# Quark Matter 2005 – Theoretical Summary

Berndt Müller [a]

[a]Department of Physics, Duke University, Durham, NC 27708, USA

This is a review of the latest developments in the theory of superdense nuclear matter, formed in relativistic heavy ion collisions or in the core of collapsed stars, as they were reported and discussed at the Quark Matter 2005 conference in Budapest (Hungary).

## 1. INTRODUCTION

The motivation for the study of superdense matter in relativistic heavy ion collisions is the prospect of observing a novel state of strongly interacting matter, the quark-gluon plasma. Experiments at the CERN Super Proton Synchrotron (SPS) and the Relativistic Heavy Ion Collider (RHIC) at Brookhaven have yielded important clues of the characteristic signatures of this new state in recent years. In fact, the RHIC experiments and Brookhaven National Laboratory (BNL) have recently announced the discovery of a new form of matter with the properties of a "perfect fluid", i. e. of a strongly interacting plasma-like state of nuclear matter with an extremely low ratio of shear viscosity to entropy.

The LHC will soon extend the range of energy densities that can be explored with measurements of hard probes (jets, heavy quarks, photons). In parallel to these large-scale experimental programs at major accelerator facilities, continuing progress in the detection and observation of pulsars holds the promise of increasing insight into the limits of stability of neutron stars and their astrophysical properties. There is good reason for hope that such studies will ultimately lead to new insights into the equation of nuclear matter at low temperature and high baryon density.

It is an understatement to say that this field of research is driven by experiments at this time. In comparison to the amazing wealth of new and exciting data from the RHIC and SPS experiments reported at this meeting, the progress in our theoretical understanding of the central questions relating to the properties of quark matter has been modest. This remark should not be taken to mean that no progress is being made on the theory front – I will discuss some important and intriguing new developments below – but as we learn more about the complexities of the problem, we realize that definitive theoretical results will require very large investments in time, manpower, and computing resources. All these are occurring at the current time, which leads to the expectation that steady theoretical progress will be made over the next few years.

The role of theory in physics is to develop ideas and concepts which allow us to quantitatively interpret the experimental data and to propose new experimental tests and analysis strategies. We need a steady stream of new ideas, both "good" and "bad" ones, because it is notoriously difficult to correctly judge the value of a new idea at an early stage in its development. We also need sustained diligent work on established approaches, which have been found to be fruitful and valid, and we must sort out and safely dispose of failed ideas. In my discussion of what was presented at this conference I will begin with the exciting new ideas, I will then report on the progress in the established areas of theoretical relativistic heavy ion physics and, finally, I will discuss the status of theoretical insight in various areas of heavy ion phenomenology. In doing so, I will not be able to cover every single theory talk at QM2005, and I apologize to all those speakers and presenters of posters whose work remains unmentioned in this summary.

## 2. NEW IDEAS

### 2.1. Thermalization

One of the greatest puzzles posed by the experimental results from RHIC has been the apparently almost instantaneous equilibration of the produced matter demanded by the flow observables. Perhaps the most exciting theoretical development relevant to this issue and reported at this meeting (by S. Mrowczyński [1], M. Strickland [2], and Y. Nara[3]) is the idea that plasma instabilities in the soft modes of the gluon field may be responsible for the rapid momentum equilibration. The fact that unstable field modes exist whenever the particle momentum distribution is anisotropic has long been known in electromagnetic plasma physics. But only in recent years has the potential relevance of these so-called Weibel instabilities [4] to relativistic heavy ion collisions been recognized.

The basic idea, nicely explained in Mrowczyński's talk, builds on the insight that the momentum spectrum of the partons, which are deposited by semihard QCD interactions during the initial impact of the two nuclei, quickly becomes locally anisotropic due to the longitudinal expansion of the system. Measured in the comoving frame, the width of the longitudinal momentum distribution becomes much narrower than the width of the transverse momentum distribution: $\langle \Delta p_L^2 \rangle \ll \langle \Delta p_T^2 \rangle$. A transverse color current fluctuation with wavelength in the beam ($z$-)direction less than the color screening length in the plasma will then induce a transverse chromomagnetic field, which tends to further amplify the current fluctuation. In a range of modes $k_z$, this leads to exponential growth of the color field at a rate $\Gamma(k_z) \sim k_z < m^*$, where $m^*$ is the effective gluon mass (proportional to the plasma frequency) in the medium.

Over the past year, the dynamical evolution of this instability has been studied in detail by means of numerical simulations of the nonlinear plasma dynamics in the presence of an anisotropic parton distribution [5–7]. The numerical studies show that the initially exponential growth of the unstable modes eventually moderates, when the magnetic energy density in the soft modes reaches the value $g^2 Q_s^4$, where $Q_s$ is the typical energy carried by the particles. Thereafter, the energy in the soft gluon modes grows only linearly, indicating a continuing cascade of field energy into short wavelength modes. At present, it is unclear which mechanism triggers the end of the exponential growth.

What is clear, however, is that the strength of the chromomagnetic field ($B \sim gQ_s^2$) is sufficient to bend the momentum of the hard partons on a time scale $t_{\rm iso} \sim p/gB \sim (g^2 Q_s)^{-1}$, which thus governs the isotropization of the parton distribution. Note that this is an expression which does not involve $\hbar$ and thus constitutes a classical time scale. It is also amusing to note that $t_{\rm iso}$ is reminiscent of the characteristic growth time of the (coarse grained) entropy of classical gauge field, which is given by the Lyapunov exponents $\lambda$: $t_s = S/(dS/dt) \sim \lambda^{-1} \sim (g^2T)^{-1}$ [8].

A detailed dynamical framework of the role of this mechanism in the process of parton thermalization has yet to be developed. Although it appears unlikely that the weak-coupling estimate of the time to full thermalization obtained in the "bottom-up" scenario [9] will be significantly modified, it is quite likely that the plasma instabilities can induce a precocious approach to isotropy of the local momentum distribution. This would already explain the rapid transition to a regime, in which fluid dynamics can be applied, as the comparison of hydrodynamic calculations of the elliptic flow $v_2$ with the RHIC data suggests. It may even be possible to devise an effective description of this early phase in terms of a type of chromo-magneto-hydrodynamics. The bending effect of the soft gluon fields would have a similar effect as collisions in limiting the rate of momentum transport. Because particle propagation in coherent fields conserves entropy, it is not clear whether such an effect can lead to an increase in the shear viscosity $\eta$ of the medium. Boldly assuming that it does, the general expression for the viscosity, $\eta \sim \rho\bar{p}\lambda_f$ (where $\bar{p} \sim Q_s$ denotes the average momentum of a particle), would be replaced with $\eta \sim \rho\bar{p}t_{\rm iso} \sim \rho\bar{p}^2/gB \sim \rho/g^2$, reducing the shear viscosity by a factor $g^2$ compared with the usual perturbative result.

One may ask whether the strong gluon fields generated by the Weibel instability can be experimentally observed. One way may be through the study of event-by-event fluctuations in $v_2$ [10]; another way could be via their effect on jets, which penetrate the plasma in the transverse direction. Since also the chromomagnetic fields are aligned in the transverse plane, they can only deflect jets in the longitudinal direction. A rough estimate suggests that the deflection could be substantial, leading to a broadening of the jet cone along the beam direction, but no effect on the azimuthal opening angle of the jet. Note that this is exactly what is observed experimentally [11].

Another interesting approach to the thermalization is to invoke the analogy to the Unruh phenomenon of the thermal vacuum in accelerated reference frames [12], presented here by D. Kharzeev [13]. As argued by Unruh and first demonstrated in detail by Fulling [14], the Minkowski space vacuum of a free relativistic quantum field theory appears to a constantly accelerated observer exactly like a thermal state. The temperature $T$ and the acceleration $a$ are related by the formula $T = \hbar a/(2\pi c)$ or $T = a/2\pi$ in natural units. The phenomenon is closely related to, though more general, than the Hawking effect [15], by which black holes radiate thermally. A constantly accelerated observer also experiences an event horizon which, in contrast to the case of black holes, is not a topological property of the space-time geometry.

