# Heavy quark diffusion and lattice correlators

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We study charmonia correlators at finite temperature. We analyze to what extent heavy quarkonia correlators are sensitive to the effect of heavy quark transport.

## 1. Introduction

There are plenty of experimental evidence that strongly interacting matter at high energy density has been produced at RHIC [1,2]. One of the most exciting results from RHIC so far is the large azimuthal anisotropy of light hadrons with respect to the reaction plane, known as elliptic flow. The observed elliptic flow is well described by ideal hydrodynamics (see e.g. Ref. [3] for review) suggesting early equilibration of the produced matter and very short transport mean free path. This interpretation of the experimental data can be challenged by measuring elliptic flow of charm and bottom mesons [4,5]. The first experimental results show a non-zero elliptic flow for these heavy mesons. Naively, since the quark mass is significantly larger than the temperature of the medium, the mean free pass of heavy mesons is $\sim M/T$ longer than the light hadron mean free path. Quantitatively the mean free path is described by the heavy quark diffusion constant which can be defined through the diffusion equation for the heavy quark number density $N(\mathbf{x}, t)$, $\partial_t N + D\nabla^2 N = 0$. If the heavy quark diffusion constant $D \geq 1/T$, the predicted heavy quark elliptic flow will be too small and in contradiction with current experimental data [6].

Kubo formulas relate hydrodynamic transport coefficients to the small frequency behavior of real time correlation functions. Correlation functions in real time are in turn related to correlation functions in imaginary time by analytic continuation. Karsch and Wyld [7] first attempted to use this connection to extract the shear viscosity of QCD

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from the lattice. More recently, additional attempts to extract the shear viscosity [8,9] and electric conductivity [10] have been made. It turns out that Euclidean correlations functions are remarkably insensitive to transport coefficients. For weakly coupled field theories this has been discussed by Aarts and Martinez Resco [11]. For this reason, only precise lattice data and a comprehensive understanding of the different contributions to the Euclidean correlator can constrain the transport coefficients. It appears that heavy quarkonia correlators are the likely candidates for meeting this conditions.

## 2. Euclidean and real time correlators

On the lattice we calculate correlation function of local meson operators ( currents ) $J^h_E(\mathbf{x},\tau)=\bar{q}(\mathbf{x},\tau)\Gamma_h q(\mathbf{x},\tau)$ at finite temperature, $G^h(\mathbf{k},\tau,T)=\int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\left\langle J^h_E(\mathbf{x},\tau)J^h_E(0,0)\right\rangle$, with $\Gamma_h$ being some combination of the Dirac matrices. This correlation function is related to the real time correlation functions $D^>_h(\mathbf{x},t,T)=\langle J^h(\mathbf{x},t)J^h(0,0)\rangle$, $D^<_h(\mathbf{x},t,T)=\langle J^h(0,0)J^h(\mathbf{x},t)\rangle$. The most important channels for our further discussion are the pseudo-scalar, $\Gamma_h=\gamma_5$ and the vector, $\Gamma_h=\gamma_\mu$ channels. In the vector channel the Euclidean correlators are related to density-density correlator $D^>_{NN}=\langle N(\mathbf{x},t)N(0,0)\rangle$ and current-current correlators $D^{>\,ij}_{JJ}=\langle J^i(\mathbf{x},t)J^j(0,0)\rangle$,

$$G^{00}(\mathbf{x},\tau,T)=-D^>_{NN}(\mathbf{x},-i\tau,T)\,,G^{ij}(\mathbf{x},\tau,T)=D^{>;ij}_{JJ}(\mathbf{x},-i\tau,T)\,. \tag{1}$$

Similarly for the pseudo-scalar channel $G^5(\mathbf{x},\tau,T)=\langle J^5_E(\mathbf{x},\tau)J^5_E(0,0)\rangle=D^>_5(\mathbf{x},-i\tau,T)$. The minus sign in Eq. (1) comes from the relation $A^0=-iA^0_E$ between the temporal component of the vector in Minkowski space and Euclidean space, in particular $x^0=-ix^0_E=-i\tau$. The spectral function is defined through Fourier transform of $D^<_h$ and $D^>_h$ or equivalently as imaginary mart of the retarded correlator $\chi_h(\mathbf{k},\omega)$: $2\pi\rho^h(\mathbf{k},\omega,T)=(D^>_h(\mathbf{k},\omega,T)-D^<_h(\mathbf{k},\omega,T))=2\mathrm{Im}\chi_h(\mathbf{k},\omega,T)$. Using the Kubo-Martin Schwinger (KMS) relation $D^>_h(\mathbf{k},t)=D^<_h(\mathbf{k},t+i/T)$, one discovers the relation between the spectral density and the Euclidean correlator,

$$G^h(\mathbf{k},\tau,T)=(-i)^r\int_0^\infty d\omega\,\rho^h(\mathbf{k},\omega,T)K(\omega,\tau,T) \tag{2}$$

Here $K(\omega,\tau,T)=\frac{\cosh\left(\omega\left(\tau-\frac{1}{2T}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$ and $r$ is number of zeros in the space-time indexes.

## 3. Lattice results on the charmonia correlators and spectral functions

Charmonia correlators have been studied in lattice QCD and the corresponding spectral functions were reconstructed using the Maximal Entropy Method (MEM) [13–15]. These studies showed that the 1S states ($\eta_c$ and $J/\psi$) survive in the plasma up to temperatures as high as $1.6T_c$. Though it is quite difficult to reliably reconstruct the spectral functions, the temperature dependence of the correlators can be determined quite precisely [15].

