Viscosities of Hot Gluon – A Lattice QCD Study –

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We present transport coefficients (shear viscosity, η, and bulk viscosity, ζ) for the gluon system obtained by the lattice QCD. This is an indispensable calculation towards the understanding of "New State of Matter" observed in RHIC. We study the temperature regions of RHIC ($1.4 \leq T/T_c \leq 1.8$) and much higher ones up to $T/T_c \sim 20$. In RHIC regions, the ratio of shear viscosity to entropy density, η/s, is around $\sim 0.1 - 0.4$, and satisfies the KSS bound. At high temperature, η becomes two or three oder of magnitude larger.

Our calculation has two limitations: (i) the use of the quench approximation, i.e., without quark pair creation-annihilation effects on vacuum, and (ii) the use of an ansatz for the spectral function. The first point has been well studied in calculations of the spectroscopy and the phase-transition behavior. To investigate the second point, we compare our results with perturbative calculations in high T-regions, and also check the effects of the modification of the spectral function on the viscosity.

1. INTRODUCTION – Matter in deconfinement region

More than twenty years ago, Gross, Pisarski and Yaffe[1] wrote as follows: "Now that we possess a theory of the strong interactions, it is natural to explore the properties of hadronic matter in unusual environments, in particular at high temperature or high baryon density. There are three places where one might look for the effects of high temperature and/or large baryon density, (1) the interior of neutron stars, (2) during the collision of heavy ions at very high energy per nucleon, and (3) about 10^-5 sec after the big bang". We are now in an excited era. At RHIC, the confinement/deconfinement transition temperature is probably exceeded, and for the first time in science history, a deconfinement system is created in a laboratory on earth.

The outcome is very surprising: The matter produced does not look as a quasi-free gas as naively expected, but rather is well described as a fluid. In SPS energy regions, the hydro-model describes well one-particle distributions, HBT etc., but fails to describe the elliptic flow data. This may be not so surprising. Fifty years ago, Landau criticized Fermi's statistical model[2], and noticed 'owing to high density of the particles and to strong interaction between them, one cannot really speak of their number' and proposed his relativistic hydro-dynamical model[3]. The first quantum field theoretical analysis of the applicability conditions of the Landau hydro-dynamical model was reported in Ref.[4].

In three-dimensional hydro-dynamical calculations to analyze RHIC data, it is assumed that the matter produced is a perfect fluid, i.e., *its viscosity is zero.* This assumption is supported by several phenomenological analyses. This also suggests that the new state of matter produced at RHIC should be treated as a strongly coupled system: The perturbative calculation results in

$$\eta = \frac{\eta_1 \cdot T^3}{g^4 \ln(\mu^*/gT)}. \tag{1}$$

The viscosity is small when g is large. This is understandable because there should be sufficient frequent momentum exchange to realize a perfect fluid. Policastro, Son and Starinets have shown an example of a strongly coupled theory in which the viscosity is indeed very small, i.e., $\eta/s = 1/4\pi$[5,6]. They stressed that this is much smaller than that of ordinary matter, such as water or liquid helium, and conjectured that this is the lowest bount (KSS bound)[7].

It is thus important now to calculate the transport coefficients from QCD, non-perturbatively.

2. Transport Coefficients on lattice

On the lattice, the calculation of the transport coefficients is formulated in the framework of the linear response theory [8,9].

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\epsilon(t_1-t)} \int_{-\infty}^{t_1} dt' \langle T_{12}(\vec{x},t) T_{12}(\vec{x'},t')\rangle_{ret}. \tag{2}$$

Here, $\langle T_{\mu\nu} T_{\rho\sigma}\rangle_{ret}$ is the retarded Green's function of energy momentum tensors $T_{\mu\nu}$ at a given temperature. In the quenched approximation, the energy momentum tensors are constructed from only gluonic field strength terms. Bulk viscosity is defined in a similar manner.

Shear viscosity in Eq.(2) is also expressed using the spectral function ρ of the retarded Green's function $\rho(\omega)$[9] as

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega} = \pi \lim_{\omega\to 0} \frac{d\rho(\omega)}{d\omega}. \tag{3}$$

It is determined by the shape of the spectral function near $\omega = 0$.

For evaluating $\rho(\omega)$, we use a well known fact that the spectral function of the retarded Green's function at temperature T is the same as that of Matsubara-Green's function. Therefore, our target is to calculate Matsubara-Green's function($G_\beta(t_n)$) on a lattice and determine ρ from it[11].

To determine the spectral function $\rho(\omega)$ from $G_\beta(t_n)$, we adopt the simplest non-trivial ansatz, i.e., a Bright-Wigner type ansatz proposed by Karsch and Wyld[12],

$$\rho_{BW}(\omega) = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} - \frac{\gamma}{(m+\omega)^2+\gamma^2}\right) \tag{4}$$

As this formula has already 3 parameters, to determine them, the lattice size in temperature direction(N_T) should be $N_T \geq 8$. Thus, the minimum lattice size should be $24^3 \times 8$, to obtain non trivial results.

