Can Isotropization and Thermalization in Heavy Ion Collisions be Obtained from Summing Feynman Diagrams?

Yuri V. Kovchegov[a] *

[a]Department of Physics, The Ohio State University
Columbus, OH 43210

We argue that isotropization and, consequently, thermalization of the system of gluons and quarks produced in an ultrarelativistic heavy ion collision does not follow from Feynman diagram analysis to all orders in the coupling constant. We conclude that the apparent thermalization of quarks and gluons, leading to success of Bjorken hydrodynamics in describing heavy ion collisions at RHIC, can only be attributed to the non-perturbative QCD effects not captured by Feynman diagrams.

1. INTRODUCTION: ISOTROPIZATION VERSUS FREE STREAMING

The results presented here are based on the work done in [1,2].

Similar to the original Bjorken hydrodynamics approach [3], let us consider a central high energy collision of two very large nuclei. For simplicity, here we will discuss the case where the distribution of particles is independent of space-time rapidity $\eta = (1/2)\ln(x_+/x_-)$, where $x_\pm = (t \pm z)/\sqrt{2}$. Since the nuclei are very large the transverse coordinate dependence can also be neglected for most physical quantities, leaving only the dependence on the proper time $\tau = \sqrt{2x_+x_-}$. For this geometry, one can show that the most general energy-momentum tensor can be written as (at $z = 0$) [1]

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & p(\tau) & 0 & 0 \\ 0 & 0 & p(\tau) & 0 \\ 0 & 0 & 0 & p_3(\tau) \end{pmatrix}, \tag{1}$$

where z-axis is taken along the beam direction, and x, y-axes are in the transverse direction. Applying the conservation of energy-momentum tensor condition

$$\partial_\mu T^{\mu\nu} = 0 \tag{2}$$

to the energy-momentum tensor that gives Eq. (1) at $z = 0$ we obtain

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + p_3}{\tau}. \tag{3}$$

There are two interesting cases one can consider:

*This work is supported in part by the U.S. Department of Energy under Grant No. DE-FG02-05ER41377.

(i) if $p_3 = 0$ longitudinal pressure vanishes and, due to Eq. (3), we get

$$\epsilon \sim \frac{1}{\tau}, \tag{4}$$

such that the total energy $E \approx \epsilon\,\tau = \text{const}$. This case is known as *free streaming*: the system expands freely without loosing any energy.

(ii) if $p_3 = p$ the energy-momentum tensor in Eq. (1) becomes *isotropic*. This is the case of ideal Bjorken hydrodynamics [3]. Eq. (3) with $p_3 = p$ was derived in [3]. If combined with the ideal gas equation of state, $\epsilon = 3\,p$, it gives

$$\epsilon \sim \frac{1}{\tau^{4/3}} \tag{5}$$

or, for other equations of state,

$$\epsilon \sim \frac{1}{\tau^{1+\Delta}} \qquad \text{with} \qquad \Delta > 0. \tag{6}$$

Eq. (3) demonstrates that changes in the total energy $E \approx \epsilon\,\tau$ (or, equivalently, deviations from $\epsilon \sim 1/\tau$ scaling) are due to work done by the longitudinal pressure p_3. The classical initial conditions in the Color Glass Condensate approach [4] yield the free streaming final state with $p_3 = 0$. A thermalized quark-gluon plasma is characterized by non-zero p_3, leading to the energy density scaling as shown in Eq. (6). Therefore, below we will understand *isotropization*, which is the necessary condition for *thermalization*, as dynamical generation of non-zero longitudinal pressure $p_3 \neq 0$, or, equivalently, deviations from the scaling of Eq. (4) leading to the scaling of Eq. (6).

2. FORMAL ARGUMENT

An extensive search of the diagrams which would bring in the desired deviations from the scaling of Eq. (4) carried out by the author did not yield any positive results: while many diagrams have contributions to ϵ scaling as Eq. (6), such terms are *always* subleading additive corrections to the leading (at late times) terms scaling as Eq. (4). In fact one can construct an argument [1] demonstrating that the leading contribution to energy density from any-order diagrams scales as $\epsilon \sim 1/\tau$. The argument is presented below.

We begin by considering a gluon field generated by an arbitrary Feynman diagram [1], illustrated in Fig. 1. In $\partial_\mu A^\mu = 0$ covariant gauge it can be written as

$$A_\mu^a(x) = -i \int \frac{d^4k}{(2\pi)^4} \, \frac{e^{-ik\cdot x}}{k^2 + i\epsilon k_0} \, J_\mu^a(k), \tag{7}$$

where the function $J_\mu^a(k)$ denotes the rest of the diagram in Fig. 1 (the truncated part), which depends on the momenta of other outgoing gluons as well. Indeed gluon field can be defined as a simple function only in the classical case: the "field" in Eq. (7) should be thought of as a Feynman diagram in Fig. 1 with one of the outgoing gluon lines being off mass-shell, i.e., a generalization of the classical field which we will need in calculating

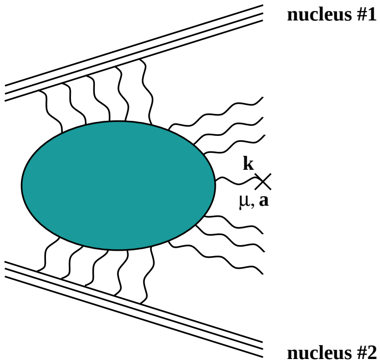

Figure 1. Gluon "field" generated by an arbitrary-order diagram (see text).

energy density [1]. (The expression in Eq. (7) can also be thought of as an operator equation.)

