Violation of $k_\perp$ factorization in quark production from the Color Glass Condensate

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We examine the violation of the $k_\perp$ factorization approximation for quark production in high energy proton-nucleus collisions. We comment on its implications for the open charm and quarkonium production in collider experiments.

1. Introduction

Semi-hard processes, where $\sqrt{s} \gg m_{q_\perp} \gg \Lambda_{\rm QCD}$, contribute significantly to particle production in high-energy collider experiments due to the large density of the small-x gluons. The $k_\perp$ factorization formalism[1] systematically resums corrections of $(\alpha_s \ln(s/q_\perp^2))^n$ from gluon branchings in perturbative QCD. In this framework, the particle production cross-section is expressed as a convolution of a hard matrix element and *unintegrated* distributions of gluons in the hadrons with definite transverse momentum $\boldsymbol{k}_{i\perp}$ and longitudinal fraction x_i in each projectile hadron (i=1, 2).

Multiple-scattering (higher twist) effects become important at small x due to the large density of small-x gluons. It is expected to be the origin of the Cronin enhancement and $p_\perp$ broadening of hadrons observed in nuclear experiments. It is also relevant for the nuclear suppression of quarkonium production.

The simplest situation for studying the impact of higher twist effects on $k_\perp$ factorization is in proton-nucleus (pA) collisions, wherein the proton is dilute and the nucleus is dense. The $k_\perp$ factorization formalism was examined in the color glass condensate framework[2]. It is shown that the factorization is recovered when one keeps only the terms that are of the lowest order in the charge sources $\rho_{p,A}$ of the projectiles[3]. The cross-sections at the leading order in ρ_p, but at all orders in the dense source ρ_A of the nucleus are obtained analytically. Gluon production by the "2-to-1" processes is shown to be $k_\perp$-factorizable [4–6] whereas the quark production is generally not[7–9].

Here we report the numerical estimates for the $k_\perp$ factorization breaking in quark production within the McLerran-Venugopalan (MV) model[10]. We briefly discuss open charm production and quarkonium suppression in pA collisions in this framework.

2. Violation of $k_\perp$ factorization in quark pair production

The quark pair production cross-section is obtained as[7]:

$$\frac{d\sigma}{d^2\boldsymbol{p}_\perp d^2\boldsymbol{q}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4(N^2-1)} \int_{\boldsymbol{k}_{1\perp},\boldsymbol{k}_{2\perp}} \frac{\delta(\boldsymbol{p}_\perp + \boldsymbol{q}_\perp - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$
$$\times \Bigg\{ \int_{\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp} \mathrm{tr_d}\Big[(\not{q}+m)T_{q\bar{q}}(\not{p}-m)\gamma^0 T'^{\dagger}_{q\bar{q}}\gamma^0\Big]\phi_A^{q\bar{q},q\bar{q}}(\boldsymbol{k}_{2\perp};\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp)$$
$$+ \int_{\boldsymbol{k}_\perp} \mathrm{tr_d}\Big[(\not{q}+m)T_{q\bar{q}}(\not{p}-m)\gamma^0 T_g^{\dagger}\gamma^0 + \text{h.c.}\Big]\phi_A^{q\bar{q},g}(\boldsymbol{k}_{2\perp};\boldsymbol{k}_\perp)$$
$$+\mathrm{tr_d}\Big[(\not{q}+m)T_g(\not{p}-m)\gamma^0 T_g^{\dagger}\gamma^0\Big]\phi_A^{g,g}(\boldsymbol{k}_{2\perp})\Bigg\}\varphi_p(\boldsymbol{k}_{1\perp})\,, \qquad (1)$$

where the explicit forms for the Dirac matrices $T_{q\bar{q}}(\boldsymbol{k}_{1\perp},\boldsymbol{k}_\perp)$ and $T_g(\boldsymbol{k}_{1\perp})$ are given in [7]. Here $\varphi_p(\boldsymbol{l}_\perp) \equiv (\pi^2 R_p^2 g^2/l_\perp^2)$ F.T. $\langle \rho_p^a(\boldsymbol{0})\rho_p^a(\boldsymbol{x}_\perp)\rangle$ is the unintegrated gluon distribution for the proton, and F.T. denotes the Fourier transformation. One needs, however, *three* nuclear distributions defined as (see Eqs. (42), (43) and (45) in [7])

