

The role of quark mass in cold and dense pQCD and quark stars

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For almost twenty years the effects of a nonzero strange quark mass on the equation of state of cold and dense QCD were considered to be negligible, thereby yielding only minor corrections to the mass-radius diagram of compact stars. By computing the thermodynamic potential to first order in α_s , and including the effects of the renormalization group running of the coupling and strange quark mass, we show that corrections can be of the order of 25%, and dramatically affect the structure of compact stars.

Compact stars provide the most promising “laboratory” to constrain the equation of state for strong interactions. In this work we compute the thermodynamic potential for cold quark matter with two light (massless) flavors, corresponding to the up and down quarks, and one massive flavor, corresponding to the strange quark, in perturbation theory to first order in α_s in the $\overline{\text{MS}}$ scheme [1]. In this fashion, we can easily include modern renormalization group running effects for both the coupling and the strange quark mass. We find that the corrections due to the running nonzero mass are sizable, and should not be neglected in the evaluation of thermodynamic quantities. Solving the Tolman-Oppenheimer-Volkov (TOV) equations using our equation of state in the presence of electrons, we show that a running strange quark mass dramatically modifies the mass-radius diagram for quark stars, even at first order in α_s .

The thermodynamic potential for cold QCD in perturbation theory to $\sim \alpha_s^2$ was first computed a long time ago [2–4]. Nevertheless, the original approach to quark stars [5,6] made use of the bag model with corrections $\sim \alpha_s$ from perturbative QCD to compute the thermodynamic potential. In the massless case, first-order corrections cancel out in the equation of state, so that one ends up with a free gas of quarks modified only by a bag constant. Finite quark mass effects were then estimated to modify the equation of state by less than 5% and were essentially ignored.

A few years ago, corrections $\sim \alpha_s^2$ with a modern definition of the running coupling constant were used to model the non-ideality in the equation of state for cold, dense QCD with massless quarks [8,9]. This approach can be compared to treatments that resort to resummation methods and quasiparticle model descriptions [10–13]. Remarkably, these different frameworks seem to agree reasonably well for $\mu \gg 1$ GeV, and point in the same direction even for $\mu \sim 1$ GeV and smaller, pushing perturbative QCD towards its border of applicability.

Even the most recent QCD approaches mentioned above generally neglected quark

masses and the presence of a color superconducting gap as compared to the typical scale for the chemical potential in the interior of compact stars, ~ 400 MeV and higher. However, it was recently argued that both effects should matter in the lower-density sector of the equation of state [14]. In fact, although quarks are essentially massless in the core of quark stars, the mass of the strange quark runs up, and becomes comparable to the typical scale for the chemical potential, as one approaches the surface of the star.

In what follows, we present an exploratory analysis of the effects of a finite mass for the strange quark on the equation of state for perturbative QCD at high density, leaving the inclusion of a color superconducting gap in this framework for future investigations. To illustrate the effects and study the modifications in the structure of quark stars, we focus on the simpler case of first-order corrections. Results including full corrections $\sim \alpha_s^2$, as well as technical details of the calculation at each order and renormalization, will be presented in a longer publication [15].

The leading-order piece of the thermodynamic potential of QCD for one massive flavor is given by [2–4]

$$\Omega^{(0)} = -\frac{N_c}{12\pi^2} \left[\mu u (\mu^2 - \frac{5}{2}m^2) + \frac{3}{2}m^4 \ln \left(\frac{\mu + u}{m} \right) \right], \quad (1)$$

where N_c is the number of colors and $u \equiv \sqrt{\mu^2 - m^2}$.

Using standard quantum field theoretical methods, one obtains the complete renormalized exchange energy for a massive quark in the $\overline{\text{MS}}$ scheme (in the limit $T \rightarrow 0$):

$$\Omega^{(1)} = \frac{\alpha_s(N_c^2 - 1)}{16\pi^3} \left[3 \left(m^2 \ln \frac{\mu + u}{m} - \mu u \right)^2 - 2u^4 + m^2 \left(6 \ln \frac{\bar{\Lambda}}{m} + 4 \right) \left(\mu u - m^2 \ln \frac{\mu + u}{m} \right) \right]. \quad (2)$$

The thermodynamic potential to order α_s for one massive flavor, given by the sum of Eqs. (2) and (1), depends on the quark chemical potential μ and on the renormalization subtraction point $\bar{\Lambda}$ both explicitly and implicitly through the scale dependence of the strong coupling constant $\alpha_s(\bar{\Lambda})$ and the mass $m(\bar{\Lambda})$. The scale dependencies of both α_s and m , which in the following we will take to be the mass of the strange quark, are known up to 4-loop order in the $\overline{\text{MS}}$ scheme [16]. Since we have only determined the free energy to first order in α_s , we choose

