Power-law tailed spectra from equilibrium

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We propose that power-law tailed hadron spectra may be viewed as stemming from a matter in an unconventional equilibrium state typical for non-extensive thermodynamics. A non-extensive Boltzmann equation, which is able to form such spectra as a stationary solution, is utilized as a rough model of quark matter hadronization. Basic ideas about a non-extensive simulation of the QCD equation of state on the lattice are presented.

1. Particle spectra

Statistical models have been often applied to hadron physics. Starting with Rolf Hagedorn's statistical model of meson resonances [1], several attempts occurred to describe hadron multiplicities in elementary collisions by means of statistical distributions. The very idea of a phase transition between confined and deconfined quark matter relies on traditional equilibrium thermodynamics. The search for quark matter began assuming a local thermal equilibrium in an otherwise exploding fireball [2,3]. Experimental particle spectra are, however, not purely exponential: both exponential and power-law functions have been fitted to pion, kaon and antiproton spectra. Although the traditional approach explains the power-law tail at very high p_T values by pQCD calculations [4], the non-extensive statistics provides a unified view for the whole spectrum. Pion spectra from heavy ion collisions at RHIC seem to contain a power-law part exceeding the scaled pQCD yield [5]. Another evidence can be extracted from the minimum bias pion p_T-spectrum form RHIC AuAu collisions at 200 GeV (Fig.1 in Ref. [6]). Here a Tsallis-distribution fit can already be made at the p_T-region between 1 and 4 GeV. The extrapolation of this fit almost coincides with the fit to the whole observed range between 1 and 12 GeV. So one concludes that the power-law behavior is not restricted hard scales. Furthermore in the non-extensive statistical approach there is a connection between the soft properties (temperature T) and the hard ones.

The experimentally measured specific hadron spectra may reflect statistical properties of the precursor matter [7]. Fortunately transverse momentum spectra are influenced only partially, at their low end, by final state interactions and late resonance decay [8]. Since the relativistic energy is given by $E = m_T \cosh y$ with transverse mass $m_T = \sqrt{p_T^2 + m^2}$ and rapidity y for a particle with mass m, the best way to study statistical equilibrium distribution of hadrons is the comparison of m_T-spectra at rapidity $y = 0$ for different particles. A universal behavior [9] indicates that the one-particle distributions depend on the energy only and not on all momentum components: a basic feature of generalized and conventional thermal distributions.

2. Non-Extensive Boltzmann Equation

Non-conventional distributions can be based on a non-conventional entropy formula, which replaces the Boltzmann entropy. Such a formula is the Tsallis entropy, discussed vividly in recent years. This non-extensive thermodynamics is intended to be an effective theory for non-equilibrium and long-range order phenomena [10]. Its canonical distribution is a power law, which occurs in particle and heavy-ion physics experiments. As nonlinear models, two generalizations of the Boltzmann equation have been investigated: The generalization of the product rule for probabilities (dropping statistical independency) leads to a non-multilinear Boltzmann equation [11], while considering two-particle energies composed by an extended addition rule mounds in the non-extensive Boltzmann equation [12].

The general structure of the Boltzmann equation describes the evolution of the probability $f_1 = f(\vec{p}_1)$ of a one-particle state by considering possible transitions to and from other states: $\dot{f}_1 = \int\limits_{234} w_{1234} \, (f_{34,12} - f_{12,34})$. Here the dot denotes a total time derivative (Vlasov operator) comprising the essential evolution of the one-particle phase space density, f_1. The indices 1234 refer to two particles before and after a microcollision. The transition probability, w_{1234} contains conditions on conserving momentum and energy:

$$w_{1234} = M_{1234}^2 \, \delta((\vec{p}_1 + \vec{p}_2) - (\vec{p}_3 + \vec{p}_4)) \, \delta(E_{12} - E_{34}), \tag{1}$$

with E_{12} total two-particle energy before and E_{34} after the collision. The particle density factors, $f_{12,34}$ and $f_{34,12}$ weight the transition yields for a $3+4 \longrightarrow 1+2$ and for a $1+2 \longrightarrow 3+4$ process, respectively. In our approach [12] we keep the statistical independency, $f_{12,34} = f_1 f_2$, but generalize the energy addition formula to a nontrivial composition rule $E_{12} = h(E_1, E_2)$. Rules not being a simple sum, $h(x,y) \neq x + y$, present a non-extensive energy composition. Associative rules can be mapped to the simple addition [13]: $X(h) = X(x) + X(y)$, unique up to a constant factor. In each microcollision $X(E_1) + X(E_2) = X(E_3) + X(E_4)$ holds, and the stationary solution is hence given by $f(p) = \frac{1}{Z} \exp(-X(E)/T)$. A statistical dispersion relation is obtained due to the mapping of the general composition: the total sum, $X(E_{tot}) = \sum_i X(E_i)$ is conserved.