If this insight could be applied to heavy ion collisions, where strong color fields lead to the coherent acceleration of many partons, the spontaneous appearance of a nearly thermal system could be neatly explained. One might even, with slight hyperbole, call the idea "thermalization by black magic"! The question is how far the analogy carries. The idea that external fields other than gravity could be interpreted as mediators of a thermal ensemble has been discussed before (see e. g. [16]). It turns out that the analogy is not perfect, because ordinary electric or chromo-electric fields do not accelerate all particles equally but differentiate between particles with different charges, in particular between particles and antiparticles. In other words, quarks and antiquarks in a given chromo-electric field do not experience the same event horizon, nor do gluons. The equivalence principle only applies to gravity, not to other interactions. Furthermore, gluon radiation can be induced not only by the acceleration of color charges, but also by their rotation in SU(3) space. This aspect, which was nicely demonstrated in Kovchegov and Rischke's derivation of the gluon spectrum emitted in the collision of two sheets of color glass condensate [17], is peculiar to QCD and does not occur in QED. Finally, even if this idea could be invoked with benefit for the explanation of a (approximately) thermal initial gluon spectrum, it would not explain the formation of a system that can be described by fluid dynamics, because it completely lacks the notion of a mean free path. Thus, at best, the concepts discussed by Kharzeev could be used to explain what has recently been called "pre-thermalization" [18].

### 2.2. Other new ideas

Let us now turn to other new ideas. I find the idea of jet-induced Mach or Cherenkov cones intriguing, because it may help us learn about the transport properties of the matter created in heavy ion collisions. But since this topic was covered in Jörg Ruppert's Focus talk [20], I refrain from summarizing it here once more.

Another new idea was discussed by J. Cramer [21], who presented his and G. Miller's calculation of identical pion correlations using the framework of the distorted wave Born approximation (DWBA). Their calculation uses the optical potential for pions interacting with a dense nuclear medium, which has long been studied in pion-nucleus scattering and pionic atoms. The potential has a large imaginary part, which describes absorption within the medium. It has been argued that such a description is inadequate, because the strongest absorptive channels – such as the $\rho$-meson and the $\Delta$-baryon – are short-lived resonances, which mostly decay again by pion emission. In other words, the absorbed pions often reappear quickly, albeit in different quantum states.

However, I believe that this critique is unfounded. The DWBA is an approximation to S-matrix theory, and if one includes absorptive effects in the final state, they really describe the time reversed process, i. e. emission. Since the source term in the HBT formalism describes the vertex positions of the last inelastic interactions of the emitted pions, the DWBA seems to be a valid approach to a quantum mechanical description that includes the effects of the in-medium propagation of the observed pions. The one issue that is a bit worrisome is that the Cramer-Miller analysis invokes both, a large pion chemical potential and a strongly attractive potential, which is interpreted as onset of chiral symmetry restoration. The combination of these two effects may bring the pion field precariously close to Bose condensation. It would be good to check whether this is a problem or not [22]. It would also be interesting to apply the DWBA approach to other hadrons, such as protons and charged kaons.

As a last example of new developments, which were discussed at this meeting, I want to mention the treatment of (soft-hard) recombination as a special case of parton fragmentation. Hwa and Yang [23] , as well as Greco, Ko and Levai [24], have argued for some time that this hadronization mode is an important source of hadrons emitted with momenta $p_T = 2 - 8$ GeV/c. E. Wang [25] showed how soft-hard parton recombination can be formulated as a special case of parton fragmentation in the presence of a medium. This represents an important step toward a unified, QCD-based theory of hadron emission at intermediate and high transverse momenta.

## 3. ESTABLISHED APPROACHES

### 3.1. Lattice QCD and the equation of state

One of the best established theoretical approaches in our field is lattice gauge theory, which is unique in its ability to provide model independent predictions for observables beyond the reach of perturbative QCD. It is no exaggeration to say that most of what we believe to know about the structure of hadronic matter at high temperature is based on results from lattice gauge theory. Lattice simulations predict that the properties of strongly interacting matter without net baryon surplus change drastically at an energy density of the order of 1 GeV/fm$^3$, corresponding to a temperature of about $T_c \approx 170$ MeV. Previous calculations also indicated that this transition is not a true phase transition (at zero net baryon density), but at rapid crossover from a low-temperature phase dominated by hadrons into a high-temperature phase of a plasma of interacting quarks and gluons. The transition is seen as a steep rise in the vicinity of $T_c$ in the effective number of degrees of freedom $\nu$ defined as $s(T) = 2\nu\pi^2 T^3/45$, where $s(T)$ is the entropy density as function of temperature.

Up to now, limitations in the available computer power forced lattice simulations to make a number of unphysical assumptions. For example, the quark masses were too large, the quark actions violated chiral symmetry, and the number of lattice points in the euclidean time direction, $N_t$ was usually quite small ($N_t = 4$). This required rather large extrapolations of the numerical results to the physical point, leading to substantial systematic uncertainties in the predictions. The recent advent of computers with multi-teraflops capability has changed this. Several calculations of QCD at nonzero temperature are presently underway with realistic quark masses, finer mesh sizes, and improved fermion actions. The first, preliminary results of one of these calculations were reported at this conference by S. Katz [26], who showed a long list of improvements compared to previous calculations, including physical quark masses, exact Monte-Carlo algorithms, and $N_t = 6$. It may be important to mention, however, that the calculation does not use one of the best improved fermion actions, nor a fermion action with an explicit chiral symmetry, such as domain wall fermions.

The results presented by Katz confirm many of the previous findings. The transition between the low- and the high-temperature phase is still a smooth, but rapid crossover. However, the new calculation seems to suggest a considerably higher value for the critical temperature. Katz [26] reported the values: $T_c = 186(3)(3)\,\text{MeV}(N_t = 4)$ and $T_c = 193(6)(3)\,\text{MeV}(N_t = 6)$ , which lie significantly above the values reported at earlier Quark Matter conferences ($T_c^{(N_f=2)} = 173 \pm 8\,\text{MeV}$ and $T_c^{(N_f=3)} = 154 \pm 8\,\text{MeV}$ [27], see also [28]). If the new values are confirmed, it would force us to rethink the message from the chemical equilibrium analyses of the hadron yields in relativistic heavy ion collisions. The temperature $T_{\text{ch}}$ obtained in these fits is consistently lower than 180 MeV (see e. g. Florkowski's talk at this conference [29], or [30]). A clear separation between $T_c$ as defined by the peak in susceptibilities and $T_{\text{ch}}$ would imply that the hadronic system does not chemically freeze out immediately after the deconfinement transition. The experimentally determined value of $T_{\rm ch}$ would then not have a universal meaning, because the chemical freeze-out would not reflect thermodynamic properties of QCD matter, but aspects of the hadrochemical kinetics, which is system dependent. This would, of course, not be against fundamental principles of physics, but it would constitute an unfortunate circumstance, because it would make an experimental determination of $T_c$ much more difficult. Given these implications, it is probably wise to wait for the results of other currently ongoing lattice calculations before revising the standard picture.

It is interesting to ask whether it is possible to put data points on the equation of state curve $\varepsilon(T)$. This would require the simultaneous measurement of the energy density $\varepsilon$ (or the entropy density $s$ or the pressure $P$) and the temperature $T$. The latter may be possible, if thermal photon radiation from the quark-gluon plasma can be detected [31]. An elegant novel experimental method, measuring low-mass dileptons at high transverse momentum as a proxy, presented here by the PHENIX collaboration [32] is a promising step in this direction. As K. Rajagopal and I have recently pointed out, an alternative way of determining the thermodynamically active number of degrees of freedom $\nu$ is to simultaneously measure $\varepsilon$ and $s$ [33]. We already have a rather precise determination of the entropy per unit of rapidity ($dS/dy = 5100 \pm 400$, corresponding to $s = (33 \pm 3)$ fm$^{-3}$ at $\tau = 1$ fm/c). What is needed now is an independent determination of the energy density at an early time. The energy loss of partons may be the best available tool for such a determination; the report at this meeting of the observation of the "punch through" of the away-side jet [34] represents an important progress toward this goal (see also below).

