Survival of Back-to-Back Correlations for Finite Expanding Fireballs

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In the late 1990's, *Back-to-Back Correlations* (*BBC*) of boson-antiboson pairs were predict to exist if the particles masses were modified in the hot and dense medium[1], expected to be formed in high energy nucleus-nucleus collisions. The BBC are related to in-medium mass-modification and squeezing of the quanta involved. Not much longer after that, it was also shown that an analogous BBC existed between fermion-antifermion pairs with medium-modified masses[2]. A similar formalism is applicable to both BBC cases, related to the Bogoliubov-Valatin transformations of in-medium and asymptotic operators. Both the bosonic (bBBC) and the fermionic (fBBC) Back-to-Back Correlations are positive and have unlimited magnitude, thus differing from the identical-particle correlations, also known as HBT (Hanbury Brown & Twiss) correlations, which are limited for both cases, being negative in the fermionic sector. BBC were expected to be significant for $p_T < 2$ GeV/c. Nevertheless, already in the Ref.[1], it was shown that, if the emission process is not sudden, even a short duration of particle emission significantly suppresses the BBC magnitude. On the other hand, the effects of finite system sizes and of collective phenomena had not been studied yet. Thus, for testing the survival and magnitude of the effect in more realistic situations, we study the BBC when mass-modification occurs in a finite sized, thermalized medium, considering a non-relativistically expanding fireball with short emission duration, and evaluating the width of the back-to-back correlation function. We show that the BBC signal indeed survives the expansion and flow effects, with sufficient magnitude to be observed at RHIC. Some preliminary results are discussed here and illustrated for particular cases.

Our analysis assumes the validity of local thermalization and hydrodynamics up to the system freeze-out. We also consider $H = H_0 - \int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x}-\mathbf{y}) \phi(\mathbf{y})$ as an effective in-medium Hamiltonian, where the first term is the asymptotic (free) Hamiltonian in the matter rest frame, and the second term describes the medium modifications. The

*SSP would like to express her gratitude to the Organizing Committee of Quark Matter 2005 and to Fundação de Amparo à Pesquisa do Estado de São Paulo (Proc. N. 05/52190-1 and 2004/10619-9 - *Projeto Temático*) for their complementary financial support.

scalar field ϕ represents quasi-particles propagating with momentum-dependent medium-modified mass $m_\star$, related to the vacuum mass, m, by $m_\star^2(|\mathbf{k}|) = m^2 - \delta M^2(|\mathbf{k}|)$. This implies that the dispersion relations in the vacuum and in-medium are given, respectively, by $\omega_k^2 = m^2 + \mathbf{k}^2$ and $\Omega_k^2 = m_\star^2 + \mathbf{k}^2 = \omega_k^2 - \delta M^2(|\mathbf{k}|)$, where Ω is the frequency of the in-medium mode with momentum $\mathbf{k}$.

The annihilation (creation) operator, b ($b^\dagger$), for the in-medium, thermalized quasi-particles is related to the annihilation (creation) operator, a ($a^\dagger$), of the asymptotic, observed quanta with momentum $k^\mu = (\omega_k, \mathbf{k})$, by the Bogoliubov-Valatin transformation: $a_k = c_k b_k + s^*_{-k} b^\dagger_{-k}$ ($a^\dagger_k = c^*_k b^\dagger_k + s_{-k} b_{-k}$), where $c_k = \cosh(f_k)$ and $s_k = \sinh(f_k)$; $f_k = \frac{1}{2}\log(\frac{\omega_k}{\Omega_k})$ is called *squeezing parameter*, since the Bogoliubov transformation creates squeezed states from coherent ones. In cases where the particle is its own anti-particle (for $\pi^0\pi^0$ or $\phi\phi$ boson pairs, for instance), the full correlation function is written as

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_c(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1) G_c(\mathbf{k}_2, \mathbf{k}_2)} + \frac{|G_s(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1) G_c(\mathbf{k}_2, \mathbf{k}_2)}, \tag{1}$$

where the first two terms correspond to the HBT correlation, and last term, represents this additional contribution to the correlation function, i.e., the squeezing part.

For a hydrodynamical ensemble, both the chaotic and the squeezed amplitudes, G_c and G_s, respectively, can be written in the special form derived by Makhlin and Sinyukov [3] (see Eqs. (22) and (23) of Ref. [1]), namely

$$G_c(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4\sigma_\mu(x)}{(2\pi)^3} K^\mu_{1,2}\, e^{i\, q_{1,2}\cdot x} \Big\{ |c_{1,2}|^2\, n_{1,2}(x) + |s_{-1,-2}|^2\, \left[n_{-1,-2}(x) + 1 \right] \Big\}, \tag{2}$$

$$G_s(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4\sigma_\mu(x)}{(2\pi)^3} K^\mu_{1,2}\, e^{2\, i\, K_{1,2}\cdot x} \Big\{ s^*_{-1,2}\, c_{2,-1}\, n_{-1,2}(x) + c_{1,-2}\, s^*_{-2,1}\, \left[n_{1,-2}(x) + 1 \right] \Big\}. \tag{3}$$