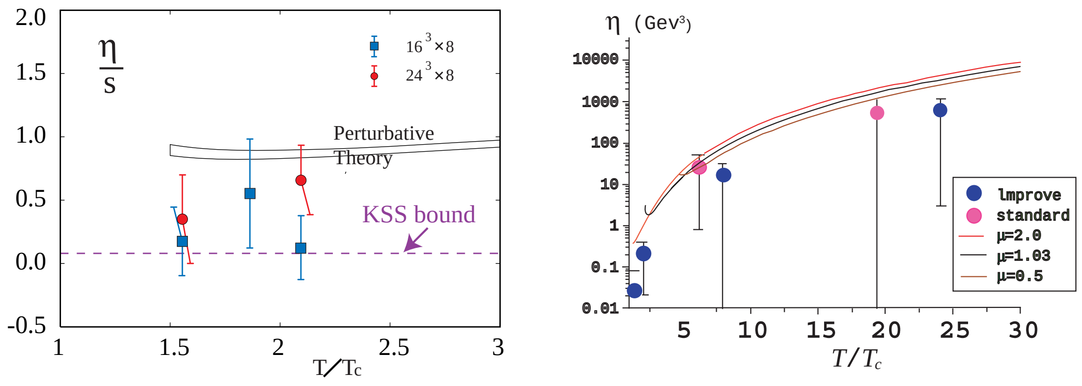

Figure 1. Shear viscosity as a function of temperature. Its ratio to entropy density in RHIC temperature regions (left) and data in physical unit in wide temperature regions together with perturbative calculations (right).

Simulations are carried out using the Iwasaki's improved action and standard Wilson action. The simulations are performed at β =3.05, 3.3, 4.5 and 5.5 for the improved action and at β=7.5 and 8.5 for Wilson action. With roughly 10^6 MC measurements at each β, we determine Matsubara-Green's functions $G_\beta(t_n)$. The errors of G_β are still large in the large t region, however, we fit them with the spectral function ρ given by Eq.(4).

The bulk viscosity is equal to zero within the range of error bars, whereas the shear viscosity remains finite.

3. Conclusions

We may compare our results with the perturbation results of η in rather high temperature regions. In perturbation, bulk viscosity becomes zero[9,10], whereas shear viscosity in the next-to-leading log is given by Eq.(1). As seen in the right-hand figure of Fig.1, in low-T regions, the perturbative calculation becomes inapplicable. At very high temperature, lattice and perturbative results are satisfactorily consistent with each other. Although our result depends on the assumption regarding ρ_{BW} given in Eq. 4, it may be a reasonable approximation of $d\rho/d\omega$ at $\omega = 0$.

Aarts and Resco has proposed an another form of ρ as [14]

$$\rho(\omega) = \rho^{low}(\omega) + \rho^{high}(\omega), \tag{5}$$

$$\rho^{high}(\omega) = \theta(\omega - 4m_{th}^2)\frac{d_A(\omega^2 - 4m_{th}^2)^{5/2}}{80\pi^2\omega}[n(\omega/2) + 0.5], \tag{6}$$

where $d_A = N_c^2 - 1$ and $n(\omega) = 1/(\exp(\omega/T) - 1)$. $\rho^{low}(\omega)$ is a rational function with coefficients as parameter.

In order to study the effect of ρ^{high} on the shear viscosity, η, we assume that ρ is given by $\rho = \rho_{BW} + \rho^{high}$, where ρ_{BW} is given by Eq.(4). By changing m_{th}, the change in η is studied at $\beta = 3.3$ of improved action. When ρ^{high} is absent($m_{th} = \infty$), ηa^3=0.00225(201). If m_{th} is set to be 5.0, 3.0 and 2.0, ηa^3 becomes 0.00223(0.00191), 0.00194(0.00194) and 0.00126(0.00204), respectively. At $m_{th} = 1.8$, the contribution from ρ^{high} becomes larger than $G_\beta(t_n)$ of simulation at $t = 1$, that fit could not be done. Generally, as m_{th} decreases, the contribution from ρ^{high} increases and ρ in the small ω region is suppressed. In this case, it results in a decrease in η.

We have calculated Matsubara-Green's function and determine the shear viscosity of gluon plasma. In the high-temperature region, the agreement of the lattice and perturbative calculation is satisfactory. The lattice result of η/s in $T/T_c \leq 3$ is smaller than that obtained by the extrapolation of the perturbative calculation and satisfies the KSS bound. From the well known relation between the mean free path and viscosity, our results also suggest that gluon plasma is strongly interactive.

Although our results depend on the form of the spectral function ρ_{BW} given by Eq.(4), we think that the qualitative features will not change, because as discussed, our results are stable if the high frequency part of the spectral function is included. We think that η and η/s will not reach 10 times of the present value when more accurate determination of the transport coefficients is carried out.

However, it is important to carry out a more reliable and accurate calculation of transport coefficients, independent of the assumption regarding the spectral function. To this purpose, we are starting the simulation on an anisotropic lattice, to apply maximum entropy method.

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