There is steady progress toward a detailed understanding of the physics of quark matter at high density, but low temperature. K. Rajagopal [35] review the general status of the theory of color superconductivity. If the existence of "gapless" color superconductors (referring to the absence of a gap in some pairing channels) is confirmed by improved calculations, it could have interesting phenomenological consequences in collapsed stars. If the ground state of the matter has a crystalline structure, of the "LOFF" type, it would allow quark stars to exhibit glitches and other phenomena, which are normally thought to require a solid neutron crust. T. Schäfer [36] discussed the progress in constructing effective theories for the dynamics near the Fermi surface when unscreened QCD forces vitiate the formalism of Fermi liquid theory. On the numerical front, S. Eijiri [37] presented new results of lattice calculations for nonzero baryon chemical potential, using the Taylor expansion around $\mu_B = 0$. The results impressively confirm the validity of the resonance gas model (in the baryonic sector) for $T < T_c$. The $\mu_B$-dependence of the color screening mass agrees well with perturbation theory for $T > 1.5T_c$. The calculations have reached a state where many model predictions can be tested against rigorous predictions of QCD.

### 3.2. Fluid dynamics

A well established approach to the treatment of the evolution of the dense matter created in relativistic heavy ion collisions is fluid dynamics. One of the core insights of the RHIC program is that fluid dynamics is successful in describing the collective flow of hadrons and hadron spectra up to $p_T = 2 - 3$ GeV/c. The strong rapidity dependence of many observables has made it clear, however, that fully three-dimensional calculations are needed. Significant progress on this front was reported at this conference. Y. Hama [38] presented results from the SPheRIO code, which is uses the particle-in-cell method and implements a continuous freeze-out model, where hadrons are decoupled on the basis of somewhat schematic mean-free path considerations. T. Hirano [39] showed results from a 3-D Eulerian hydrodynamics code, which allows for deviations from chemical equilibrium in the hadronic phase. Hirano, as well as C. Nonaka [40], whose code is based on Lagrangian fluid dynamics, also presented first results from so-called "hydro+micro" models, in which the evolution above $T_c$ is described by 3-D hydrodynamics, and the hadronic phase is treated by a cascade simulation. The few glimpses we have seen here raise the expectation that the new codes will yield a much improved understanding of the rapidity dependence of soft observables.

These are important developments for our field, and we can look forward with great interest to the results of further systematic comparisons of these improved transport models with the experimental data, similar to the studies by Hirano and Nara [41] of the interplay between jet propagation and hydrodynamical evolution. It will be highly desirable to see direct comparisons between the different codes for the same initial conditions, equation of state, and other assumptions. The increasing complexity of the various implementations of what amounts to essentially identical physics makes it crucially important to establish the validity of these codes before we use them to draw firm conclusions. (For such a study of the performance of the SPHERIO code, see [42].) The immediate goals are the determination from the data of the equation of state and the initial conditions reached at RHIC. Other important applications will be calculations of the effect of the medium on hard probes, such as jets, photons, and lepton pairs.

### 3.3. Parton transport

Several speakers reported substantial progress in microscopic approaches to parton transport. D. Teaney's [43] discussed the momentum evolution of charm quarks in a quark-gluon plasma by means of a collisional Langevin equation of the form

$$\frac{dp}{dt} = \eta_D p + \xi(t) \tag{1}$$

with Gaussian white noise $\langle \xi(t)\xi(t') \rangle = (2T^2/3D)\delta(t-t')$ and the friction coefficient $\eta_D = \frac{T}{MD}$, where $D$ is the diffusion coefficient of charm and $M$ the charm quark mass. The diffusion coefficient is related to the shear viscosity $\eta$ by a relation of the form $D \approx \frac{3}{4\pi\alpha_s T} \approx 6\frac{\eta}{sT}$. Teaney showed that the PHENIX data on the suppression coefficient $R_{AA}$ of non-photonic electrons (presumably originating from semileptonic charm decays) can be well described by a value $D = (3\cdots 6)/(2\pi T)$, consistent with a very low value of the shear viscosity $\eta/s = (1\cdots 2)/(4\pi T)$, near its conjectured lower bound [70].

Continuing with the topic of charm transport, R. Rapp presented results from a model calculation assuming that charm quarks can scatter from D-meson like resonances with various hadronic quantum numbers in the plasma slightly above $T_c$ [44]. He and his collaborators (see also the talk by M. Mannarelli [45]) find that this mechanism strongly accelerates the momentum equilibration of $c$-quarks, even for a single resonant state and its partners dictated by the heavy quark and chiral symmetries. M. Djordjevic showed the results of a complete radiative energy loss calculation for heavy quarks in a quark-gluon plasma [46]. On the basis of a schematic space-time model for the absorbing medium she concluded that the $R_{AA}$ value for electrons from charm decays cannot be lower than twice the $R_{AA}$ value for pions. This limit is incompatible with the preliminary results from STAR and only marginally compatible with the PHENIX data. It would be good to explore whether the limit holds up in the context of more realistic models of the space-time evolution. The importance of comparing the effects of energy loss on light and heavy quarks was also emphasized in N. Armesto's talk [47].

There also was news on the front of partonic cascade codes. Z. Xu [49] reported results from a new cascade code, which implements $gg \leftrightarrow gg$ and $gg \leftrightarrow ggg$ scattering by means of local rates with detailed balance. The inelastic process turns out to be crucial, as argued by S. Wong almost a decade ago [48], not only for chemical but also for thermal equilibration. It still needs to be understood in more detail, how the $2 \leftrightarrow 3$ process manages to speed up thermalization more than the increased transport cross section from its more isotropic angular distribution would naively suggest. Other implementations of radiative gluon processes in parton transport codes did not show a similarly large effect [50].

Several speakers, such as D. Molnar [51] and B. Zhang [52] presented calculations of binary parton cascades with hugely cranked up binary cross sections. The studies may serve some purpose in schematic explorations of the origin of flow and energy loss at the partonic level. However, the question needs to be asked what exactly these studies are telling us, beside the obvious (that larger partonic cross sections lead to faster equilibration and to increased collective flow), and whether their conclusions are not misleading due to the unrealistic assumptions.

### 3.4. Parton saturation

Another established fruitful idea is parton saturation with the asymptotic approach to a universal structure, the color glass consensate (CGC). K. Itakura [53] gave an impressive review of the recent conceptual and formal developments in this field. The question whether this asymptotic physics is relevant to the phenomena observed at forward rapidities in d+Au collisions at RHIC is still under discussion (see the talks by G. Veres [54], D. Röhrich [55], and J. Jalilian-Marian [56]). The problem is that the phenomenology is complex, involving a mixture of initial-state and final-state effects. It appears to be difficult to cleanly separate different sources of nuclear modifications from each other.

One important new result in the context of the CGC, reported here by T. Lappi [57], is that quark-pair production in the strong gluons fields generated by the shattering CGC is quite large. For reasonable values of the parameters, Lappi finds that several hundred light quark pairs are produced. On the one hand, this result is welcomed, because it suggests that a chemically equilibrated quark-gluon plasma is originally produced. On the other hand, it poses a potential problem for the standard jet quenching picture, because the total number of gluons and quarks is limited by the total entropy observed at freeze-out. More quarks imply fewer gluons. But quarks are much less efficient at causing parton energy loss, which would imply that the slight underestimate of the measured hadron suppression by many calculations (see e. g. B. Cole's lecture [58]) would be further enhanced. It is too early to draw firm conclusion, but the result suggests that a detailed analysis of the possible implications for jet quenching of different abundance ratios of quarks and gluons in the medium is needed.

