameter) is a function of the reduced temperature $r = (T - T_c)/T_c$ and the external magnetic field h with T_c being the critical temperature. The CEP is located at the origin $(r, h) = (0, 0)$. At $r < 0$ the order of the phase transition is first and at $r > 0$ it is crossover.

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In order to determine the singular part of the entropy density, we start from the Gibbs free energy density $G(h, r)$,

$$G(h, r) = F(M, r) - Mh, \quad (1)$$

where $F(M, r)$ is the free energy density. Differentiating the Gibbs free energy by the temperature, we obtain the singular part of the entropy density s_c ,

$$s_c = - \left(\frac{\partial G}{\partial T} \right)_{\mu_B}. \quad (2)$$

Note that T in Eq. (2) is the temperature on the QCD side.

Next, the mapping between the r - h plane in the three dimensional Ising model and the T - μ_B plane in QCD needs to be specified. The CEP in the three dimensional Ising Model, which is the origin in the r - h plane, is mapped to the CEP in QCD, $(T, \mu_B) = (T_E, \mu_{BE})$. The r axis is tangential to the first order phase transition line at the CEP [6]. There is no general rule about how the h axis is mapped in the T - μ_B plane. For simplicity, we set the h axis perpendicular to the r axis. For quantitative construction of equations of state with the CEP, we fix the relation between the scales in (r, h) and (T, μ_B) variables, which provides the size of the critical region around the CEP in the T - μ_B plane, as follows: $\Delta r = 1$ (r - h plane) $\leftrightarrow \Delta\mu_{B\text{crit}}$ (T - μ_B plane) and $\Delta h = 1$ (r - h plane) $\leftrightarrow \Delta T_{\text{crit}}$ (T - μ_B plane). In order to connect the equations of state in the singular region and the non-singular region smoothly, we define the dimensionless variable $S_c(T, \mu_B)$ for the singular part of the entropy density s_c , which has the dimension [energy] $^{-1}$,

$$S_c(T, \mu_B) = A(\Delta T_{\text{crit}}, \Delta\mu_{B\text{crit}})s_c(T, \mu_B), \quad (3)$$

where $A(\Delta T_{\text{crit}}, \Delta\mu_{B\text{crit}}) = \sqrt{\Delta T_{\text{crit}}^2 + \Delta\mu_{B\text{crit}}^2} \times D$ and D is a dimensionless constant. The extension of the critical domain around the CEP is specified by the parameters ΔT_{crit} , $\Delta\mu_{B\text{crit}}$, and D .

Using the dimensionless variable $S_c(T, \mu_B)$, we define the entropy density in the T - μ_B plane,

$$s(T, \mu_B) = \frac{1}{2} (1 - \tanh[S_c(T, \mu_B)]) s_H(T, \mu_B) + \frac{1}{2} (1 + \tanh[S_c(T, \mu_B)]) s_Q(T, \mu_B), \quad (4)$$

where s_H and s_Q are the entropy densities in the hadron phase and QGP phase away from the CEP, respectively. This entropy density includes both singular and non-singular contributions, and more importantly, gives the correct critical exponents near the QCD critical end point. All thermodynamical quantities are obtained from the entropy density. In the following, we use s_H calculated from the equation of state of the hadron phase in the excluded volume approximation [7] and s_Q obtained from the equation of state of the QGP phase in the Bag model.

2. RESULTS and DISCUSSIONS

When entropy production can be ignored, the entropy and baryon number are conserved in each volume element and, therefore, the temperature and chemical potential in a given volume element change along the contour lines specified by the initial condition.

Figure 1 shows the isentropic trajectories in the T - μ_B plane, i.e., contour lines of n_B/s . The values of $(\Delta T_{\text{crit}}, \Delta \mu_{B\text{crit}}, D)$ are (100 MeV, 200 MeV, 0.15). The trajectories are focused to the CEP. Thus, the CEP acts as an attractor of isentropic trajectories. Figure 2 shows isentropic trajectories in the bag plus excluded volume model, which is currently employed in most of hydrodynamical calculations. There is no focusing effect on the isentropic trajectories in this case. Instead, the trajectories are just shifted to the left on the phase transition line. Thus, the hydrodynamical evolution in the case with the CEP is very different from the one in the case with the equation of state given by the bag plus excluded volume model. This attractor character of the CEP leads to the following consequences: it is not needed to fine-tune the collision energy to make the system pass near the CEP and the effect of the CEP in observables, if any, changes only slowly as the collision energy is changed. We note that the entropy densities in both the hadron phase and QGP phase must be carefully taken into account in order to consider this focusing effect. It is because the baryon number density is given by the integral of $\partial s/\partial \mu_B$ with regard to the temperature. If one fails to reproduce the entropy density in the hadron phase, it in turn affects the baryon number density in the QGP phase, and the focusing property of the isentropic trajectories. See, for example, Reference [8].