In Eq. (2) and (3), $d\sigma^4_\mu(x) = d^3\Sigma_\mu(x,\tau) F(\tau) d\tau$ is the product of the normal-oriented volume element depending parametrically on τ (freeze-out hyper-surface parameter) and on its invariant distribution, $F(\tau)$; σ^μ is the hydrodynamical freeze-out surface. In Eq. (2), the pair momentum difference and the pair average momentum are given, respectively, by $q^\mu_{i,j}(x) = k^\mu_i(x) - k^\mu_j(x)$, and $K^\mu_{i,j}(x) = \frac{1}{2}\left[k^\mu_i(x) + k^\mu_j(x) \right]$, as in HBT; $c_{i,j} = \cosh(f_{i,j})$ and $s_{i,j} = \sinh(f_{i,j})$, with $f_{i,j}(x) = \frac{1}{2}\log\left[\frac{K^\mu_{i,j}(x)\, u_\mu(x)}{K^{*\nu}_{i,j}(x)\, u_\nu(x)}\right] = \frac{1}{2}\log\left[\frac{\omega_{\mathbf{k}_i}(x) + \omega_{\mathbf{k}_j}(x)}{\Omega_{\mathbf{k}_i}(x) + \Omega_{\mathbf{k}_j}(x)}\right] \equiv f_{\pm i, \pm j}(x)$. We are using a short-hand notation for the momenta, $\pm i, \pm j \equiv k_{\pm i, \pm j}$, with $i, j = 1, 2$.

For studying the expansion of the system we adopt the non-relativistic hydrodynamical model of Ref. [4]. In this model the fireball expands in a spherically symmetric manner with a local flow vector given by the four-velocity $u^\mu = \gamma\,(1, \mathbf{v})$, assumed to be non-relativistic, with $\gamma = (1 - \mathbf{v}^2)^{-1/2} \approx 1 + \mathbf{v}^2/2$, where $\mathbf{v} = \langle u \rangle \mathbf{r}/R$, being $\langle u \rangle$ and R the mean expansion velocity and the radius of the fireball, respectively.

In addition, we consider the Boltzmann limit of the Bose-Einstein distribution for n_k, i.e., $n_{i,j} \sim \exp\left[-(K^\mu_{i,j} u_\mu - \mu(x))/T(x)\right]$, and assume a time-dependent parametric solution of the hydrodynamical equations, i.e., $\mu(x)/T(x) = \mu_0(x)/T(x) - r^2/(2R^2)$, as in Ref. [4]. Furthermore, we consider a smeared freeze-out, for which $\frac{\theta(\tau-\tau_0)}{\Delta\tau} e^{-(\tau-\tau_0)/\Delta\tau}$, with short emission Δt. This more realistic scenario has a dramatic effect on the Back-to-Back Correlation function, as already showed in Ref.[1], by reducing severely the signal's magnitude, even for a smearing of about $\Delta\tau \sim 2$ fm/c. A clear illustration of the finite emission duration as compared to the sudden freeze-out can be seen in Ref.[5,6].

For discussing finite-size effects, we distinguish between the volume of the entire thermalized medium, denoted by V (with radius R), and the volume filled with mass-shifted quanta, denoted by V_s (with radius R_s). Naturally, $V_s \leq V$ in the general case. In the derivation of the expressions for $G_c(1,2)$ and $G_s(1,2)$, for simplicity, we consider that the volumetric region where the mass $m_\star$ is significantly modified is smooth and Gaussian in shape, i.e., we introduce a three-dimensional Gaussian profile, $\sim \exp\left[-\mathbf{r}/(2R^2)\right]$, for representing the system volume.

In the non-relativistic limit, the accounting for the squeezing effects can be simplified for small mass shifts $(m_\star - m)/m \ll 1$, such that the squeezing parameter can be written simply as $f(i,j,\mathbf{r}) \approx \frac{1}{2}\log(m/m_\star)$. This limit is important, because in this case the coordinate dependence enters the squeezing parameter f only through the possible position dependence of the mass-shift which, in principle, could be calculated from thermal field models in the local density approximation. Therefore, in an approximation such that the position dependence of the in-medium mass can be neglected, the $c(i,j) = c_0$ and $s(i,j) = s_0$ factors can be removed from the integrands in Eq. (2) and (3) and all what remains to be done are Fourier transforms of Gaussian functions.