### 3.5. The strongly coupled quark-gluon plasma

Practitioners of lattice theory have told us for quite some time that QCD matter at temperatures not far above $T_c$ is *not* a perturbative quark-gluon plasma. The impressive work which has been done over the past five years on perturbative resummation techniques for the QCD equation of state has confirmed this insight [59,60]. The perturbative expansion based on hard-thermal loop quasiparticles works well for $T > (2 \cdots 3)T_c$, but it fails in the region near $T_c$, which is explored by the RHIC experiments. If the plasma below $2T_c$ is a strongly coupled one (a "sQGP" in the current terminology), the question is whether it contains any long-lived quasiparticles and if so, what their structure is.

We may look for guidance to strongly coupled electromagnetic plasmas, which have been thoroughly studied experimentally and theoretically. The physics of these plasmas and its possible relevance to our field was superbly reviewed by M. Thoma [61]. The two parameters controlling electromagnetic plasmas are temperature and charge carrier density. A line defined by the condition $\Gamma_{\rm EM} = q^2/(Ta) = 1$, where $a$ is the average distance between particles and $q$ their electric charge, divides the phase diagram into regions of weak ($\Gamma < 1$) and strong ($\Gamma > 1$) coupling. The precise translation of this condition to a quark-gluon plasma is not known, but it must roughly have the form $\Gamma_{\rm QGP} \sim C_2\alpha_s/(Ta) \approx 1$, where $C_2$ is the eigenvalue of the quadratic Casimir operator for the color charges making up the plasma. If one inserts numbers, one finds that $\Gamma_{\rm QGP}$ is, indeed, somewhat larger than unity for the conditions prevailing in the range $T_c < T < 2T_c$. The experience from electromagnetic physics suggests that a plasma in this range of parameters is no longer perturbative, but still far away from the region where regular short-range correlations or even quasi-crystalline structue are found. Still, the analogy to electromagnetic plasmas is very useful, because it suggests phenomena and properties to look for, and maybe even theoretical tools to apply, in theoretical investigations of the quark-gluon plasma near the phase boundary.

An conjecture, whose ultimate value is still uncertain, is the dominance of partonic bound states in the quark-gluon plasma in the region near $T_c$, which was discussed by E. Shuryak [62]. Such substructures are difficult to track in lattice simulations, because they are not color singlets. Their calculation on the lattice, therefore, requires gauge fixing with all its concomitant pitfalls. It is much easier, although possibly less conclusive, to look for indirect signals of the abundant presence of partonic bound states on the lattice. Two such studies were discussed by V. Koch [63] and F. Karsch [64] in their talks. Koch and collaborators focus on the correlations between strangeness and baryon number. If all quasiparticles in the plasma have the quantum numbers of quarks or gluons, only quarks carry strangeness. If bound states of quarks and antiquarks exist as quasiparticles, strangeness is also carried by states without baryon number, as in a hadronic gas. The $\langle BS \rangle$ correlator can be expressed in terms of quark susceptibilites, where the presence of $q\bar{q}$ bound states should show up as an enhancement of flavor-off-diagonal elements. Existing lattice calculations rule these out with great precision. Karsch and collaborators have done a similar study for electric charge correlations, which are sensitive to the presence of quasiparticles carrying diquark quantum numbers. Again, the lattice results indicate a rapid transition from a hadron resonance gas to a perturbative quark-gluon plasma in the range $T_c < T < 1.4T_c$.

While it may be too early to proclaim the demise of the "bound state quark-gluon plasma" model, these new results are not encouraging for the model. It is true that the lattice results mentioned above have nothing to say about gluonic bound states. It would be worthwhile to explore generalizations of these tests to gluons. A direct application is impossible, because one cannot introduce a gluonic chemical potential, but maybe it would be possible to define a chemical saturation factor $\gamma_g$, similar to the strangeness saturation factor $\gamma_s$ which is introduced in hadro-chemical equilibrium models. One would never have to do a lattice calculation with $\gamma_g \neq 1$, the parameter would only be used to define derivatives for infinitesimal deviations from equilibrium. Anyway, model builders will have to abide by the constraints introduced by these and future lattice results.

Quite generally, it appears to me that a low viscosity (relative to the entropy density) of the quark-gluon plasma argues against structure formation, except possibly at the nearest neighbor level. Materials with long-distance particle correlations, such as polymers, generally have large viscosity. The reason is that, although transport cross sections can be huge, momentum transfer in these materials is not dominated by particle transport, but by the transport of momentum along molecular chains or via collective modes. The mean free path for momentum exchange is therefore large. In the extreme case of crystals, momentum transfer is caused by phonons, which may travel freely over extremely large distances, even if the atoms are immobilized. A small viscosity is thus indicative of a liquid without strong correlations, except possibly on the scale of the interparticle separation.

Is there any approach, other than euclidean lattice simulations, which would allows us to explore the strongly coupled quark-gluon plasma? The experience from electromagnetic plasmas is that molecular dynamics might be a promising method. Two speakers at this conference, E. Shuryak [62] and P. Hartmann [65], presented first results from what I would call "QCD inspired" molecular dynamics calculations. As intriguing as their results are, the fact that these models are constructed in the framework of direct two-particle interactions makes them suspect, for several reasons: Such models make it very difficult, if not impossible, to implement local gauge invariance; also, the perturbatively unscreened sector in high-temperature QCD is the chromomagnetic sector, which is not easily encoded in two-body forces.

It is perhaps worth mentioning that a numerical approach to parton molecular dynamics already exists, which encodes hard thermal loops in Minkowski space via a coupled system of colored partons and a spatial lattice [66]. In this formulation, which was put to use in the work on color instabilities presented here by Nara [3], the partons propagate according to Wong's equation

$$\frac{dp_i^\mu}{d\tau} = gQ_i^a F^{a\mu\nu} v_\nu, \qquad \frac{dQ_i^a}{d\tau} = gf_{abc} A^{b\mu} Q_i^c v_{i\mu}, \tag{2}$$

while the soft classical fields satisfy the Yang-Mills equation with the particles as sources:

$$\partial_\mu F^{a\mu\nu} = \int d\tau \sum_i gQ_i^a v_i^\nu \delta\left(x - \xi_i(\tau)\right). \tag{3}$$

At weak coupling, this system of equations (first proposed by Heinz [67]) is known [68] to reproduce the HTL effective theory, and this has also been demonstrated numerically [66]. No one has explored what these equations predict for strong coupling. Although it is clear that the Heinz model does not reproduce full QCD when the coupling becomes strong, the fact that we know and understand its weak coupling limit provides confidence. Note that the Heinz model only contains a mean field and no two-body forces, which may become increasingly important at stronger coupling. It would be straightforward and logical to include such short-distance interactions as scatterings involving color exchange among partons contained in the same lattice cell [69].

An important theoretical playground for those interested in strongly coupled gauge theories are the supersymmetric gauge theories, for which a dual gravity-like theory exists. A. Starinets [70] reported about the progress in calculating corrections to the results obtained in the extreme strong coupling limit. For strongly coupled $N = 4$ supersymmetric Yang-Mills theory we now know that the asymptotic value of the shear viscosity / entropy density ratio is approached as ($N_c$ denotes the number of colors)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{135\,\zeta(3)}{(8g^2 N_c)^{3/2}} + \cdots\right]. \tag{4}$$

Note that $\eta/s$ does not grow for very strong coupling in this theory, presumably because there is no formation of long-range structure. Unfortunately, the mapping between the quasiparticles at weak coupling and those at strong coupling is not well understood in these SUSY models, and it thus remains unclear to what extent these beautiful results apply to real QCD. However, if it were possible to apply molecular dynamics methods developed for strongly coupled QCD to the SUSY theories, the analytical results could provide rigorous tests for the numerical methods.