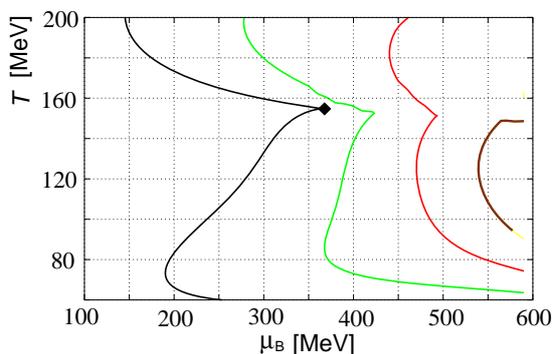


Figure 1. Isentropic trajectories in the cases with the CEP. The CEP is located at $(T_E, \mu_{BE}) = (154.7 \text{ MeV}, 367.8 \text{ MeV})$. The values of n_B/s on the trajectories are 0.01, 0.02, 0.03, and 0.04 from left to right.

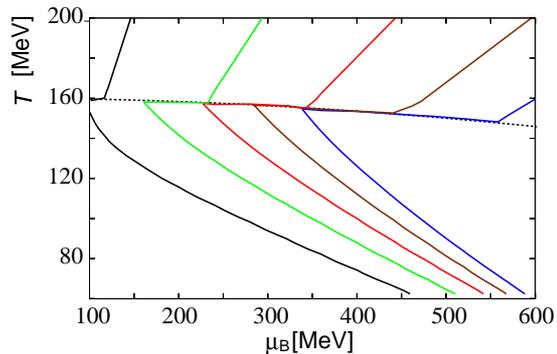


Figure 2. Isentropic trajectories (solid lines) in the bag plus excluded volume model. The values of n_B/s on the trajectories are 0.01, 0.02, 0.03, 0.04, and 0.05 from left to right. The dashed line stands for the first order phase boundary.

Finally, we present the time evolution of the correlation length. Figure 3 shows the correlation length as a function of L/L_{total} , where L is the path length to a point along the isentropic trajectory with a given n_B/s from a reference point on the same isentropic trajectory on the T - μ_B plane and L_{total} is the one to another reference point along the trajectory. The dashed and solid lines stand for the correlation lengths in equilibrium at $n_B/s = 0.008$ and $n_B/s = 0.01$, respectively. The thin and thick lines are the equilibrium correlation length ξ_{eq} and correlation length ξ , respectively. We assume that the thermal

equilibrium is established soon after collisions and that the medium follows Bjorken's scaling solution. The initial temperature and proper time are set to 200 MeV and 1 fm/c, respectively. The other parameters are the same as for Figure 1. The maximum value of ξ_{eq} along the former trajectory is larger than that along the latter, because the former approaches the CEP more closely than the latter.

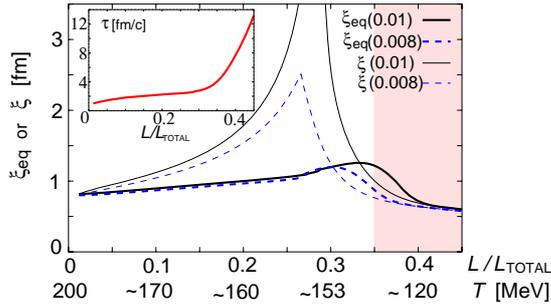


Figure 3: Equilibrium correlation length ξ_{eq} and non-equilibrium correlation length ξ on the isentropic trajectories with $n_B/s = 0.008$ and 0.01 , together with τ as functions of L/L_{total} (inlet).

While the equilibrium correlation length diverges at the CEP, the actual correlation length does not diverge because of the critical slowing down. Furthermore, the correlation length at freezeout does not have to show substantial enhancement even if the system passes through the CEP before it freezes out. It is due to the final state interaction in the hadron phase. Figure 3 shows this clearly. The non-equilibrium correlation length ξ is smaller than ξ_{eq} at the beginning. Then, ξ becomes larger than ξ_{eq} later. These are both due to the critical slowing down around the CEP, as pointed out in Reference [9]. However, the difference becomes small by the time the system gets to the kinetic freezeout point.

If the transverse expansion is taken into account, the time scale in the hadron phase becomes much shorter, but ξ_{eq} is already small in the hadron phase and the difference is expected to remain small. The dependence on the non-universal constant A , the definition of which is given in References [5,9], is very weak. Thus, even if there is a CEP in the QCD phase diagram, sudden increase in the correlation length and fluctuation as a function of collision energy is not expected. Instead, the low kinetic freezeout temperature like that observed at RHIC is anticipated on the left hand side of the CEP. See Reference [5] for the details. Finally, we note that the focusing of the isentropic trajectories does not necessarily lead to the focusing of the chemical freezeout points, since the critical region around the CEP, where the phase transition is of second order, is the region where the free-resonance gas model is by no means valid.

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