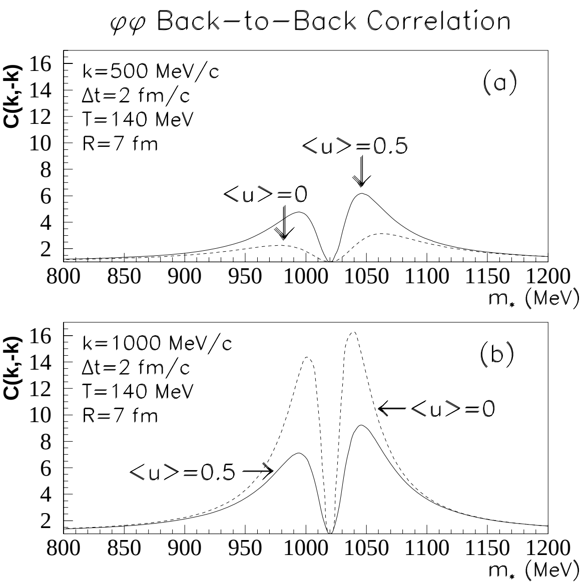

Figure 1. The maximal BBC is illustrated vs. $m_\star$, when the mass-shift occurs in the entire system with radius R, i.e., $V_s = V$, for two values of the momentum k of the back-to-back pair, with and without flow.

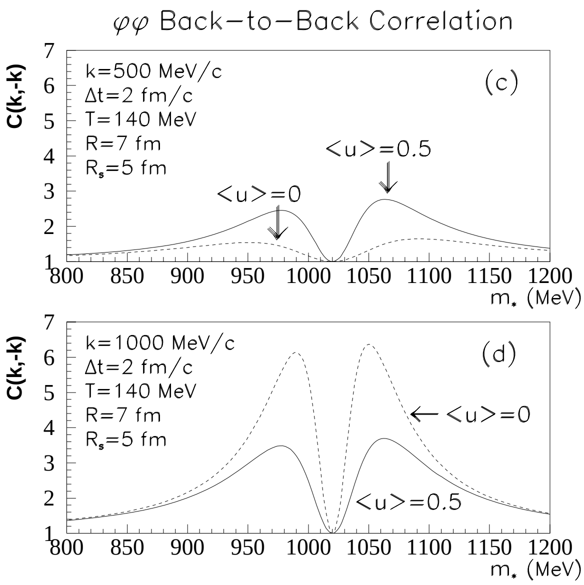

Figure 2. The maximal BBC vs. $m_\star$ is shown, for mass-shift occurring only in a smaller portion of the system, with radius $R_s < R$, i.e., $V_s < V$. All the other variables are similar to the ones in Figure 1.

In Figures 1 and 2, we illustrate some of the results found in the non-relativistic approximation, in the particular case of weak flow coupling. Figure 1 corresponds to the case in which the mass-shift is extended over the entire system volume, whereas in Figure

2, we show results for the squeezing occurring in a smaller portion of the system region. In the plots, T stands for the freeze-out temperature. We see that the cases corresponding to the absence of flow and to its inclusion produce similar results within the limits of our illustration, and that the strength of the squeezed BBC function is proportional to the size of the mass-shifting region. However, depending on the value of $k_1 = -k_2 = k$, there are noticeable differences. Being so, we see that, for smaller values of k, the presence of flow seems to slightly enhance the signal, whereas at large values of k, the non-flow case wins. Nevertheless, the non-flow case grows faster with increasing k.

In summary, our main goal on presenting these new results here was to revive the discussion on the search of the squeezed BBC. For fulfilling this purpose, we estimated the strength of the squeezed BBC signal in a more realistic situation, considering the mass-shifting in a finite region, and the particle emission occurring during a short interval. We also considered that the system expands non-relativistically and analyze the simplified situation of weak flow dependence of the squeezed BBC. For illustrating the effects, we considered $\phi\phi$ pairs. We showed in Figures 1 and 2 the back-to-back correlation function versus the in-medium shifted mass, $m_\star$, with pronounced maxima around $m \approx m_\star$. We also saw that both the non-flow and the flow cases produced similar results, and that the BBC magnitude increases proportionally to the size of the mass-shifting region. However, for reducing the BBC magnitude, we saw that the effect of decreasing the system size is far less significant than the sensitivity to the spread in the time emission interval. Our main conclusion, nevertheless, is that in any of the two cases discussed above, a sizeable strength of the squeezed BBC signal could be seen, making it a promising effect to be searched for experimentally at RHIC.

Naturally, in a more refined calculation, it would be mandatory to introduce a model-based mass-shift. On top of that, it would also be essential to perform more realistic calculations with flow, in a less constrained kinematical region, while simultaneously searching for those windows which could optimize the observation of the squeezed BBC signal. Also, an estimate of the shape and width of the BBC around the direction $k_1 = -k_2 = k$ should be performed. Finally, for being able to make predictions closer to the experimental conditions, it will be extremely important to obtain some feed-back on the experimental acceptance, conditions, and restrictions that could finally lead to the BBC discovery.

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