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 L} \left[1 - 2 \frac{\beta_1 \ln L}{\beta_0^2 L} \right], \quad m_s(\bar{\Lambda}) = \hat{m}_s \left(\frac{\alpha_s}{\pi} \right)^{4/9} \left[1 + 0.895062 \frac{\alpha_s}{\pi} \right], \quad (3)$$

where $L = 2 \ln(\bar{\Lambda}/\Lambda_{\overline{\text{MS}}})$, $\beta_0 = 11 - 2N_f/3$, and $\beta_1 = 51 - 19N_f/3$ and we take $N_f = 3$. The scale $\Lambda_{\overline{\text{MS}}}$ and the invariant mass \hat{m}_s are fixed by requiring [17] $\alpha_s \simeq 0.3$ and $m_s \simeq 100$ MeV at $\bar{\Lambda} = 2$ GeV; one obtains $\Lambda_{\overline{\text{MS}}} \simeq 380$ MeV and $\hat{m}_s \simeq 262$ MeV. With these conventions, the only freedom left is the choice of $\bar{\Lambda}$.

To study the effect of the finite strange quark mass on the equation of state for electrically neutral quark matter with 2 light (massless) flavors (up and down quarks) and one massive flavor (strange quark), we have to include electrons, with chemical potential μ_e and assume beta equilibrium. Since neutrinos are lost rather quickly, one may set their chemical potential to zero, so that chemical equilibrium yields $\mu_d = \mu_s = \mu$ and $\mu_u + \mu_e = \mu$, with μ_u, μ_d and μ_s the up, down and strange quark chemical potentials, respectively.

On the other hand, overall charge neutrality requires $(2/3)n_u - (1/3)n_d - (1/3)n_s - n_e = 0$, where n_i is the number density of the particle species i . Together, the above equations insure that there is only one independent chemical potential, which we take to be μ . Number densities are determined from the thermodynamic potential by $n_i = -(\partial\Omega/\partial\mu)$ and the total energy density is given by $\epsilon = \Omega + \sum_i \mu_i n_i$, where $\Omega = \sum_i (\Omega_i^{(0)} + \Omega_i^{(1)})$ and again i refers to the particle species. The pressure is $P = n_B \frac{\partial\epsilon}{\partial n_B} - \epsilon$, where $n_B = \frac{1}{3}(n_u + n_d + n_s)$ is the baryon number density and the Gibbs potential per particle is given by $\frac{\partial\epsilon}{\partial n_B} = (\mu_u + \mu_d + \mu_s)$. We restrict the freedom of choice for $\bar{\Lambda}(\mu_u, \mu_d, \mu_s)$ by requiring that in case of vanishing strange quark mass all quark chemical potentials and number densities become equal so that $P^{(m_s=0)} = -\Omega^{(m_s=0)}$ and, consequently, one has $\mu_e \rightarrow 0$. Furthermore, in order to compare our findings to existing results in the literature [8,11], we require that in the massless case $\bar{\Lambda} = 2\mu$. Consequently, we choose $\bar{\Lambda} = \frac{2}{3}(\mu_u + \mu_d + \mu_s)$, but have tested that our results are not much affected by other choices obeying the above conditions.

The effects of the finite strange quark mass on the total pressure and energy density for electrically neutral quark matter (plus electrons) are given in Fig. 1. There we show results for 3 light flavors and running coupling, corresponding to the case considered in [8], and for 2 light flavors and one massive flavor, with both running coupling and strange quark mass (which reaches $m_s \sim 137$ MeV at $\mu = 500$ MeV).

