The QCD equation of state for two flavours at non-zero chemical potential

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We present results of a simulation of 2 flavour QCD on a $16^3 \times 4$ lattice using p4-improved staggered fermions with bare quark mass $m/T = 0.4$. Derivatives of the thermodynamic grand canonical partition function $Z(V, T, \mu_u, \mu_d)$ with respect to chemical potentials $\mu_{u,d}$ for different quark flavours are calculated up to sixth order, enabling estimates of the pressure and the quark number density as well as the chiral condensate and various susceptibilities as functions of $\mu_{u,d}$ via Taylor series expansion. Results are compared to high temperature perturbation theory as well as a hadron resonance gas model. We also analyze baryon as well as isospin fluctuations and discuss the relation to the chiral critical point in the QCD phase diagram. We moreover discuss the dependence of the heavy quark free energy on the chemical potential.

1. Introduction

It is important to study QCD at high temperature and high density by numerical simulations of lattice QCD. In particular, studies of the equation of state (EoS) can provide basic input for the analysis of the experimental signatures for QGP formation, e.g. the EoS will control the properties of any hydrodynamic expansion. We extend studies of the EoS to non-zero baryon number density.

Simulation at non-zero baryon density is known to be difficult; however recent studies have found that a Taylor expansion with respect to chemical potential $\mu_{u,d}$ is an efficient technique to investigate the low density regime, interesting for heavy-ion collisions [1,2]. In the calculation of the Taylor expansion coefficients, i.e. calculation of the derivatives at $\mu_{u,d} = 0$, the technical difficulty for non-zero $\mu_{u,d}$ does not arise and a quantitative

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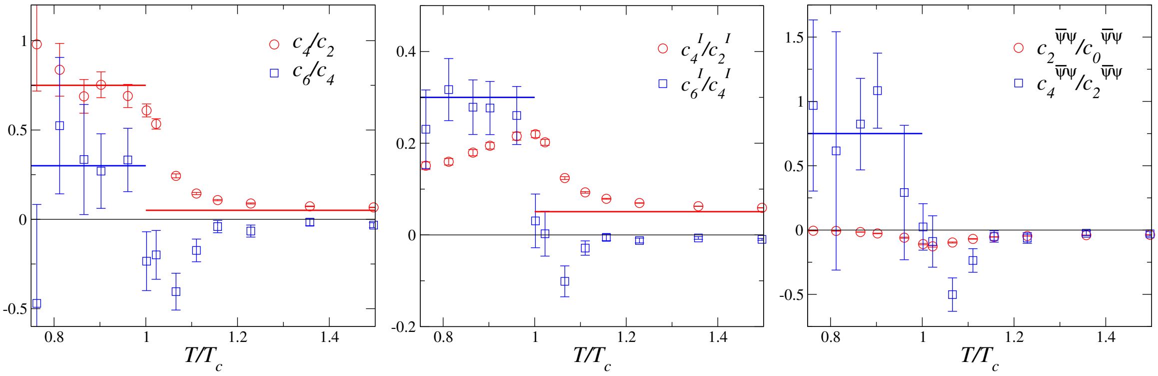

Figure 1. The ratios of Taylor expansion coefficients for pressure (quark number susceptibility), isospin susceptibility and chiral condensate.

study becomes possible within the error by truncation of higher order terms. Since thermodynamic quantities can be defined by derivatives of the partition function, e.g. chiral condensate $\langle\bar{\psi}\psi\rangle = (T/V)(\partial \ln Z/\partial m)$, quark number density $n_q = (T/V)(\partial \ln Z/\partial \mu_q)$, quark number susceptibility $\chi_q = (T/V)(\partial^2 \ln Z/\partial \mu_q^2)$, and isospin susceptibility $\chi_I = (T/V)(\partial^2 \ln Z/\partial \mu_I^2)$, where $\mu_q = (\mu_u + \mu_d)/2$ and $\mu_I = (\mu_u - \mu_d)/2$, the calculation of first derivatives yields basic QCD thermodynamics observables, and the calculation of higher derivatives is a natural extension.