Substituting Eq. (7) into the expression for energy-momentum tensor

$$T^{\mu\nu} = \left\langle -F^{a\,\mu\rho}\, F^{a\,\nu}{}_{\rho} + \frac{1}{4} g^{\mu\nu}\, (F^{a}_{\rho\sigma})^2 \right\rangle, \tag{8}$$

averaging over the nuclear wave functions and employing the symmetries of the collision of two identical nuclei we obtain the energy density due to the gluon field [1]

$$\epsilon = \int \frac{d^4k\, d^4k'}{(2\pi)^8} \frac{e^{-ik\cdot x - ik'\cdot x}}{(k^2 + i\epsilon k_0)(k'^2 + i\epsilon k'_0)} \left\{ \frac{1}{2} \left[\left(\frac{\tau}{x_+} \right)^2 k_+ k'_+ - \underline{k}^2 \right] f_1(k^2, k'^2, k\cdot k', k_T) + \ldots \right\} \tag{9}$$

where $f_1(k^2, k'^2, k\cdot k', k_T)$ is some unknown function (a "form-factor") and the ellipsis indicate addition of two more similar terms with different "form-factors" f_2 and f_3.

Rewriting each "form-factor" as

$$\begin{aligned} f_i(k^2, k'^2, k\cdot k', k_T) \;&=\; f_i(k^2 = 0, k'^2 = 0, k\cdot k' = 0, k_T) + \\ +[f_i(k^2, k'^2, k\cdot k', k_T) &- f_i(k^2 = 0, k'^2 = 0, k\cdot k' = 0, k_T)] \end{aligned} \tag{10}$$

and using the fact that the square of truncated part of the diagram gives a cross section

$$\frac{dN}{d^2k\, dy} = \frac{1}{2(2\pi)^3} \left\langle\left\langle J^{a\,\rho}(k)\, J^{a}_{\rho}(-k) \right\rangle\right\rangle\Big|_{k^2=0} \tag{11}$$

we conclude that, keeping only the first term on the right hand side of Eq. (10) for all "form-factors" in Eq. (9) yields

$$\epsilon = \frac{\pi}{2} \int d^2k\, \frac{dN}{d^2k\, d\eta\, d^2b}\, k_T^2 \left\{ [J_1(k_T\tau)]^2 + [J_0(k_T\tau)]^2 \right\} \approx \frac{1}{\tau} \int d^2k\, \frac{dN}{d^2k\, d\eta\, d^2b}\, k_T, \tag{12}$$

where the last equality is valid for late proper times τ. Since, as was shown in [1], each factor of k^2, k'^2 or $k\cdot k'$ gives a factor of $1/\tau$, the terms in the square brackets of Eq. (10) give a subleading (compared to Eq. (12)) contribution to energy density at late times τ and can be safely neglected. We have shown that any diagram and/or any set of diagrams contributing to gluon production cross section lead to energy density scaling as in Eq. (4), i.e., that isotropization and, consequently, thermalization do not take place in perturbation theory analysis of the collisions.

The main assumption of the argument presented above is the existence of multiplicity of produced gluons $dN/d^2k\, dy$, which is the essential assumption of QCD perturbation theory. This is what makes our argument perturbative.

3. PHYSICAL ARGUMENT

Now let us present a physical argument demonstrating the origin of the power of 4/3 in Eq. (5) and explaining why it is impossible to achieve in perturbation theory [2]. Let us assume that thermalization does take place at some time τ_{th}. If a gauge invariant time τ_{th} exists, we can put the QCD coupling constant $g = 0$ for all times $\tau > \tau_{th}$. Bjorken hydrodynamics in the $g = 0$ limit is governed by the ideal gas equation of state $\epsilon = 3\,p$, which leads to the energy density scaling as shown in Eq. (5). (For small but non-zero g, Eq. (5) would get an $o(g^2)$ negative correction to 4/3: the expansion in g would still be around the power of 4/3.) Due to Eq. (3), the scaling of Eq. (5) in the $g = 0$ limit of Bjorken hydrodynamics means that $p_3 \neq 0$ and the gas of *non-interacting* particles is *doing work* in the longitudinal direction! What causes such a behavior of the system? The problem lies in the ideal gas equation of state, $\epsilon = 3\,p$, which assumes that the ideal gas is in contact with some *external thermal bath*. Such external thermal bath could be a background field or a box containing the gas: the ideal gas of non-interacting particles stays thermal through the interactions between the gas particles and the thermal bath. This is the only interaction allowed and it is responsible for the work done by the non-interacting gas. Since there is no such external thermal bath in heavy ion collisions, the scaling of Eq. (5) is impossible to achieve at small coupling.

Without the external thermal bath the particles in the gas would be just free streaming, giving the physically correct energy density scaling of Eq. (4). Of course, at a fixed time τ hydro is not applicable in the $g = 0$ limit, since the mean free path of the particles would exceed the longitudinal size of the system. However, *if* thermalization does happen, for any fixed arbitrary small g, if we wait long enough hydrodynamics should become applicable, leading to the scaling arbitrary close to that of Eq. (5) and doing work in the longitudinal direction which would be mostly due to contact with the non-existing external thermal bath. Therefore, we arrive at a contradiction, demonstrating that hydrodynamics is not achievable at small coupling. At large coupling, non-perturbative effects may mimic the external thermal bath, possibly leading to energy density scaling shown in Eq. (6).

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