## 4. PHENOMENOLOGY

### 4.1. Electromagnetic probes

There is little to add to C. Gale's beautiful review of electromagnetic probes of dense matter [71]. The amazing data presented at this conference hold promise for more to come. The di-lepton data from NA60 [72] demonstrate that excellent resolution and high statistics are crucial prerequisites for progress. The invariant mass spectra of di-muons from In+In shown by NA60 exhibit not only a broad source of prompt di-leptons, but even show the narrower peak from $\rho$-meson decays in vacuum. Future calculations will thus have to describe the magnitude of this peak. The data are good enough to discriminate between different models. Even a superficial glance suggests that any model, which predicts only a mass shift in medium, without broadening, will face great difficulties.

It would be desirable, if theorists would not only compare the new data with various specific models of the in-medium modifications of the $\rho$-meson. Obviously, these modifications can be broken down into the contributions of various hadronic channels contributing to the changes in the spectral function of the $\rho^0$. As interesting as the obtained insights are, they may obscure the answer to more general questions, such as: What is the route to chiral symmetry restoration and deconfinement? Is it the general broadening of hadronic states until they dissolve into a continuum? Or does the transition occur primarily through shifts in hadron masses or mass splittings between chiral partners? It would be useful to interpret the di-lepton data in terms of such deeper concepts in a model independent way.

**4.2. Jet quenching**

We have seen a lot of (by now) routine phenomenology on jet quenching, but also some interesting new ideas. N. Borghini [73] presented an approach, in which the medium effects are simply encoded in a one-parameter change of the splitting function within the well established modified leading logarithmic approximation (MLLA) theory of jet fragmentation. The resulting $dN/dz$ distribution of jet fragments looks just like what is observed in Au+Au collisions when the threshold for back-to-back coincidences is set low. It will be interesting to see how far this idea can be pushed and whether the new parameter in the splitting function can be microscopically calculated. Another promising new development is the theory of di-hadron fragmentation functions, discussed by A. Majumder [74].

There is a sense among theorists now – especially after the Cu+Cu data did not generate a new surprise – that jet quenching at RHIC is sufficiently well understood to justify detailed predictions for the LHC, as presented in C. Salgado's talk [75]. Such efforts should be encouraged, because a detailed set of prediction will allow us to assess the quality and predictiveness of the theory when the LHC data arrive in a few years.

But let me inject a word of caution. As first demonstrated by Armesto et al. [76], collective flow can influence the energy loss of a parton in the quark-gluon plasma. This is especially true, if the longitudinal expansion is not boost invariant, leading to a modified density profile with time. As Renk and Ruppert recently showed [77], the deduced energy loss parameter $\hat{q}$ can differ by a factor of five (!) between a boost invariant scenario and one with longitudinal acceleration and strong transverse flow. The fact that the latter would give a much larger suppression for the same value of $\hat{q}$ could help counteract, e. g., the effect of a larger initial quark-to-gluon ratio as suggested by Lappi's calculations [57] mentioned above. As the phenomenology of jet quenching becomes increasingly quantitative, it will be important to compare different calculations of parton energy loss for the same space-time evolution model.

On a personal note, I found the results on single hadron suppression ($R_{AA}$) and additional back-to-back suppression ($I_{AA}$) shown by the STAR collaboration [34], especially exciting. For the specific cuts set by STAR, the two factors were equal: $R_{AA} \approx I_{AA} \approx 0.23$. Why should this be the case? As I and others have argued, $R_{AA}$ is a geometric suppression factor which represents the size of the surface layer from which observed leading hadrons are emitted, compared with the total volume in which jets are produced. Similarly, $I_{AA}$ can be viewed as an additional reduction factor describing the width of the "equatorial" torus from which the observed di-jets are tangentially emitted with an approximately equal and modest energy loss (see Figure). I predicted this relationship in a somewhat schematic treatment of jet quenching a few years ago [78] and am happy to see the prediction substantiated. Clearly, the relative value of $R_{AA}$ and $I_{AA}$ will depend on the selected momentum thresholds, but the general argument suggests that the connection could be another tool in the analysis of dense matter effects on hard partons.

Some of the lectures at this meeting, experimental as well as theoretical ones (see e. g. [58,79], have conveyed the message that jet quenching in Au+Au and Cu+Cu collisions at RHIC is quantitatively well understood. How sure are we of this? Besides the uncertainties already mentioned (influence of flow, gluon dominance of the early quark-gluon plasma), there is a glaring discrepancy between different approaches, which has not been resolved. Those who use the data to deduce an effective value of the energy loss coefficient $\hat{q} = 5 \cdots 15\,\mathrm{GeV}^2/\mathrm{fm}$ [80] (effective, because the quoted value approximately corrects for longitudinal expansion) have noted that this value is significantly larger, by a factor five or so, than the value predicted years ago by Baier [81] on the basis of perturbative QCD. On the other hand, those who apply perturbative calculations of radiative energy loss to calculate $R_{AA}$ in nuclear collisions [79,82] find good agreement with the data.

There are two possible resolutions to this conundrum. One would be that the standard fits of $\hat{q}$ overestimate the value of this parameter due to unrealistic assumptions, e. g. about the space-time evolution of the energy density, as discussed above. Another one would be that Baier's estimate of $\hat{q}$ as a function of the energy density represents a serious underestimate of the true effective value of $\hat{q}$ in perturbative QCD. I believe that both sides owe us a careful analysis of the problem, on which everyone can agree. Because jet quenching is such a central aspect of the new discoveries by the RHIC physics program, it is important that the apparent contradiction be resolved soon.

Another challenge for theorists is, of course, the development of techniques for a non-perturbative calculation of $\hat{q}$ on the lattice. This does not seem totally hopeless, because (a) gluon dominance makes quenched lattice simulations suitable for this property of the medium, and (b) the parameter $\hat{q}$ can be formulated in terms of correlators along the light cone [83] of the form

$$\int_0^{x_-} dx'_- \,\left\langle F_{+i}(x_-)W(x_-,x'_-)F^{+i}(x'_-)W^\dagger(x_-,x'_-)\right\rangle , \tag{5}$$

which could perhaps be calculated by means of analytic continuation using maximum entropy techniques. Even if this turns out to be a very hard computational problem, it needs to be attacked, because the claim that the quark-gluon plasma seen at RHIC is strongly coupled and nonperturbative in view of its low viscosity and the claim that perturbative QCD describes the data on jet quenching are not easily reconciled. At the very least, it would be important to get a quantitative understanding of the relevant QCD scale for $\hat{q}$, in order get better control of its value in perturbation theory. This would require a next-to-leading order calculation of radiative energy loss in the medium.

### 4.3. Charmonium

Quite possibly the most challenging set of data for theorists, which was first presented at this conference, are the PHENIX results on charmonium production in Au+Au and Cu+Cu. Even with their limited statistics, they rule out any simple extrapolation of the models of comover suppression developed to explain the CERN-SPS data. Two diametrically opposed conclusions have been drawn from the data, as discussed by M. Nardi [84]: one possibility is that the $J/\Psi$ is not "anomalously" suppressed, both at the SPS and RHIC, the other one is that there is a fair amount of regneration of the $J/\Psi$ at RHIC by $c-\bar{c}$ recombination. The first alternative is supported by the recent results of quenched lattice calculations of the $c-\bar{c}$ spectral functions, which show the survival of a pronounced resonance in the vector meson channel up to $T > 1.5T_c$ [85]. The second alternative is supported by the growing evidence from the RHIC experiments that $c$-quarks participate in the collective flow, suggesting that they are also subject to statistical hadronization by coalescence. Finally, there is the possibility that what has been called "anomalous" suppression at SPS energies is caused by the absorption of excited charmonium states, especially the $\chi_c$, feeding into the observed $J/\Psi$ yield. If these states are completely absorbed in the most central events at the SPS, then no additional suppression due to this mechanism can occur at RHIC.