$$\phi_A^{g,g}(\boldsymbol{l}_\perp) \equiv \frac{\pi^2 R_A^2 l_\perp^2}{g^2 N}\text{ F.T. tr}\left\langle U(\boldsymbol{0})U^\dagger(\boldsymbol{x}_\perp)\right\rangle\,,$$
$$\phi_A^{q\bar{q},g}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp) \equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N}\text{ F.T. tr}\left\langle \widetilde{U}(\boldsymbol{x}_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)t^b U_{ba}(\boldsymbol{0})\right\rangle\,,$$
$$\phi_A^{q\bar{q};q\bar{q}}(\boldsymbol{l}_\perp;\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp) \equiv \frac{2\pi^2 R_A^2 l_\perp^2}{g^2 N}\text{ F.T. tr}\left\langle \widetilde{U}(\boldsymbol{0})t^a\widetilde{U}^\dagger(\boldsymbol{y}_\perp)\widetilde{U}(\boldsymbol{x}'_\perp)t^a\widetilde{U}^\dagger(\boldsymbol{y}'_\perp)\right\rangle\,, \qquad (2)$$

where U and $\widetilde{U}$ denote the path-ordered exponentials of the gauge fields in the nucleus in the adjoint and fundamental representations, respectively, and describe the multiple scatterings of the gluon and the quarks. The average $\langle\cdots\rangle$ is taken over the Gaussian distribution of the color charge sources characterized by the saturation scale Q_s^2.

$k_\perp$ factorization is violated by the transverse structure of the quark pair probed by the momentum $\boldsymbol{k}_\perp{}^{(\prime)}$ from the nucleus since each quark from the pair can resolve and interact with several gluons from the nucleus. If any of the transverse masses $m_{q_\perp}$ and $m_{p_\perp}$ of the produced quarks is large compared with the typical rescattering scale, Q_s, we can neglect $\boldsymbol{k}_\perp^{(\prime)}$ in $T_{q\bar{q}}(\boldsymbol{k}_{1\perp},\boldsymbol{k}_\perp^{(\prime)})$ and recover the $k_\perp$ factorized formula thanks to the sum rule for ϕ_A's; $\int_{\boldsymbol{k}_\perp,\boldsymbol{k}'_\perp}\phi_A^{q\bar{q},q\bar{q}} = \int_{\boldsymbol{k}_\perp}\phi_A^{q\bar{q},g} = \phi_A^{g,g}$.

In Fig. 1 we compare the exact result with the $k_\perp$ factorized approximation for single charm quark production. The breaking is relatively small for the saturation momentum Q_s^2=1 GeV^2, which may be the relevant scale for RHIC at central rapidity. At Q_s^2=15, 25 GeV^2 (corresponding to very forward rapidities in the proton fragmentation region at RHIC and LHC) the correction can be as large as 40% at $q_\perp \sim Q_s$. For the bottom quark production the violation is smaller. To assess the model-dependence of our results, we compute them now, shown in Fig. 2, with a non-local Gaussian model known to be the

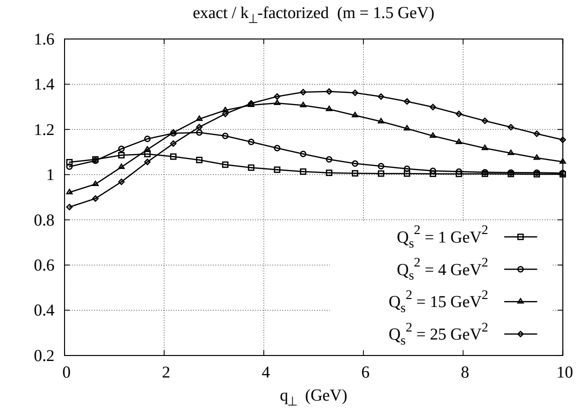

Figure 1. Breaking of $k_\perp$ factorization in single charm quark production.

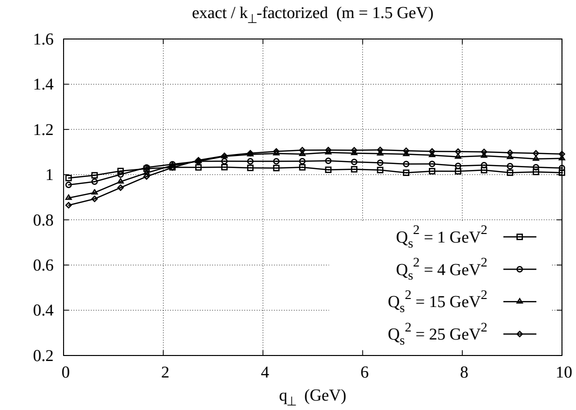

Figure 2. The same as in Fig. 1 but in the nonlocal Gauss model.

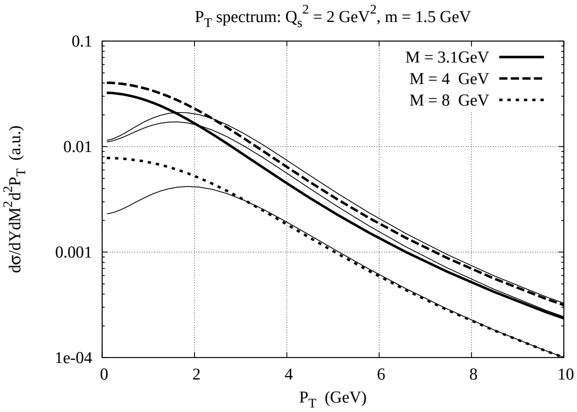

Figure 3. $P_\perp$ spectrum of the quark pair with fixed invariant mass M.

