Isotropization by QCD Plasma Instabilities

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Numerical solutions of the Wong-Yang-Mills equations with anisotropic particle momentum distributions are presented. Their isotropization by collective effects due to the classical Yang-Mills field is shown.

1. INTRODUCTION

It is of great importance to understand the non-equilibrium dynamics in the early stage of high energy heavy-ion collisions and if, how, and when the produced partons thermalize. Boltzmann transport approaches with inelastic processes have been employed to follow the scattering of the hard gluons among themselves [1]. However, the soft classical color fields may give rise to non-perturbative collective processes and hence could also play an important role. Specifically, QCD plasma instabilities may develop, e.g. due to anisotropic distributions of released hard partons [2]. As a consequence, the "bottom-up scenario" [3] for thermalization of the Color Glass Condensate would be modified significantly [4].

Indeed, within the hard loop effective theory unstable soft modes were found [5] if the hard modes exhibit an anisotropic distribution in momentum space. The time evolution has been studied via the full non-linear hard loop effective action [6]. $3 + 1$D simulations of that theory [7] show that instabilities grow more slowly once the field strength becomes large and non-Abelian self-interactions become nonperturbative. Possible effective potentials for anisotropic QCD plasmas beyond the hard loop approximation are discussed in Ref. [8].

In the present work we follow numerically the evolution of the hard modes, represented by particles, coupled to a classical SU(2) Yang-Mills field. The back-reaction of the fields on the particles is taken into account fully. We present some additional results as compared to ref. [9], such as the modification of the particle spectra.

2. Model

We solve Wong's equations [10]

$$\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i, \qquad \frac{d\boldsymbol{p}_i}{dt} = gQ_i^a \left(\boldsymbol{E}^a + \boldsymbol{v}_i \times \boldsymbol{B}^a \right), \qquad \frac{dQ_i}{dt} = igv_i^\mu [A_\mu, Q_i], \tag{1}$$

for the i-th (test) particle, coupled to the Yang-Mills equation

$$D_\mu F^{\mu\nu} = j^\nu = g \sum_i Q_i v^\nu \delta(\boldsymbol{x} - \boldsymbol{x}_i). \tag{2}$$

These equations reproduce the "hard thermal loop" effective action [11] near equilibrium. Numerical techniques to solve the classical field equations coupled to particles have been developed in Ref. [12]. Our update algorithm is closely related to the one explained there.

In the following, we assume that the fields only depend on time and on one spatial coordinate, x, which reduces the Yang-Mills equations to 1+1 dimensions. The particles are allowed to propagate in three spatial dimensions. For simplicity, we also restrict ourselves to the case without expansion here; the more realistic case with longitudinal expansion (solved without particles in [13]) will be addressed in the future. The initial anisotropic phase-space distribution of hard gluons is taken to be $f(\boldsymbol{p}, \boldsymbol{x}) \propto \exp(-\sqrt{p_y^2 + p_z^2}/p_{\text{hard}}) \, \delta(p_x)$. This represents a quasi-thermal distribution in two dimensions, with "temperature" $= p_{\text{hard}}$. It is assumed to come about by free streaming of the particles for a time of order $\sim 1/p_{\text{hard}}$ after the collision of the nuclei. In a comoving frame (with the central rapidity region) then, the longitudinal momenta are red-shifted to much below the typical transverse momentum.

The initial field amplitudes are sampled from a Gaussian distribution with a width tuned to a given initial energy density. We solve the Yang-Mills equations in $A^0 = 0$ gauge and also set $\boldsymbol{A} = 0$ (i.e. all gauge links =1) at time $t = 0$; the initial electric field is taken to be polarized in a random direction transverse to the x-axis.
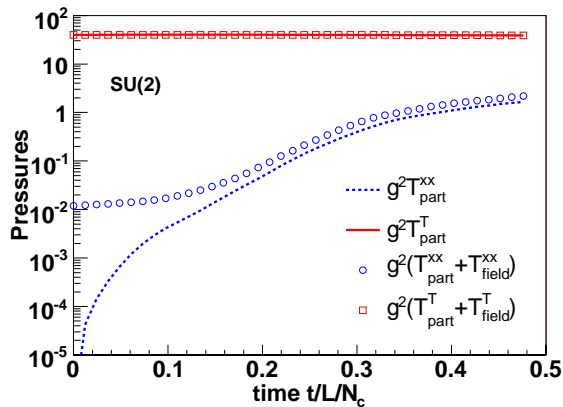
3. RESULTS

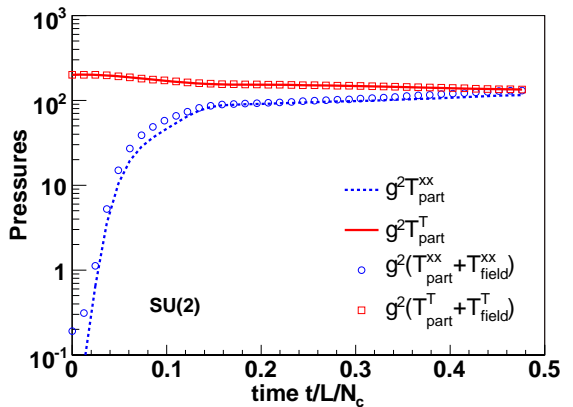Figure 1. Time evolution of the pressures for weak initial fields.

