Hot matter from exploding black holes

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The relativistic viscous fluid equations describing the outflow of high temperature matter created via Hawking radiation from microscopic black holes are solved numerically for a realistic equation of state. We focus on black holes with initial temperatures greater than 100 GeV and lifetimes less than 6 days. The spectra of photons and neutrinos are calculated for energies greater than 1 GeV. The most promising route for the observation of these black holes is to search for point sources emitting gamma rays or neutrinos of ever-increasing energy.

1. INTRODUCTION

Hawking radiation from black holes [1] is of fundamental interest because it relies on the application of relativistic quantum field theory in the presence of the strong field limit of gravity, a situation that could potentially be observed. It is also of great interest because of the temperatures involved. A black hole with mass M radiates thermally with a Hawking temperature $T_{\rm H} = m_{\rm P}^2/8\pi M$ where $m_{\rm P} = G^{-1/2} = 1.22 \times 10^{19}$ GeV is the Planck mass. (Units are $\hbar = c = k_{\rm B} = 1$.) In order for the black hole to evaporate rather than accrete it must have a temperature greater than that of the present-day black-body radiation of the universe of 2.7 K = 2.3×10^{-4} eV. This implies that M must be less than 1% of the mass of the Earth. Such small black holes most likely would have been formed primordially; there is no other mechanism known to form them. As the black hole radiates, its mass decreases and its temperature increases until $T_{\rm H}$ becomes comparable to the Planck mass, at which point the semi-classical calculation breaks down and the regime of full quantum gravity is entered. We shall focus on Hawking temperatures greater than 100 GeV (corresponding to a Schwarzschild radius of 1.6×10^{-4} fm) where new physics is likely to be discovered. It is highly unlikely that such temperatures could ever be achieved in accelerator based heavy ion collisions, not even with the LHC. The fact that microscopic black holes have not yet been observed should not be viewed as a deterrent, but rather as a challenge for the new millennium!

2. VISCOUS FLUID DYNAMICS

Heckler has shown that when the Hawking temperature is sufficiently high so many particles are emitted that they scatter from each other, creating something akin to a wind [2]. He showed that it is inconsistent to assume that they do not scatter and, at

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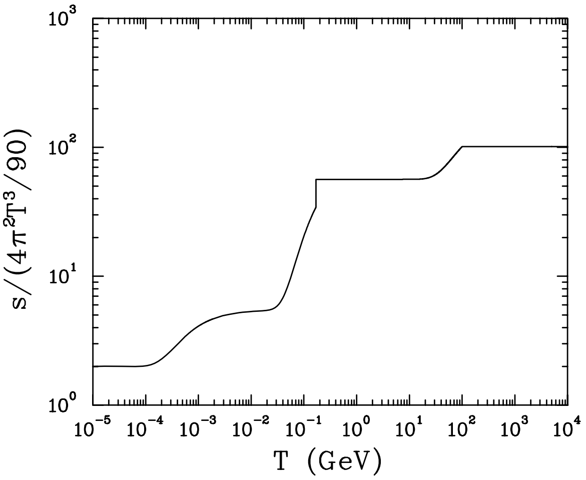

Figure 1. Entropy density as a function of temperature, excluding neutrinos and gravitons. It is assumed that the QCD phase transition is first order and the EW phase transition is second order.

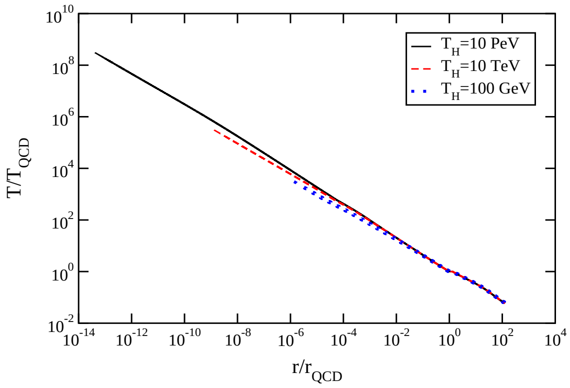

Figure 2. The radial dependence of T for three different Hawking temperatures.

the other extreme, that it is inconsistent to assume that they scatter so frequently that perfect fluid dynamics applies. Therefore we have used relativistic viscous fluid dynamics to describe the outflow of emitted particles [3–5]. Under the assumptions of steady-state, spherically symmetric flow, with no net baryon number or electric charge, and working at distances at least several times the Schwarzschild radius so that gravity may be neglected, the equations of motion are:

$$4\pi r^2 \left[\gamma u T s - \frac{4}{3}\eta\gamma u \left(\frac{du}{dr} - \frac{u}{r}\right) - \zeta\gamma u \left(\frac{du}{dr} + \frac{2u}{r}\right)\right] = L \tag{1}$$

$$\frac{d}{dr}\left(4\pi r^2 u s\right) = \frac{4\pi r^2}{T}\left[\frac{8}{9}\eta\left(\frac{du}{dr} - \frac{u}{r}\right)^2 + \zeta\left(\frac{du}{dr} + \frac{2u}{r}\right)^2\right] \tag{2}$$

Here s is the entropy density, η is the shear viscosity, ζ is the bulk viscosity (typically very small compared to η), and $u = v\gamma$ where v is the radial flow velocity. The first equation is energy conservation with L being the luminosity of the black hole as calculated from Hawking radiation, and the second equation gives the rate of increase of entropy with radius. The functions $s(T)$, $\eta(T)$, and $\zeta(T)$ are the input, and this pair of equations is solved for $T(r)$ and $u(r)$.