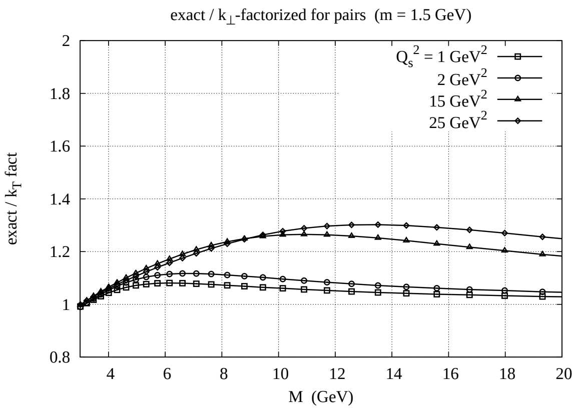

Figure 4. Breaking of $k_\perp$ factorization in charm quark pair production.

asymptotic solution of renormalization equations for x evolution[11]; non-linear evolution effects reduce the magnitude of the violation of $k_\perp$ factorization.

In Fig. 3 shown is the total $P_\perp$ distribution of the charm quark pair with the fixed invariant masses M=3.1, 4, 8 GeV. In the $k_\perp$ factorized approximation (thin curves), either quark or antiquark exchanges all the momentum from the nucleus and we see the bump structure near Q_s, reflecting the gluon distribution of the nucleus. The bump is smeared out due to multiple scatterings of both the quark and antiquark in the full formula. Integrating over $P_\perp$, we show in Fig. 4, the magnitude of factorization breaking in the invariant mass spectrum of the pair.

3. Phenomenology

We study the importance of small-x distributions in D meson production by convoluting the single quark spectrum with an appropriate fragmentation function[12]. We find, however, the production spectrum is determined not by the quark distribution with $q_\perp \lesssim Q_s$, but largely by the tail part $\propto 1/q_\perp^4$ of the MV model. Moreover, in order to assess

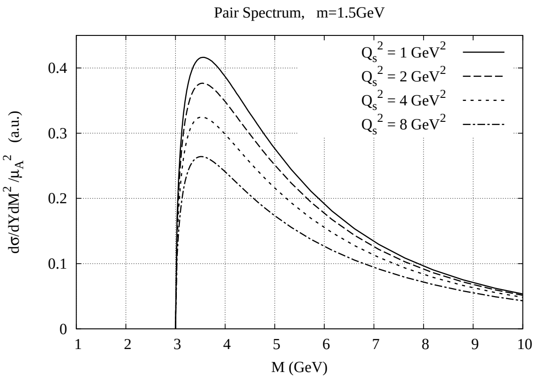

Figure 5. Q_s^2 dependence of the charm pair production.

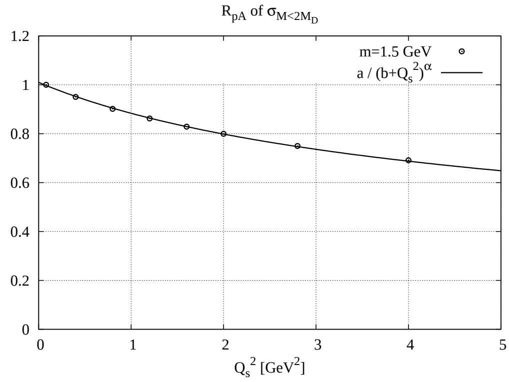

Figure 6. Suppression of low mass pairs in pA collisions.

the rapidity dependence of open charm production, the x-dependence of the unintegrated gluon distributions should be taken into account, which requires going beyond the MV model. Our results on open charm production will be reported elsewhere[13].

The Q_s^2-dependence of the pair spectrum (divided by the charge density μ_A^2) is diplayed in Fig. 5. At larger M, where the high-density effects are diminished, all curves converge to a single one. The multiple scatterings of the pair quarks suppress the yield in the low M region. (The overall cross-section is of course enhanced with increasing Q_s^2.) One can get an idea about the normal suppression of the quarkonium production in the pA collisions, relying on the color evaporation picture. We show the nuclear modification ratio, R_{pA}, for the pairs with M less than the open charm threshold $2M_D$, as a function of Q_s^2. The suppression pattern fits the form $1/(Q_s^2)^\alpha$ with $\alpha \sim 0.42$, and not the frequently assumed exponential form. One should note here that $Q_s^2 \sim A^{1/3}$ in the MV model.

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