The problem, in my view, is that we still do not have a comprehensive and generally accepted theoretical framework, in which to discuss $J/\Psi$ suppression. The reason for this lack is not clear. If the $J/\Psi$ acts as a "hard" probe, it should be possible to derive a systematic formulation of its production and destruction in heavy ion collisions within the framework of perturbative QCD and effective field theory (e. g., nonrelativistic QCD). The heavy quark mass provides a large scale, which can serve to separate nonperturbative physics of the dense medium from the dynamics of the heavy quark. Such a formulation might only be marginally valid for $c$-quarks, but it would surely work well for $b$-quarks, which will soon become the center of attention at the LHC. On the other hand, if such a formalism cannot be found, we will need to give up on the notion that the $J/\Psi$ (or the $\Upsilon$!) is a "hard" probe. The ball is clearly in the theorists' court, and the new data provide ample motivation to address this issue vigorously.

On specific issues, the failure of direct lattice calculations of the $c-\bar{c}$ spectral functions [86] and potential models motivated by the lattice results [87] is puzzling. The spectral functions derived from the potential models do not agree with those obtained by analytic continuation directly from the lattice. I am not sure we fully understand the reason for this discrepancy. The spectral functions derived from the lattice do not indicate the presence of a continuum in the vector meson channel. Because the $D-\bar{D}$ channel can be easily incorporated into potential models, but remains absent in quenched lattice calculations of the spectral functions, the role of dynamical quarks in the survival of heavy quarkonium states at $T > T_c$ remains an issue of concern. It is unclear, however, why the lattice results do not reveal a gluonic continuum at $T \geq 1.5\,T_c$.

The existence of a possibly broadened peak in the $J/\Psi$ spectral function does not necessarily mean that the initially created $J/\Psi$ mesons survive in the medium. The present lattice calculations may not reflect the true width of the state and thus may not give a good estimate of its lifetime inside the quark-gluon plasma. On the topic of regeneration, more detailed calculations of $J/\Psi$ formation by $c-\bar{c}$ recombination are urgently needed. Given the success of recombination models for light hadrons, reliable calculations should be within reach. Overall, it is clear that there is much room for theoretical improvements in the area of charmonium suppression.

### 4.4. Hadronization

As W. Florkowski's [29] talk aptly demonstrated, the dispute about the relationship between chemical and kinetic ("thermal") freeze-out of hadrons at the end of a heavy ion collision remains unresolved. Clearly, there exist strong theoretical arguments for a differential freeze-out of various hadrons depending on their mean free paths in an equilibrated hadron gas. Hadrons with no or few known resonant excited states, such as the $\phi$-meson or the $\Omega$-hyperon should freeze out early. On the other hand, a common general freeze-out of hadron abundances and momentum spectra would require a specific mechanism forcing a sudden termination of efficient equilibration mechanisms. If such a scenario were realized in nature, I can only imagine it to be associated with the hadronization transition itself. The fact that, according to lattice QCD, this transition is smooth and not discontinuous in thermal equilibrium, such a scenario will require strongly off-equilibrium conditions during hadronization (such as the sudden decay of a deeply supercooled state [88,89]).

One can ask whether the available data already provide conclusive evidence for or against a common freeze-out. The new high resolution di-lepton data from NA60 should be sufficient to set a lower bound on the duration of the strongly interacting hadron gas phase. The idea that the $\rho$-meson yield seen through the lepton-pair decay channel measures the lifetime of the hadronic phase is not new [90]. But only now do we have data from the SPS, which tell us how many $\rho^0$'s decay inside a hadronic medium, which is dense enough to modify the spectral function. This broadening is related to the rate of binary collisions which, in turn, maintain the kinetic (but not necessarily the chemical) equilibrium of the hadronic medium. Single freeze-out models are thus challenged to describe the observed yield of low-mass di-lepton emission. Other data, such as the results from STAR on the yield of strange resonances as function of centrality (see S. Salur's talk [91]) should also provide serious tests for freeze-out models. The proponents of these models need to address these results.

Let me now turn to strangeness. Most fits to the data yield an almost perfect abundance equilibration of strange hadrons ($\gamma_s = 1$), which was predicted long ago as one of the characteristic signatures of deconfinement and chiral symmetry restoration. However, there exist other high-quality fits of hadron yields [92], which invoke a significant oversaturation of strangeness. Furthermore, because of the different masses of the carriers of strangeness in the quark-gluon plasma and the hadron gas phase, the value of $\gamma_s$ does not have to remain the same during the transition. This suggests that it might be an interesting intellectual exercise to study the equation of state of QCD as a function of $\gamma_s$, the overall strangeness abundance. As in the case of $\mu_B$, one would probably want to calculate the derivatives with respect to $\ln(T\gamma_s)$ at the equilibrium point $\gamma_s = 1$. In many respects, this method would correspond to studying the effects of a varying number of flavors on the equation of state, which we know to be nontrivial.

The recombination model for hadron production at intermediate transverse momenta continues to work amazingly well. The extended range of particle identification in Runs 4 and 5 is finally allowing the data to probe the predicted transition between the region dominated by recombination and the high-momentum region dominated by fragmentation. Results shown at this conference [93] confirm the theoretical predictions of a transition between 4 and 7 GeV/c, and theorists are eagerly waiting for more analyzed data.

How quickly the experiments move to challenge theoretical predictions was demonstrated by STAR's presentation of data on the systematic baryon/meson violations of the valence quark number scaling of the $p_T$-dependence of the elliptic flow. Including higher Fock states in the hadron wave functions, we had recently predicted deviations at the few-percent level [94]. A second source of such deviations from the scaling law can be caused by the internal momentum distribution of constituent quarks [95]. How exciting to see that the data confirm the existence of such deviations with the correct sign [96]! More work will need to be done to determine whether the deviations from universality provide another confirmation of the recombination model, or whether they constitute a challenge. Other theoretical progress reported at this conference were the effects of parton correlations on same-side di-hadron yields [97] and the possible origin of the pedestal effect in the same data [98].

### 4.5. Fluctuations and Correlations

As R. Lednicky's talk [99] made clear, the "HBT puzzle" of the RHIC data is still with us. Generally the measured correlations indicate a short overall duration of the reaction, strong transverse flow, and a very brief, almost sudden freeze-out. Theory still has great trouble getting the small value $R_{\rm out}/R_{\rm side} \approx 1$ right. As the success of hydro-motivated parametrizations of the freeze-out shows [100,101], it is not impossible to reproduce the data. The problem lies in finding a dynamical evolution model, which yields the right freeze-out parameters. There is justified hope that the new 3-D hydro plus cascade codes may change the situation. Stay tuned to see whether this venerable problem finds a resolution soon.

## 5. CONCLUSIONS

This conference has been a showcase of significant progress in the theory of superdense hadronic matter and its formation and evolution in relativistic heavy ion collisions. It is certainly a very productive time for theorists in our field, because the experiments provide for a wealth of data which require interpretation and explanation. The data also make it easier than ever before to verify or repudiate theoretical approaches and models. As I have tried to point out, the successes of theory are not evenly distributed. There are some areas, where theory is well on its way toward definite descriptions, and others, where even a comprehensive framework is still lacking and it is time to catch up. Above all, however, it is upon us theorists to embrace the offer made by one of the earlier speakers in this session [102]: "We look forward to working with the theory community to extract the properties of the matter (produced in relativistic heavy ion collisions)."

## 6. ADDITIONAL COMMENTS

The following remarks were motivated by comments received after posting a draft version of my summary talk.