In order to utilize such a non-extensive Boltzmann equation for the stationary state Tsallis-distributed parton matter, we consider an evolution from initially boosted Fermi spheres. During the simulation we take out pairs of partons with a color singlet against octet probability 1/9. This rate turns out to be low enough not to disturb the Tsallis distribution remarkably. Fig.1 shows the resulting energy (m_T) distribution of such pairs.

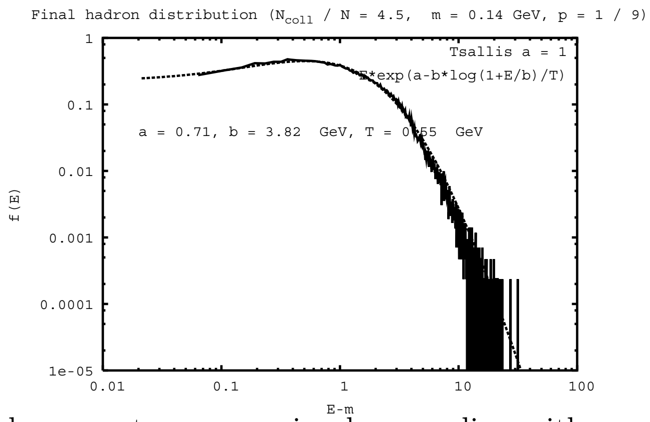

Figure 1. A model hadron spectrum emerging by sampling with a probability 1/9 from the non-extensive parton cascade simulation with massless partons.

It is interesting to note that an exponentially growing mass spectrum, originally proposed by Hagedorn and recently checked against latest experimental data in Ref. [14], with its famous consequence of having a limiting (or Hagedorn-) temperature for such a system, can be reconstructed on the basis of Tsallis distributed quark constituents. This approach [15] assumes that the Tsallis distribution of the quarks and antiquarks is folded into mesonic and baryonic distributions of the conserved total energy satisfying $X(E) = \sum_i X(E_i)$. The Hagedorn temperature is given by $T_H = E_c/d$.

3. Lattice strategy

The implementation of the Tsallis distribution in lattice field theory can be based on the superstatistics approach [16]. The Tsallis expectation value of an observable $\hat{A}[U]$ over lattice field configurations U is of interest. It may include the timelike link length, say on the power v: $\hat{A} = \theta^{\,v} A$. The Tsallis expectation value then is an average over all possible a_t link lengths according to a Gamma distribution of a_t/a_s. We obtain:

$$\langle A \rangle_{TS} \;=\; \frac{1}{Z_{TS}} \frac{c^c}{\Gamma(c)} \int d\theta \, \theta^{\,c-1} e^{-c\,\theta} \int \mathcal{D}U A\,[U]\, \theta^{\,v} e^{-S[\theta,U]} \tag{2}$$

with Z_{TS} obtained by requiring $\langle 1 \rangle_{TS} = 1$. The θ dependence of the lattice gauge action is known long. Due to the time derivatives the electric("kinetic") part scales like $a_t a_s^3/(a_t^2 a_s^2) = a_s/a_t$, and the magnetic ("potential") part like $a_t a_s^3/(a_s^2 a_s^2) = a_t/a_s$. This leads to the following expression for the general lattice action: $S\,[\theta, U] = a\,\theta + b/\theta$, where $a = S_{ss}[U]$ sums space-space, and $b = S_{ts}[U]$ time-space oriented plaquettes. In the $c \to \infty$ limit the scaled Gamma distribution approximates $\delta(\theta - 1)$, (its width narrows extremely, while its integral is normalized to one), and one gets back the traditional lattice action $S = a + b$, and the traditional averages. For finite c, one can exchange the θ integration and the configuration sum (path integral) and obtains exactly the power-law-weighted expression: $\langle A \rangle_{TS} \;=\; \int \mathcal{D}U \, W_{v,c}\,[U] A\,[U] \;/\; \int \mathcal{D}U \, W_{0,c}\,[U]$, with the Gamma fluctuating time-link averaged general weight factor,

$$W_{v,c} = \frac{c^c}{\Gamma(c)} \int d\theta \, \theta^{\,v+c-1} e^{-c\,\theta} e^{-S[\theta,U]}. \tag{3}$$

The θ integration can be carried out analytically using the replacement $\theta = e^t\sqrt{b/(a+c)}$. The result contains the K Bessel function:

$$W_{v,c} = \frac{c^c}{\Gamma(c)}\left(\frac{b}{a+c}\right)^{\frac{c+v}{2}} 2\,K_{v+c}\left(2\sqrt{b(a+c)}\right). \tag{4}$$

The K-Bessel function has an exponentially decreasing asymptotics, so we are in principle able to utilize known Monte Carlo techniques in order to calculate Tsallis expectation values. On the other hand we cannot simply use old data, produced according to the weight $e^{-(a+b)}$, because the argument of the K-Bessel function is not $a+b$. This makes it necessary to redo lattice calculations – but only with a slightly increased effort.

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