Quark-antiquark production from classical fields and chemical equilibration

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We compute by numerical integration of the Dirac equation the number of quark-antiquark pairs produced in the classical color fields of colliding ultrarelativistic nuclei. The backreaction of the created pairs on the color fields is not taken into account. While the number of $q\bar{q}$ pairs is parametrically suppressed in the coupling constant, we find that in this classical field model it could even be compatible with the thermal ratio to the number of gluons. After isotropization one could thus have quark-gluon plasma in chemical equilibrium.

1. Introduction

The initial stages of an ultrarelativistic heavy ion collision are believed to be dominated by strong classical color fields. There is a twofold interest in calculating the production of quark–qntiquark pairs from these classical fields. Firstly, although heavy quark production is in principle calculable perturbatively, it would be interesting to understand whether these strong color fields influence the result. Secondly, being able to compute both gluon and quark production in the same framework would give insight into the chemical equilibration of the system and test the consistency of the assumption of gluon dominance. The number of quark pairs present in the early stages of the system has observable consequences in the thermal photon and dilepton spectrum.

In this talk we shall present first results [1] of a numerical computation of quark antiquark pair production from the classical fields of the McLerran-Venugopalan (MV) model. The equivalent calculation, although in another gauge, has been carried out analytically to lowest order in the densities of both color sources ("pp"-case) in Ref. [2] and to lowest order in one of the sources ("pA"-case) in Ref. [3]. The corresponding calculation in the Abelian theory [4,5], of interest for the physics of ultraperipheral collisions, can be done analytically to all orders in the electrical charge of the nuclei. Quark pair production has also been studied in a related "CGC"-approach in Refs. [6,7] and in a more general background field in Ref. [8].

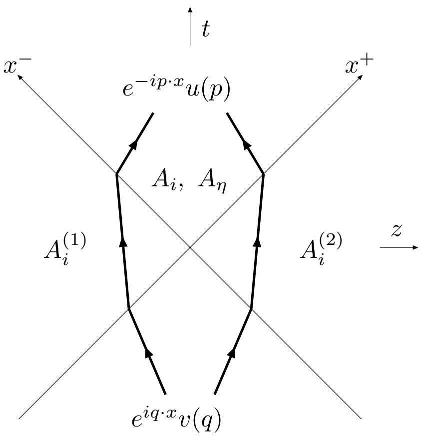

Figure 1. Domains of different time dependences. The fermion amplitude is a sum of two terms: one with interaction first with the left moving nucleus, then the right moving one, and vice versa. $A_i^{(1,2)}$ are pure gauges and A_i, A_η is a numerically computed color field.

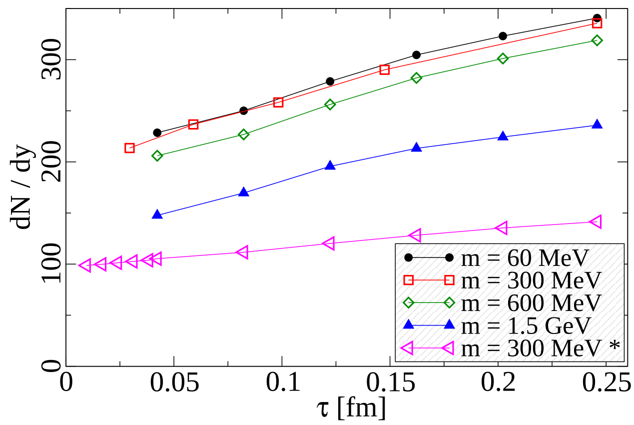

Figure 2. Dependence on proper time τ of the number of pairs of one flavor per unit rapidity $\mathrm{d}N/\mathrm{d}y$ for $g^2\mu = 2$ GeV and for values of quark mass marked on the figure. The lowest curve corresponds to $g^2\mu = 1$ GeV.

2. The numerical calculation

Our calculation of pair production relies on the numerical calculation of the classical background color field in which we solve the Dirac equation.

In the classical field model the background gluon field is obtained from solving the Yang-Mills equation of motion with the classical color source J^ν given by transverse color charge distributions of the two nuclei boosted to infinite energy:

$$[D_\mu, F^{\mu\nu}] = J^\nu = \delta^{\nu+}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\nu-}\rho_{(2)}(\mathbf{x}_T)\delta(x^+). \tag{1}$$

In the MV model [9] the color charges are taken as random variables with a Gaussian distribution

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)\rangle = g^2\mu^2\delta^{ab}\delta^2(\mathbf{x}_T - \mathbf{y}_T), \tag{2}$$

depending on the coupling g and a phenomenological parameter μ. The combination $g^2\mu$ is closely related to the saturation scale Q_s. Collisions of two ions were first studied analytically using this model in Ref. [10] and the way of numerically solving the equations of motion was formulated in Ref. [11].

Our method of solving the Dirac equation is explained in more detail and the numerics tested in a 1+1-dimensional toy model in Ref. [12]. The domains of spacetime involved in the calculation are illustrated in Fig. 1. One starts in the the infinite past $t \to -\infty$ with a negative energy plane wave solution $\psi(x) = e^{iq\cdot x}v(q)$. The Dirac equation can then be integrated forward in time analytically to the future light cone ($\tau^2 = 2x^+x^- = 0,\ x^\pm > 0$) because the background field in the intermediate region is a pure gauge. This gives an

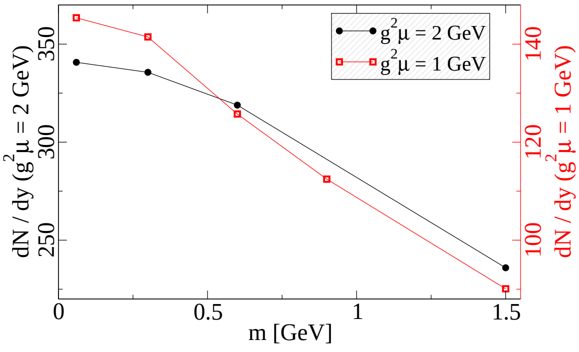

Figure 3. Dependence of the number of quark pairs on quark mass at a fixed proper time, $\tau = 0.25$ fm, and for two values of $g^2\mu$.

