From Color Fields to Quark Gluon Plasma

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We discuss a model for the energy distribution and the early space-time evolution of a heavy ion collision. We estimate the gluon field generated in the wake of hard processes and through primordial fluctuations of the color charges in the nuclei. Without specifying the dynamical mechanism of thermalization we calculate the energy momentum tensor of the following plasma phase. The results of this model can be used as initial conditions for a further hydrodynamic evolution.

1. INTRODUCTION

Our knowledge about the early phase of a heavy ion collision, is still incomplete. Hydrodynamic calculations compared with data collected at the Relativistic Heavy Ion Collider (RHIC) suggest a rather early thermalization time $\tau_f < 1$ fm/c, at which a deconfined quark gluon plasma is created [1]. After the time τ_f ideal hydrodynamics delivers a successful description of the dynamics of the fireball. The initial conditions for the hydrodynamic evolution however remain obscure and are often subject to an educated guess.

Here we propose a simple model for the initial state that focuses on two important questions: how much energy is deposited in the fireball between the receding nuclei, and how is this energy distributed in space-time at an early time τ_f? The model uses three simple ingredients: an initial distribution of color sources in the colliding nuclei, (ii) the evolution of a classical McLerran-Venugopalan gluon field, (iii) energy-momentum conservation.

2. OUTLINE OF THE MODEL

Let us choose a momentum scale Q_0 (which should be much larger than $\Lambda_{\rm QCD}$). We divide gluon production into hard modes (hard processes with transverse momentum $p_T > Q_0$) and soft modes (modes of the gluon field with $p_T < Q_0$). Hard modes can be rigorously treated within perturbative QCD (pQCD) and implemented in a parton cascade. For the soft modes we take guidance from the McLerran-Venugopalan (MV) model and describe them by classical solutions to the Yang-Mills equation $[D_\mu, F^{\mu\nu}] = J^\nu$ and the continuity equation $[D_\mu, J^\nu] = 0$ [2]. In the MV model the field is generated by local color fluctuations in the nuclei which are otherwise color neutral.

In our case the classical $SU(3)$ current J^μ contains these primordial color distributions ρ_1 and ρ_2. They are infinitely thin sheets of charge propagating with the speed of light along the light cone from $t = -\infty$ to $t = 0$. Hard processes take place on very short time scales $\tau \sim 1/p_T$. They happen almost instantaneous for the soft fields. For consistency color charges induced by hard processes have to be taken into account as additional sources of the soft gluon field, which are switched on at $t = 0$. This is similar to a color ionization process: a parton with a certain color is participating in a hard process, leaving a hole with the corresponding anticolor. The parton can instead be found in the forward light cone. Therefore J^μ contains modified color distributions ρ_1' and ρ_2' propagating on the light cone from $t = 0$ to $t = +\infty$ and a charge ρ_3 in the forward light cone.

Here we shall not discuss the most general case, but demonstrate the basic features of the model for the simple case of a boost invariant current. In an axial gauge, defined by the condition $x^+A^- + x^-A^+ = 0$ for the gauge potential A, we consider the current $J^i(x) = 0$ $(i = 1, 2)$ and

$$J^\pm(x) = \delta(x^\mp)\left[\rho_{1,2}(\vec{x}_\perp) \pm \Theta(x^\pm)\hat{\rho}(\vec{x}_\perp)\right], \tag{1}$$

with $\rho_{1,2}' = \rho_{1,2} \pm \hat{\rho}$, see Fig. 1. We solve the Yang-Mills equations for small times τ. Causality ensures that only charges with a transverse distance less than $c\tau$ can interact during this time. This suggests to discretize the charge distributions in the transverse plane. We use a rectangular grid of size a for both nuclei. In each bin the classical charge with an effective size $1/\mu < a$ $(\mu \sim Q_0)$ is put at the center. For $c\tau < a$ only the interaction of charges in the two nuclei which are opposite to each other (i.e. go through each other at the time of overlap, $t = 0$) have to be taken into account. The fireball between the receding nuclei is then described by a set of parallel tubes of width a with the bins of color charges at both ends (Fig. 2).