In this study, we evaluate the Taylor expansion coefficients of thermodynamic quantities and compare these results to the predictions from the perturbation theory in the high temperature limit and from the hadron resonance gas model in the low temperature hadron phase. Also fluctuations of quark number, isospin number and electric charge are discussed near the transition temperature T_c. They are estimated by χ_q, χ_I and charge susceptibility $\chi_C = \chi_q/36 + \chi_I/4$, and are related to the critical endpoint expected at non-zero μ_q. Moreover we study the Taylor expansion coefficients of chiral condensate and heavy quark free energy. The details of simulations are given in [2].

2. Quark gluon gas and hadron resonance gas

We define the expansion coefficients by $p/T^4 = (\ln Z)/(VT^3) \equiv \sum_{n=0}^{\infty} c_n(\mu_q/T)^n$, $\chi_q[\chi_I]/T^2 \equiv \sum_{n=2}^{\infty} n(n-1)c_n[c_n^I](\mu_q/T)^{n-2}$ and $\langle\bar{\psi}\psi\rangle/T^3 \equiv \sum_{n=0}^{\infty} c_n^{\bar{\psi}\psi}(\mu_q/T)^n$ for $\mu_I = 0$. We expect the equation of state to approach that of a free quark-gluon gas (Stefan-Boltzmann (SB) gas) in the high temperature limit. The coefficients in the SB limit for 2 flavour QCD with $\mu_I = 0$ are well known as $c_2 = c_2^I = 1, c_4 = c_4^I = 1/(2\pi^2)$ and $c_n = c_n^I = 0$ for $n \geq 6$. Moreover the sign of c_6 is negative starting at $O(g^3)$ in perturbation theory. On the other hand, in the low temperature phase QCD is well described by a hadron resonance gas model in which the pressure is obtained by summing over the contributions from all resonance states of hadrons. In this model, the contribution to p/T^4 from each individual hadron which has baryon number B_i is in proportion to $\exp(3B_i\mu_q/T)$, hence the pressure can be written as $p/T^4 = G(T) + F(T)\cosh(3\mu_q/T)$, where $G(T)$ and $F(T)$ are the mesonic and baryonic components of p/T^4 at $\mu_q = 0$. Similarly, we obtain $\chi_q/T^2 = 9F(T)\cosh(3\mu_q/T)$, $\chi_I/T^2 = G^I(T) + F^I(T)\cosh(3\mu_q/T)$