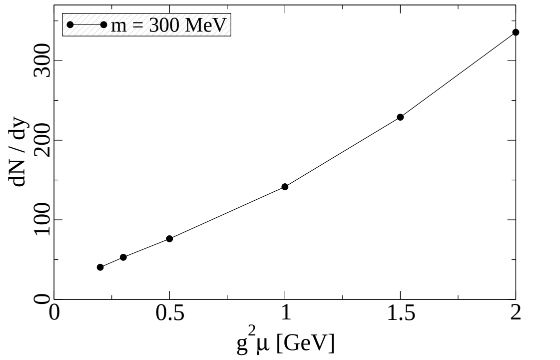

Figure 4. Dependence of the number of quark pairs on $g^2\mu$ at a fixed proper time, $\tau = 0.25$ fm, and for quark mass $m = 0.3$ GeV.

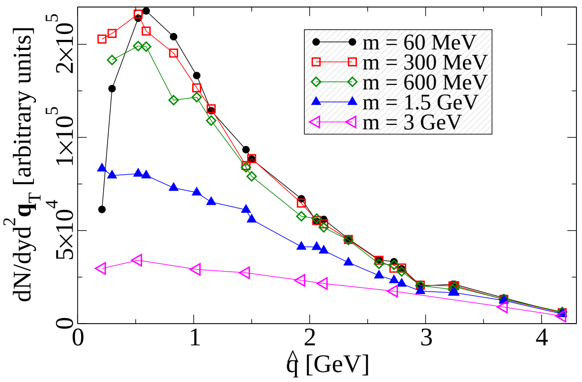

Figure 5. Transverse momentum spectrum of (anti)quarks for $g^2\mu = 2$ GeV at a fixed proper time, $\tau = 0.25$ fm, and for different quark masses.

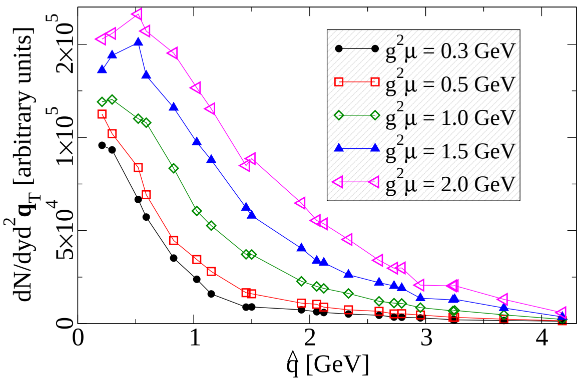

Figure 6. Transverse momentum spectrum of (anti)quarks for quark mass $m = 0.3$ GeV and for different $g^2\mu$ at a fixed proper time, $\tau = 0.25$ fm.

initial condition for numerically solving the Dirac equation for $\tau \geq 0$ using the coordinate system $\tau, z, \mathbf{x}_T$. The resulting spinor wavefunction $\psi(\tau, z, \mathbf{x}_T)$ is then projected onto positive energy states $e^{-ip\cdot x}u(p)$ at time τ to obtain the amplitude M_τ. For times larger than the formation time of the quark pair $\tau \gtrsim 1/\sqrt{m^2 + \mathbf{q}_T{}^2}$ this amplitude can be interpreted as the amplitude for producing quark antiquark pairs. The resulting number of quark pairs is shown in Fig. 2.

The physical parameters of the calculation are $g^2\mu$ characterising the strength of the background field, the nuclear radius R_A and the quark mass m. The dependence on $g^2\mu$ and m of the number of pairs at $\tau = 0.25$ fm is shown in Figs. 3 and 4. The transverse momentum spectra of the (anti)quarks as a function of $\mathbf{q}_T$ is shown for different quark masses and saturation scales in Figs. 5 and 6.

3. Discussion

According to conventional wisdom the initial state of a heavy ion collision is dominated by gluons. Assuming that the subsequent evolution of the system conserves entropy this would mean $\sim$ 1000 gluons in a unit of rapidity. In the classical field model this corresponds [13] to $g^2\mu \approx 2$ GeV. Our results seems to point to a rather large number of quark pairs present already in the initial state. One could envisage a scenario where for $g^2\mu \approx 1.3$ GeV these 1000 particles could consist of $\gtrsim 400$ gluons, $\gtrsim 300$ quarks and $\gtrsim 300$ antiquarks (take the lowest curve from Fig. 2 and multiply by $N_\mathrm{f} = 3$). This would be close to the thermal ratio of $N_g/N_q = 64/(21N_\mathrm{f})$.

4. Conclusions

We have calculated quark pair production from classical background field of McLerran-Venugopalan model by solving the 3+1–dimensional Dirac equation numerically in this classical background field. We find that number of quarks produced is large, pointing to a possible fast chemical equilibration of the system. The mass dependence of our result is surprisingly weak and we are not yet able to make any conclusions on heavy quarks until studying the numerical issues involved.

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