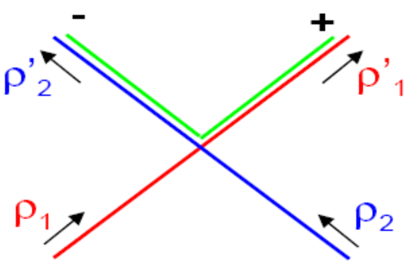

Figure 1. The propagation of the color charges defined above on the lightcone.

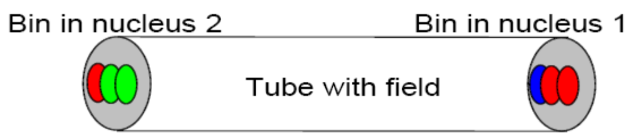

Figure 2. Charges in opposite bins in both nuclei and the tube connecting them which contains the gluon field generated by the interaction. The partons in both bins are represented by effective color charges T_1 and T_2.

In each bin in the nuclei the color charges of the partons are described by an effective classical color charge. It was shown that this classical charge is given by a random walk in $SU(3)$ space [3]. The probability distribution for the classical charge T is given by

$$w_N(|T|^2) = \left(\frac{N_c}{N\pi}\right)^4 e^{-N_c|T|^2/N} \tag{2}$$

where the length N of the random walk is determined by the number of quarks, antiquarks and gluons as $N = N_q + N_{\bar{q}} + (C_A/C_F)N_g$.

The Yang-Mills equations can now be solved for each tube using classical charges T_1 and T_2 as sources in the bins on either side. We solve them to all orders in the coupling constant g using an expansion in powers of $\tau\mu$. At small times the dominating field component is the longitudinal chromoelectric field F^{+-} in the tube. It has a finite value for $\tau = 0$, while the transverse chromoelectric and magnetic fields $F^{i\pm}$ vanish initially and then increase linearly at early times. The longitudinal chromomagnetic field F^{ij} is zero at least up to order τ^3. We have solved the field equations up to order τ^3 for the longitudinal fields and up to order τ^4 for the transverse fields.

Thus at early times the field between the receding nuclei is analogous to a capacitor as discussed in [4]. The longitudinal chromoelectric field will slow down the charges. The trajectory of each bin of partons for $t > 0$ can be calculated to lowest order by assuming that it initially travels with the rapidity y_0 of the beam. We can follow the trajectory to where it intersects the hypersurface $\tau = \tau_f$ at space-time rapidities η_1^* and η_2^*. These values limit the longitudinal extent of the tube with the gluon field or plasma (see Fig. 3).

At times $c\tau \approx a$ the fields of neighboring tubes start to overlap. We add them coherently, and calculate the resulting energy-momentum tensor $T_f^{\mu\nu}$ from the gluon field. It turns out that the energy-momentum tensor can be parameterized in the following way:

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} A + C\cosh 2\eta & B_1 \cosh\eta & B_2\cosh\eta & C\sinh 2\eta \\ B_1\cosh\eta & A + D & E & B_1\sinh\eta \\ B_2\cosh\eta & E & A - D & B_2\sinh\eta \\ C\sinh 2\eta & B_1\sinh\eta & B_2\sinh\eta & -A + C\cosh 2\eta \end{pmatrix} \tag{3}$$

where A, $\vec{B} = (B_1, B_2)$, C, D and E are functions of time τ and the transverse coordinate $x_\perp$ but independent of space-time rapidity η. At early times they behave like $A \sim \tau^0$, $\vec{B} \sim \tau^1$ and $C \sim \tau^2$ respectively, while the functions D and E will not play any further role. Using the expansion in τ for the gluon field as above we can ensure that $\partial_\mu T_f^{\mu\nu} = 0 + \mathcal{O}(\tau^2)$.

We can now match this field tensor with the tensor of the plasma phase, $T_{\mathrm{pl}}^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ characterized by a local energy density ϵ, pressure p and a 4-velocity $u = \gamma(1, \vec{v}_\perp, v_L)$ with longitudinal and transverse flow v_L and $\vec{v}_\perp$. Without detailed knowledge of the decay and thermalization of the gluon field we can still make use of the fact that $\partial_\mu T^{\mu\nu} = 0$ where $T_{\mu\nu}$ is the total energy momentum tensor of the fireball which is equal to $T_f^{\mu\nu}$ up to some time before the thermalization time τ_f and equal to $T_{\mathrm{pl}}^{\mu\nu}$ after that.