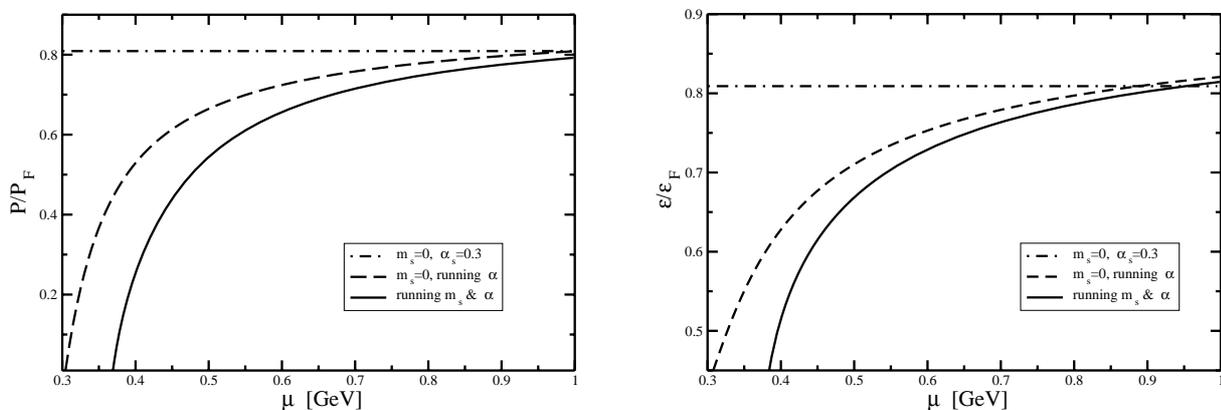


Figure 1. Pressure and energy density scaled by Stefan-Boltzmann values for $\bar{\Lambda} = \frac{2}{3}(\mu_u + \mu_d + \mu_s)$. We show results without renormalization group improvements (dash-dotted lines), running coupling (dashed lines), both for $m_s = 0$, and results with running mass and coupling (full lines).

As can be seen from this Figure, there is a sizable difference between zero and finite strange quark mass pressure and energy density for the values of the chemical potential in the region that is relevant for the physics of compact stars. As has been noticed by several authors [9,10,14], the resulting equation of state, $\epsilon = \epsilon(P)$, can be approximated by a non-ideal bag model form $\epsilon = 4B_{eff} + aP$. Here $a \sim 3$ is a dimensionless coefficient while B_{eff}

is the effective bag constant of the vacuum. Concentrating on the low-density part of the equation of state, one finds for massless strange quarks the parameters $B_{eff}^{1/4} \simeq 117$ MeV and $a \simeq 2.81$ while the inclusion of the running mass raises these values to $B_{eff}^{1/4} \simeq 137$ MeV and $a \simeq 3.17$ (all values having been obtained by including a running α_s in the equations of state). Therefore, we expect important consequences in the mass-radius relation of quark stars due to the inclusion of a finite mass for the strange quark.

The structure of a quark star is determined by the solution of the TOV equations. Corrections to the mass and radius of quark stars due to a running strange quark mass can be very large, $\sim 25\%$ [1].

Also, while the most massive star for the $m_s=0$ equation of state (with $M/M_\odot \simeq 3.2$ and radius ~ 17 km) has a central density of $n_B \simeq 0.5$ fm $^{-3}$, this number increases to $n_B \simeq 0.83$ fm $^{-3}$ (at $\mu = 470$ MeV) for the heaviest star ($M/M_\odot \simeq 2.16$ at ~ 12 km) of the massive equation of state. The inclusion of $\sim \alpha_s^2$ corrections to the pressure will increase its non-ideality and produce quark stars which are smaller, denser and less massive [8,9,15].

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REFERENCES

1. E. S. Fraga and P. Romatschke, Phys. Rev. D **71**, 105014 (2005).
2. B. A. Freedman and L. D. McLerran, Phys. Rev. **D16**, 1130 (1977); *ibid.*, **D16**, 1147 (1977); *ibid.*, **D16**, 1169 (1977); *ibid.*, **D17**, 1109 (1978).
3. V. Baluni, Phys. Rev. D **17**, 2092 (1978).
4. T. Toimela, Int. J. Theor. Phys. **24**, 901 (1985) [Erratum-*ibid.* **26**, 1021 (1987)].
5. E. Witten, Phys. Rev. D **30**, 272 (1984).
6. C. Alcock, E. Farhi and A. Olinto, Astrophys. J. **310**, 261 (1986); P. Haensel, J. L. Zdunik, and R. Schaeffer, Astron. Astrophys. **160**, 121 (1986).
7. A. R. Bodmer, Phys. Rev. D **4**, 1601 (1971).
8. E. S. Fraga, R. D. Pisarski and J. Schaffner-Bielich, Phys. Rev. D **63**, 121702 (2001).
9. E. S. Fraga, R. D. Pisarski and J. Schaffner-Bielich, Nucl. Phys. A **702**, 217 (2002).
10. A. Peshier, B. Kämpfer and G. Soff, Phys. Rev. **C61**, 045203 (2000); Phys. Rev. D **66**, 094003 (2002).
11. J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D **63**, 065003 (2001).
12. J. O. Andersen and M. Strickland, Phys. Rev. D **66**, 105001 (2002).
13. A. Rebhan and P. Romatschke, Phys. Rev. D **68**, 025022 (2003).
14. M. Alford and S. Reddy, Phys. Rev. D **67**, 074024 (2003); M. Alford, M. Braby, M. Paris and S. Reddy, nucl-th/0411016.
15. E. S. Fraga and P. Romatschke, in preparation.
16. J. A. M. Vermaseren, S. A. Larin and T. van Ritbergen, Phys. Lett. B **405**, 327 (1997).
17. S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004).