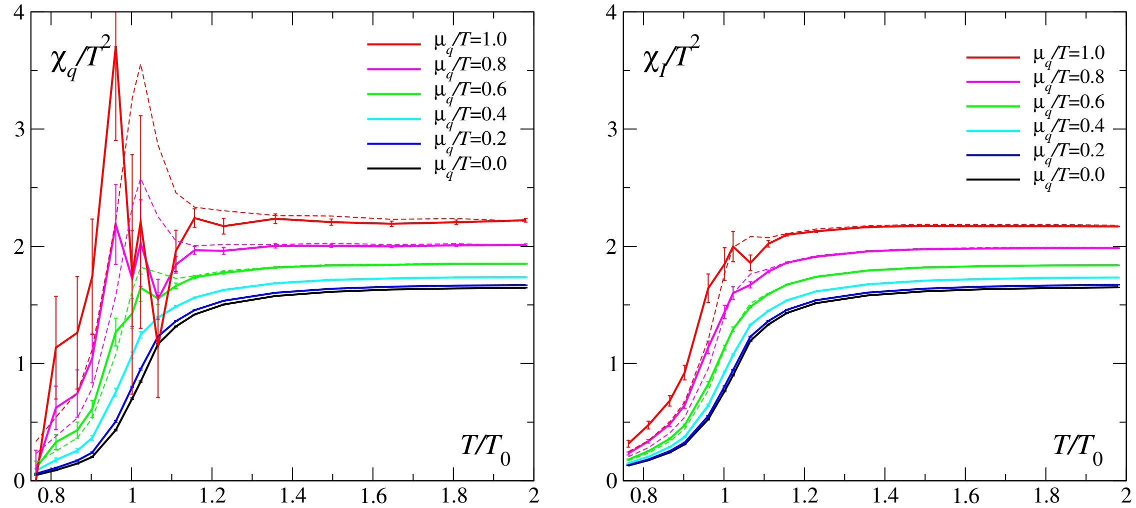

Figure 2. The quark number susceptibility χ_q/T^2 (left) and isospin susceptibility χ_I/T^2 (right) for various μ_q/T. T_0 is T_c at $\mu_q = 0$.

and $\langle\bar{\psi}\psi\rangle/T^3 = G^{\bar{\psi}\psi}(T) + F^{\bar{\psi}\psi}(T)\cosh(3\mu_q/T)$. Here, the mesonic component for χ_q is zero because mesons have baryon number zero. Therefore, $c_4/c_2 = 3/4$, $c_6/c_4 = 3/10$, $c_6^I/c_4^I = 3/10$ and $c_4^{\bar{\psi}\psi}/c_2^{\bar{\psi}\psi} = 3/4$ in the region where the hadron resonance gas provides a good approximation.

We investigate these coefficients. The results for c_{n+2}/c_n, c_{n+2}^I/c_n^I and $c_{n+2}^{\bar{\psi}\psi}/c_n^{\bar{\psi}\psi}$ are shown in Fig. 1. We find that these results are consistent with the prediction from the hadron resonance gas model for $T/T_c \leq 0.96$ and approach the SB values, i.e. $c_4/c_2 = c_4^I/c_2^I = 1/2\pi^2$, $c_6/c_4 = c_6^I/c_4^I = 0$, in the high temperature limit. Also c_6 is negative at high temperature as expected in the perturbation theory. These results suggest that the models of free quark-gluon gas and hadron gas seem to explain the behaviour of thermodynamic quantities well except in the narrow regime near T_c.

3. Susceptibilities at non-zero μ_q

Next, we calculate quark number and isospin susceptibilities in a range of $0 \leq \mu_q/T \leq 1$. The data connected by solid lines in Fig. 2 are obtained by $\chi_q/T^2 = 2c_2 + 12c_4(\mu_q/T)^2 + 30c_6(\mu_q/T)^4$ and the corresponding equation for χ_I. Dashed lines are the results from $O(\mu_q^2)$ expansion. Since the statistical error of c_6 is still large near T_c, the location and height of the peak are less accurately determined, and also the error from the truncation of higher order terms of μ_q/T seems to be visible for large μ_q/T. However, as seen in Fig. 1, c_6 changes its sign at T_c. This means the peak position of χ_q moves left, which is corresponding to the change of T_c as a function of μ_q. $T_c(\mu_q/T = 1)/T_c(\mu_q/T = 0)$ in [3] is about 0.93. Moreover, χ_q increases with higher orders of the expansion for $T \leq T_c$ which confirms the existence of a peak. This suggests the presence of a critical endpoint in the (T, μ_q) plane. On the other hand, χ_I in Fig. 2 does not show any singular behaviour. This is consistent with the sigma model prediction that only isosinglet degrees of freedom become massless at the critical endpoint.