Here we discuss the case of an instantaneous matching for which $T^{\mu\nu} = \Theta(\tau_0 - \tau)T_f^{\mu\nu} + \Theta(\tau - \tau_0)T_{\mathrm{pl}}^{\mu\nu}$. Energy-momentum conservation implies a set of equations for the plasma:

$$\vec{v}_\perp = \frac{1}{\cosh\eta}\frac{\vec{B}}{A + C + p)}, \quad v_L = \tanh\eta, \quad \epsilon + p = (A + C)\left(1 - \frac{\vec{B}^2}{(A + C + p)^2}\right) \tag{4}$$

These equations fix transverse flow to be in the direction of $\vec{B} \propto T_f^{0i}$. We also note that the longitudinal flow leads to a boost-invariant scenario where rapidity y is equal to η up to the limiting values $\eta_{1,2}^*$. Of course this is a consequence of the boost-invariant charge distribution in this example. We need another equation to unambiguously determine the magnitude of $v_\perp$, p and ϵ. This can be the equation of state which is used for the hydrodynamic evolution for $\tau > \tau_f$. E.g. an ideal gas equation of state would lead to $\epsilon = -(A+C) + \sqrt{4(A+C)^2 - 3B^2}$.

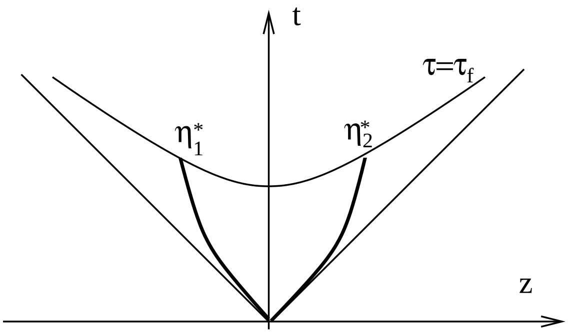

Figure 3. The longitudinal structure of a tube in the fireball. The partons in the bins to the left and right are decelerated. They reach final rapidities η_1^* and η_2^* at the thermalization time τ_f.

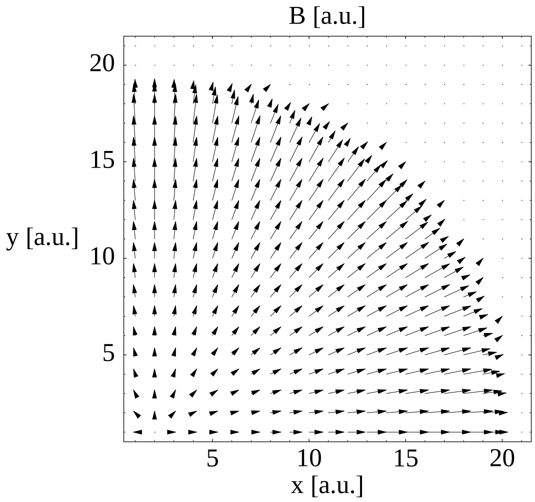

Figure 4. A sketch of the 2-vector $\vec{B}$ in the transverse plane which determines the radial flow in the plasma phase.

3. SUMMARY

To summarize, we introduce a model for the early phase of a heavy ion collision that can be used to estimate how much energy is deposited by the production of gluon fields and how the energy is distributed once the system thermalizes. In this model it is natural to have transverse flow and also elliptic flow already at the beginning of the plasma phase. Flow originates from the transverse energy flow T_f^{0i} of the early gluon phase, see Fig. 4. Preliminary results have been obtained using a boost invariant charge distribution in the nuclei. Some of the details remain to be carried out in more detail. In particular we aim for a consistent implementation of hard processes which minimizes the dependence of the final results on the hard scale Q_0. Hydrodynamic simulations using our results are in preparation.

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