Figure 2. Time evolution of the pressures for the strong field case.

Fig. 1 shows the time evolution of the particle and field pressures for the initial condition $p_{\text{hard}} = 10$ GeV and initial field energy density $\approx 10^{-2}$ GeV/fm^3/g^2, for which instabilities have been seen in Ref. [9]. We observe a rapid exponential growth of the longitudinal pressure, approaching an isotropic configuration. This is due to deflection of the particles in the exponentially growing field, which randomizes their momenta. Note also that the dominant contribution to the build-up of longitudinal pressure is from deflection of the

particles, not from the fields themselves. In our 1+1D simulations the transverse pressure of the fields is zero $T_{\text{field}}^{T} = 0$. Furthermore, we find that $T_{\text{field}}^{00} = T_{\text{field}}^{xx}$, i.e. equal energy density and pressure, which is due to the dominance of transverse magnetic fields. Since periodic boundary conditions are used, no collective flow is generated: $T^{0i} = 0$.

The separation between hard and soft modes may not be very large at RHIC energy [13]. In Fig 2, we show the time evolution of the pressures for $p_{\text{hard}} = 1$ GeV and initial field energy density $\approx 10^{-1}$ GeV/fm^3/g^2; the time scale is set by the lattice size L, =10 fm for this simulation. When the separation between hard and soft modes is small, strong instabilities are not seen, but the system still approaches isotropy [9]. In fact, the rapid isotropization of the particle momenta by the strong fields prevents the occurrence of pronounced instabilities.
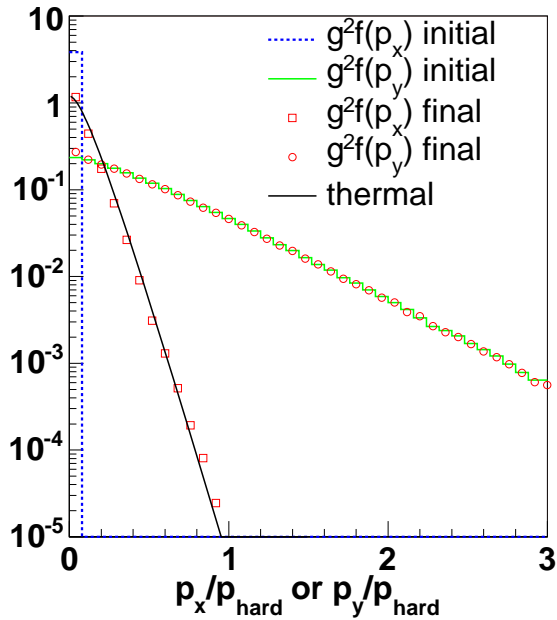

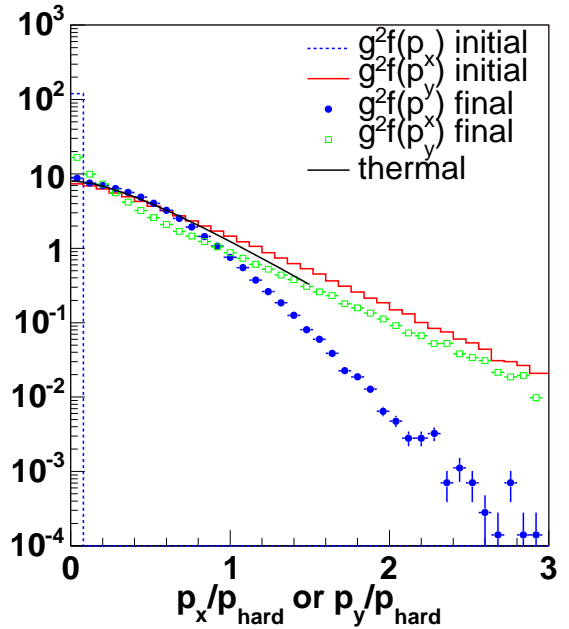Figure 3. Particle distribution functions for the weak field case.

Figure 4. Particle distribution functions for the strong field case.

It is interesting to look at the particle distribution function itself, even though we presently restrict ourselves to simulations of collisionless plasmas. Field fluctuations generate an effective collision term [14] mediating soft exchanges. We plot initial as well as final $(t/L/N_c = 0.5)$ particle distribution functions for weak initial field in Fig. 3, and for strong field in Fig. 4. For the former case, we observe that the initial δ-function in the longitudinal direction becomes a thermal distribution extending over several orders of magnitude. However, the longitudinal direction is still much colder than the transverse directions, in agreement with the behavior of the pressures above. This is because of the assumption of transversally homogeneous fields. The fluctuations of the gauge field induce fluctuations in the motion of the quasiparticles. The presence of fluctuations is

closely linked to dissipative processes.

In the future, full 3D simulations of the Wong-Yang-Mills equations are required. They will also allow us to study the effects due to a (azimuthally asymmetric) boundary. We also need to include longitudinal expansion of the system to check whether the distribution *stays* isotropic. Finally, collisions with momentum transfer above that mediated by the classical field have to be included for a complete description of the non-perturbative plasma in weak coupling.

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