### 6.1. QGP or "sQGP"?

The characterization of the matter produced in nuclear collisions at RHIC is an experimental issue. On the other hand, the predictions of QCD for the properties of thermal QCD matter in the temperature range $T_c \leq T \leq 2T_c$ are a theoretical issue. As discussed above, it has been widely argued that the quark-gluon plasma in this temperature range is a strongly coupled state of matter – a "sQGP" [103,62]. How compelling are the theoretical arguments for this claim?

It is possible to derive an effective theory for the long-distance dynamics (momenta $k < T$) of thermal gauge theories. If the dynamics at momentum scales $k \geq T$ is weakly coupled, the effective theory can be derived perturbatively. Two forms of this effective theory are known. One is the hard-thermal loop (HTL) effective theory [104], the other is the dimensionally reduced theory based on local interactions of the gauge field [105,106]. In both frameworks one can ask how strong the effective coupling $\alpha_s^{\rm eff} = g^2/4\pi$ is as a function of temperature. This question has recently been studied by Laine and Schröder [107], who calculated the effective action for QCD in the dimensionally reduced theory up to two-loops. They found that the effective coupling constant $\alpha_s = g_E^2/4\pi$ is well controlled down to $T = T_c$ and remains remarkably small: $\alpha_s^{\rm eff}(T_c) \approx 0.28$. These authors also found that the effective theory agrees very well with full thermal QCD for the spatial string tension, which is an important measure of the long distance dynamics. Similar conclusions are reached in the framework of the HTL approach [60] which, however, has not yet been carried out to full two-loop order and thus does not yet permit the extrapolation all the way down to $T_c$.

These studies suggest that the quark-gluon plasma in the temperature region near $T_c$ is well described by an effective theory with a modestly strong coupling constant. This raises the question, where the transition between the weak coupling and strong coupling regimes of thermal QCD occurs: at $T_c$ or at some $T > T_c$? This cannot be decided by numerology, because the perturbative expansion parameter is, generally, $\alpha_s N_c/\pi \approx \alpha_s$, while the expansion parameter of the strong coupling expansion of string duals of nonabelian gauge theories is $\lambda = g^2 N_c$ [108], and the conditions $\alpha_s \ll 1$ and $\lambda \gg 1$ are both fulfilled in the vicinity of $T_c$. Turning to physical arguments, it is noteworthy that the perturbative quasiparticles of the effective theory, thermal gluons and plasmons, are short-lived. The collisional width of a gluon/plasmon at rest is given by [109]

$$\Gamma_g(0) \approx \frac{1}{2\pi} g^2 N_c T \approx 1.5\,T. \tag{6}$$

On the other hand, the effective mass of a gluon/plasmon at rest is $m_g^* = gT\sqrt{N_c/9} \approx T$. Roughly the same relationship holds for thermal quarks [110]. In other words, all slowly moving quasiparticles are strongly damped. The characteristic nature of the temperature region near $T_c$ may thus be that the quasiparticles, gluons and quarks above $T_c$ and hadrons (such as the $\rho$-meson) below $T_c$, are strongly collision broadened. Such a property would be indicative of a liquid, which is characterized by the absence of long-lived quasiparticles and long-range order. Because the effective coupling $g_E$ changes only slowly with temperature, it is by no means clear how for above $T_c$ one needs to go before the widths of the plasma quasiparticles become a higher-order effect compared to their masses.

### 6.2. Viscosity

Does the apparent nearly "perfect fluid" property of the matter observed at RHIC ($\eta/s \ll 1$) imply that the quark-gluon plasma has, by some measure, a very low shear viscosity or is it simply a signature of a large number of degrees of freedom, i. e. a large entropy density, as Hirano and Gyulassy [39] have recently argued? The general expression for the shear viscosity is $\eta \sim \rho\bar{p}\lambda_f$, where $\rho$ is the particle density, $\bar{p}$ the average momentum of a partice, and $\lambda_f$ the mean free path. Since $\lambda_f = (\sigma_T\rho)^{-1}$, where $\sigma_T$ denotes the transport cross section, the density cancels from the expression, and we have $\eta \sim \bar{p}/\sigma_T$. Because cross sections are unitarity bounded ($\sigma_T \leq 4\pi/\bar{p}^2$ for s-wave scattering), we find approximately

$$\eta \geq \bar{p}^3/4\pi \sim 27\,T^3/4\pi,$$

where we made use of the relation $\bar{p} \approx 3T$ for relativistic matter. It is against this value that the smallness of the viscosity of the quark-gluon plasma should be judged.

On the other hand, the entropy density of a relativistic medium is approximately $s \sim 4\rho$, allowing us to write $\eta/s \sim \bar{p}\lambda_f/4$. The smallness of $\eta/s$ is thus a measure of the smallness of the mean free path, compared with the thermal wavelength $\lambda_{\rm th} \sim 2\pi/\bar{p}$ of a particle. As such, it is not directly sensitive to the number of degrees of freedom in the medium.


### Acknowledgements

This work was supported in part by the U. S. Department of Energy under grant DE-FG02-05ER41367. My special thanks go to the organizers of this conference for making it such a stimulating and productive experience. I am grateful for comments from M. Asakawa, T. Csörgö, I. Dremin, D. d'Enterria, T. Hirano, C.M. Ko, S. Mrowczyński, P. Petreczky, R. Pisarski, R. Rapp, S. Wong, and W. Zajc on an earlier version of this manuscript. I also thank M. Strickland for helpful communications.


### REFERENCES


1. S. Mrowczyński, these proceedings;
2. M. T. Strickland, these proceedings.
3. Y. Nara, these proceedings.
4. E. S. Weibel, Phys. Rev. Lett. **2**, 83 (1959).
5. P. Arnold, G. D. Moore and L. G. Yaffe, Phys. Rev. D **72**, 054003 (2005); P. Arnold, J. Lenaghan, G. D. Moore and L. G. Yaffe, Phys. Rev. Lett. **94**, 072302 (2005).
6. A. Rebhan, P. Romatschke and M. Strickland, JHEP **509**, 41 (2005); Phys. Rev. Lett. **94**, 102303 (2005).
7. A. Dumitru and Y. Nara, Phys. Lett. B **621**, 89 (2005).
8. T. S. Biró, S. G. Matinyan and B. Müller, *Chaos and gauge field theory*, World Sci. Lect. Notes Phys. **56** (1994) 1.
9. R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B **502**, 51 (2001).
10. S. Mrowczyński, arXiv:hep-ph/0506179.
11. J. Adams *et al.* [STAR Collaboration], arXiv:nucl-ex/0411003.
12. W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
13. D. Kharzeev, these proceedings; D. Kharzeev and K. Tuchin, Nucl. Phys. A **753**, 316 (2005).
14. S. A. Fulling, Phys. Rev. D **7**, 2850 (1973).
15. S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].
16. B. Müller, W. Greiner and J. Rafelski, Phys. Lett. A **63**, 181 (1977).
17. Y. V. Kovchegov and D. H. Rischke, Phys. Rev. C **56**, 1084 (1997) [arXiv:hep-ph/9704201].
18. J. Berges, S. Borsanyi and C. Wetterich, Phys. Rev. Lett. **93**, 142002 (2004).
19. Y. V. Kovchegov, these proceedings; Y. V. Kovchegov, Nucl. Phys. A **762**, 298 (2005).
20. J. Ruppert, these proceedings, and references therein.
21. J. G. Cramer, these proceedings; J. G. Cramer, G. A. Miller, J. M. S. Wu and J. H. S. Yoon, Phys. Rev. Lett. **94**, 102302 (2005); G. A. Miller and J. G. Cramer, arXiv:nucl-th/0507004.
22. This may not be a problem for the model parameters required to describe the RHIC data (J. G. Cramer, private communication).
23. R. C. Hwa and C. B. Yang, Phys. Rev. C **70**, 054902 (2004).