For a semi-realistic equation of state, with $s = aT^3$, $\eta = b_\eta T^3$, and $\zeta = b_\zeta T^3$, where a, b_η, and b_ζ are constants, there is a scaling solution when $u \gg 1$.

$$T(r) = T_0(r_0/r)^{2/3} \qquad u(r) = u_0(r/r_0)^{1/3} \tag{3}$$

It can be shown that this leads to an internally self-consistent description of the outflow of hot matter, with local thermal equilibrium being maintained due to viscosity. (An internally self-consistent description is not possible if the viscosities are taken to be zero, as for perfect fluid flow.) A more realistic equation of state is shown in Figure 1. Taking $\eta/s \sim$ constant and $\zeta \ll \eta$, one can solve the equations numerically. A plot of T versus r is shown in Figure 2. The numbers and scales are quite remarkable when compared to heavy ion collisions.

3. GAMMA RAY AND NEUTRINO SPECTRA

The primary sources of photons are direct emission from the decoupling surface and the decay $\pi^0 \to \gamma\gamma$ at the surface. Local thermal equilibrium is maintained as long as the local volume expansion rate $u^{\mu}{}_{;\mu}$ is less than the local particle collision rates. Our computations show that local thermal equilibrium is lost when the local temperature falls to somewhere in the range of 120 to 180 MeV. Figure 3 shows the instantaneous gamma ray spectrum for Hawking temperatures of 100 GeV, 1 TeV and 10 TeV with a decoupling/freeze-out temperature of 140 MeV. These correspond to a remaining lifetime of the black hole of 5.4 days, 7.7 minutes, and 0.5 seconds, respectively. They are approximately, but not exactly, exponential in shape. If the emission rate is integrated over time until the black hole disappears, from $T_{\rm H} = T_0$ to $T_{\rm H} = \infty$, the spectrum is well-approximated by

$$\frac{dN}{dE} = \frac{m_{\rm P}^2 T_f}{26 E^4} \tag{4}$$

which applies for $E > T_0$.

High energy neutrinos will also be copiously emitted. There are a number of sources. Neutrinos can come from charged pion decay, from the decay of directly produced muons, or from the decay of muons arising from charged pion decay. Directly produced neutrinos will be emitted from the neutrino-sphere, which is determined by the temperature at which neutrinos lose thermal equilibrium. Since neutrino interactions above a center-of-momentum energy of about 100 GeV interact as strongly as quarks and gluons, we estimate the decoupling/freeze-out temperature for them to be about 100 GeV. The time-integrated neutrino spectra from various sources are shown in Figure 4. Also shown is the spectrum arising from Hawking radiation without any rescattering (one should consider one or the other but not the sum). Compared to direct Hawking radiation, viscous flow increases the total number of neutrinos and shifts their average energy to a lower value. Like photons, the spectrum has the shape $\sim E^{-4}$ at high energies.

4. CONCLUSION

The local rate density of exploding microscopic black holes has an upper limit determined by Page and Hawking [6] to be about 1 to 10 per cubic parsec per year on the basis of the observed diffuse gamma ray spectrum for photons with energies on the order of 100 MeV. More recently Wright [7] used EGRET data to search for an anisotropic component to gamma rays with energies in the range of 30 MeV to 100 GeV to place an upper limit of about 0.4 per cubic parsec per year. If the actual rate density is close to these upper limits, a high energy neutrino detector would see about 1 event per year, and

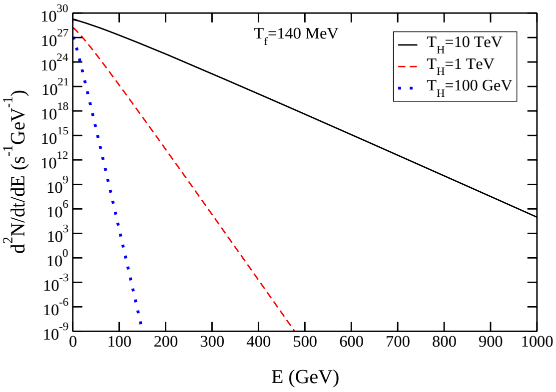

Figure 3. The instantaneous gamma ray spectrum for three different Hawking temperatures assuming a decoupling/freeze-out temperature of 140 MeV.

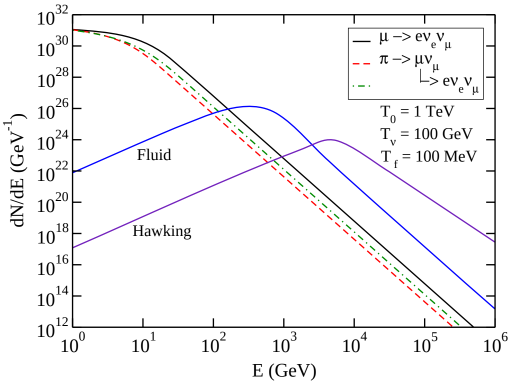

Figure 4. The time integrated spectra for one species of neutrino or anti-neutrino. The calculation begins when the black hole temperature is $T_0 = 1$ TeV. Either the direct Hawking radiation of neutrinos or the direct neutrino emission from a neutrinosphere at a temperature of 100 GeV should be used.

a high energy gamma ray detector would see more. Their characteristic signatures would be the spectra calculated here and elsewhere, but more significantly, they would be the only type of object known that would get brighter with time and then suddenly disappear. Observation of high energy gamma rays and neutrinos, especially in conjunction with one another, from exploding microscopic black holes may provide a window on physics well beyond the 1 TeV scale.

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