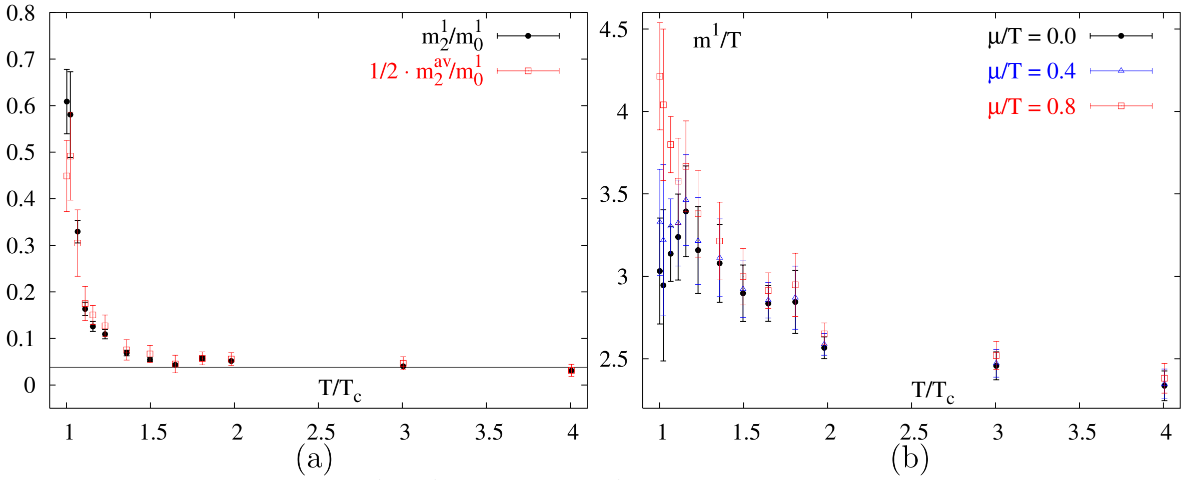

Figure 3. (a) The ratios m_2^1/m_0^1 and $m_2^{\rm av}/m_0^1$ as functions of T/T_c and (b) the singlet screening mass at non-zero μ_q.

4. Screening mass at non-zero μ_q

Finally we want to discuss the free energy of a static quark anti-quark pair at finite temperature and density. We extract the singlet free energy $F^1_{Q\bar{Q}}$ and colour averaged free energy $F^{\rm av}_{Q\bar{Q}}$ by the Polyakov loop correlation functions. For $T > T_c$, the free energy is expected to be exponentially screened at large distances, $\Delta F^{\rm av,1}_{Q\bar{Q}}(r,T,\mu_q) \equiv F^{\rm av,1}_{Q\bar{Q}}(\infty,T,\mu_q) - F^{\rm av,1}_{Q\bar{Q}}(r,T,\mu_q) \sim e^{-m^{\rm av,1}(T,\mu_q)r}$. We calculate the Taylor expansion coefficients of the Debye screening mass $m^{\rm av,1}$ in terms of μ_q/T at $\mu_q = 0$, $m^{\rm av,1} \equiv \sum_{n=0}^{\infty} m_n^{\rm av,1}(\mu_q/T)^n$. We plot the data of m_2^1/m_0^1 and $m_2^{\rm av}/m_0^1$ in Fig. 3(a). We find $m_2^1 = (1/2)m_2^{\rm av}$. This is expected from perturbation theory, which suggests that the leading order contribution to the colour singlet free energy is given by one gluon exchange while the colour averaged free energy is dominated by two gluon exchange. Moreover the perturbative Debye screening mass m_D is given by $m_D(T,\mu_q) = m_{D,0}(T)\sqrt{1 + 3N_f/[(2N_c+N_f)\pi^2](\mu_q/T)^2}$, with $m_{D,0}(T) = g(T)T\sqrt{(2N_c+N_f)/6}$. Here N_c and N_f are the number of colour and flavour respectively. The solid line in Fig. 3(a) is the perturbative prediction for m_2^1/m_0^1. The ratio m_2^1/m_0^1 is found to be consistent with perturbation theory for $T \geq 2T_c$. In Fig. 3(b) we show the μ_q-dependence of the singlet screening mass $m^1(\mu_q,T)/T$ for a small values of μ_q/T, where only contributions from m_0^1 and m_2^1 are included. Further details of this study are given in [4].

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