24. V. Greco, C. M. Ko and P. Levai, Phys. Rev. Lett. **90**, 202302 (2003).
25. E. Wang, these proceedings; A. Majumder, E. Wang and X. N. Wang, arXiv:nucl-th/0506040.
26. S. Katz, these proceedings.
27. F. Karsch, Nucl. Phys. A **698** (2002) 199.
28. Z. Fodor and S. D. Katz, JHEP **0404**, 050 (2004).
29. W. Florkowski, these proceedings; W. Florkowski and W. Broniowski, Acta Phys. Polon. B **35**, 2895 (2004).
30. J. Cleymans, B. Kampfer, M. Kaneta, S. Wheaton and N. Xu, Phys. Rev. C **71**, 054901 (2005); P. Braun-Munzinger, D. Magestro, K. Redlich and J. Stachel, Phys. Lett. B **518**, 41 (2001).
31. D. d'Enterria and D. Peressounko, arXiv:nucl-th/0503054.
32. S. Bathe, these proceedings.
33. J. Ruppert and B. Müller, Phys. Lett. B **618**, 123 (2005).
34. P. Jacobs and M. van Leeuwen, these proceedings.
35. K. Rajagopal, these proceedings.
36. T. Schäfer, these proceedings.
37. S. Eijiri, these proceedings.
38. Y. Hama, these proceedings; O. J. Socolowski, F. Grassi, Y. Hama and T. Kodama, Phys. Rev. Lett. **93**, 182301 (2004).
39. T. Hirano, these proceedings;
T. Hirano and M. Gyulassy, arXiv:nucl-th/0506049.
40. C. Nonaka, these proceedings.
41. T. Hirano and Y. Nara, Phys. Rev. C **66**, 041901 (2002); Phys. Rev. C **69**, 034908 (2004).
42. C. E. Aguiar, T. Kodama, T. Osada and Y. Hama, J. Phys. G **27**, 75 (2001).
43. D. Teaney, these proceedings; G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
44. R. Rapp, these proceedings; H. van Hees and R. Rapp, Phys. Rev. C **71**, 034907 (2005); M. Mannarelli and R. Rapp, arXiv:hep-ph/0505080.
45. M. Mannarelli, these proceedings.
46. M. Djordjevic, these proceedings; M. Djordjevic, M. Gyulassy and R. Vogt, arXiv:nucl-th/0507019.
47. N. Armesto, these proceedings.
48. S. M. H. Wong, Phys. Rev. C **54**, 2588 (1996); Phys. Rev. C **56**, 1075 (1997).
49. Z. Xu, these proceedings; Z. Xu and C. Greiner, Phys. Rev. C **71**, 064901 (2005).
50. G. R. Shin and B. Müller, J. Phys. G **28**, 2643 (2002); J. Phys. G **29**, 2485 (2003).
51. D. Molnar, these proceedings; D. Molnar, arXiv:nucl-th/0503051.
52. B. Zhang, these proceedings.
53. K. Itakura, these proceedings.
54. G. Veres, these proceedings.
55. D. Röhrich, these proceedings.
56. J. Jalilian-Marian, these proceedings; A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, arXiv:hep-ph/0506308.
57. T. Lappi, these proceedings; F. Gelis, K. Kajantie and T. Lappi, arXiv:hep-

ph/0508229.
58. B. Cole, these proceedings.
59. J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D **68**, 025011 (2003).
60. J. O. Andersen, E. Petitgirard and M. Strickland, Phys. Rev. D **70**, 045001 (2004).
61. M. Thoma, these proceedings.
62. E. V. Shuryak, these proceedings; J. Liao and E. V. Shuryak, arXiv:hep-ph/0508035. E. V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004).
63. V. Koch, these proceedings; V. Koch, A. Majumder and J. Randrup, arXiv:nucl-th/0505052.
64. F. Karsch, these proceedings.
65. P. Hartmann, these proceedings;
66. C. R. Hu and B. Müller, Phys. Lett. B **409**, 377 (1997); G. D. Moore, C. R. Hu and B. Müller, Phys. Rev. D **58**, 045001 (1998).
67. U. W. Heinz, Phys. Rev. Lett. **51**, 351 (1983).
68. P. F. Kelly, Q. Liu, C. Lucchesi and C. Manuel, Phys. Rev. D **50**, 4209 (1994).
69. S. A. Bass, B. Müller and W. Pöschl, J. Phys. G **25**, L109 (1999).
70. A. Starinets, these proceedings; P. Kovtun, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005)
71. C. Gale, these proceedings.
72. E. Scomparin, these proceedings; S. Damjanovic, these proceedings.
73. N. Borghini, these proceedings; N. Borghini and U. A. Wiedemann, arXiv:hep-ph/0506218.
74. A. Majumder, these proceedings; A. Majumder and X. N. Wang, Phys. Rev. D **72**, 034007 (2005).
75. C. Salgado, these proceedings.
76. N. Armesto, C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **93**, 242301 (2004).
77. T. Renk and J. Ruppert, Phys. Rev. C **72**, 044901 (2005).
78. B. Müller, Phys. Rev. C **67**, 061901 (2003).
79. X. N. Wang, these proceedings.
80. A. Dainese, C. Loizides and G. Paic, Eur. Phys. J. C **38**, 461 (2005); K. J. Eskola, H. Honkanen, C. A. Salgado and U. A. Wiedemann, Nucl. Phys. A **747**, 511 (2005);
81. R. Baier, Nucl. Phys. A **715**, 209 (2003).
82. I. Vitev, J. Phys. G **30**, S791 (2004).
83. A. Kovner and U. A. Wiedemann, Phys. Rev. D **64**, 114002 (2001).
84. M. Nardi, these proceedings
85. M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004); S. Datta, F. Karsch, P. Petreczky and I. Wetzorke, Phys. Rev. D **69**, 094507 (2004).
86. P. Petreczky, these proceedings.
87. A. Mocsy, these proceedings.
88. T. Csörgö and L. P. Csernai, Phys. Lett. B **333**, 494 (1994).
89. J. Rafelski and J. Letessier, Phys. Rev. Lett. **85**, 4695 (2000).
90. U. W. Heinz and K. S. Lee, Phys. Lett. B **259**, 162 (1991).
91. S. Salur, these proceedings.
92. J. Letessier and J. Rafelski, arXiv:nucl-th/0506044.
93. O. Barannikova, these proceedings.

94. B. Müller, R. J. Fries and S. A. Bass, Phys. Lett. B **618**, 77 (2005).
95. V. Greco and C. M. Ko, Phys. Rev. C **70**, 024901 (2004).
96. P. Sorensen, these proceedings.
97. S. A. Bass, these proceedings; R. J. Fries, S. A. Bass and B. Müller, Phys. Rev. Lett. **94**, 122301 (2005).
98. R. Hwa, these proceedings; C. B. Chiu and R. C. Hwa, Phys. Rev. C **72**, 034903 (2005).
99. R. Lednicky, these proceedings.
100. F. Retiere and M. A. Lisa, Phys. Rev. C **70**, 044907 (2004);
101. M. Csanád, T. Csörgö, B. Lörstad and A. Ster, J. Phys. G **30**, S1079 (2004).
102. Y. Akiba, these proceedings.
103. M. Gyulassy and L. McLerran, Nucl. Phys. A **750**, 30 (2005).
104. E. Braaten and R. D. Pisarski, Nucl. Phys. B **337**, 569 (1990); Phys. Rev. D **45**, 1827 (1992).
105. K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B **458**, 90 (1996).
106. E. Braaten and A. Nieto, Phys. Rev. D **53**, 3421 (1996).
107. M. Laine and Y. Schröder, JHEP **0503**, 067 (2005).
108. see, e. g.: I. R. Klebanov, arXiv:hep-ph/0509087.
109. E. Braaten and R. D. Pisarski, Phys. Rev. D **42**, 2156 (1990). [Note that in this article $\gamma_g$ denotes half the decay width of a quasiparticle.]
110. E. Braaten and R. D. Pisarski, Phys. Rev. D **46**, 1829 (1992).