We calculated charmonia correlators on quenched anisotropic lattices using the Fermilab formulation for heavy quarks [12]. Calculation were done at $\beta=6.5$ and $\xi=a_s/a_t=4$, corresponding to temporal lattice spacing $a_t^{-1}=14.12$GeV when we set the spatial lattice spacing $a_s$ using the Sommer scale $r_0=0.5$fm. We collected about 1000 gauge configurations at each temperature. From Eq. (2) it is clear that the temperature dependence of the correlator $G(\mathbf{k},\tau,T)$ comes from temperature dependence of the spectral function and temperature dependence of the kernel $K(\tau,\omega,T)$. To separate out the trivial temperature dependence due to the kernel $K(\tau,\omega,T)$, following [15] we introduce the reconstructed correlator $G^h_{\rm rec}(\mathbf{k},\tau,T) = \int_0^\infty d\omega\, \rho^h(\mathbf{k},\omega,T=0)\, K(\tau,\omega,T)$. If the charmonia spectral function do not change across the deconfinement transition temperature $T_c$ we expect $G^h/G^h_{\rm rec} \simeq 1$. In Fig. 1 we show the temperature dependence of $G^h/G^h_{\rm rec}$ for pseudo-scalar and vector channels at zero spatial momentum $\mathbf{k}=0$. In the vector channel we show both sum over all spatial components $\sum_i G^{ii}$ and the sum over all four components $\sum_\mu G^{\mu\mu}$. We see that the temperature dependence of the vector and pseudo-scalar correlarors is quite different. For $T=1.5T_c$ we see only very small deviations from unity for $G^h/G^h_{\rm rec}$ in the pseudo-scalar channel while significant deviations are seen in the vector channel. In fact similar temperature dependence of the vector correlator was seen in the previous study based on fine isotropic lattices [15,16]. This is quite unexpected as $\eta_c$ and $J/\psi$ should have similar properties both in the vacuum and in the medium. We will give an explanation for this difference in the next section in terms of heavy quark transport.

## 4. Spectral functions at low energies and heavy quark transport

As vector current is a conserved current there should be transport contribution to the corresponding spectral function. In general the vector spectral function can be decomposed in terms of transverse $\rho^T(\mathbf{k},\omega)$ and longitudinal $\rho^L(\mathbf{k},\omega)$ components. Since the heavy quark mass is much larger than the temperature, $M \gg T$ we can write $\rho^{L,T}(\mathbf{k},\omega) = \rho^{L,T}_{\rm low}(\mathbf{k},\omega) + \rho^{L,T}_{\rm high}(\mathbf{k},\omega)$, where $\rho_{\rm high}(\mathbf{k},\omega)$ contains the resonances and the continuum, and is non-zero for energies $\omega \sim 2M$, and $\rho_{\rm low}(\mathbf{k},\omega)$ is the transport contribution. The simplest way to estimate $\rho_{\rm low}(\mathbf{k},\omega)$ is to evaluate the vector correlator at 1-loop level [17]. In the $\mathbf{k}\to 0$ limit we have $\rho^T(0,\omega) = \rho^L(0,\omega) = \rho^{ii}(0,\omega)$, and considering small energies, $\omega \ll T$ we get $\rho^{ii}_{\rm low}(0,\omega) = \chi_s(T)\frac{T}{M}\omega\delta(\omega)$, $\rho^{00}_{\rm low}(0,\omega) = \chi_s(T)\omega\delta(\omega)$. Here $\chi_s(T)$ is the static charm number susceptibility, which in the limit $M \gg T$ is given by $\chi_s(T) = 12\left(\frac{MT}{2\pi}\right)^{3/2} e^{-M/T}$. Thus at finite temperature we expect that the $\sum_i G^{ii}$ should be enhanced by a constant contribution $3\chi_s(T)T/M$ relative to its $T=0$ value, while the $\sum_\mu G^{\mu\mu}$ should reduced by $-\chi_s(T)(1-3T/M)$ (recall Eq. (1)). This is exactly what the lattice data in Fig. 1 show. Furthermore, from data on $\sum_i G^{ii}$ and $\sum_\mu G^{\mu\mu}$ we can estimate that $M/T \simeq 6$ at $1.5T_c$. The 1-loop result for the vector correlator can be also obtained using collisionless Boltzmann equation describing free streaming of heavy quarks with no interaction with the plasma [17]. This 1-loop contribution happens to dominate the transport part of the Euclidean correlator [17]. To get the transport coefficient we need to include the effect of heavy quark interactions with the medium. It is very difficult problem in general. Luckily, the case of heavy quarks is special since the time scale for diffusion, $M/T^2$, is much longer than any other time scale in the problem. For this reason we can assume that the Langevin equations provide a good macroscopic description of the dynamics of charm quarks [6]. The Langevin equations make a definite prediction for the retarded correlator $\chi_h$ at small $\omega$ and thus for the transport part of the spectral functions [17]. For the case of zero spatial momentum $\mathbf{k}=0$ we have $\frac{\rho^{ii}_{\rm low}(0,\omega)}{\omega} = \chi_s \frac{T}{M}\frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2}$, $\frac{\rho^{00}_{\rm low}(0,\omega)}{\omega} = \chi_s\delta(\omega)$. From this equation it is clear that to calculate the transport coefficient we have to determine the curvature of of

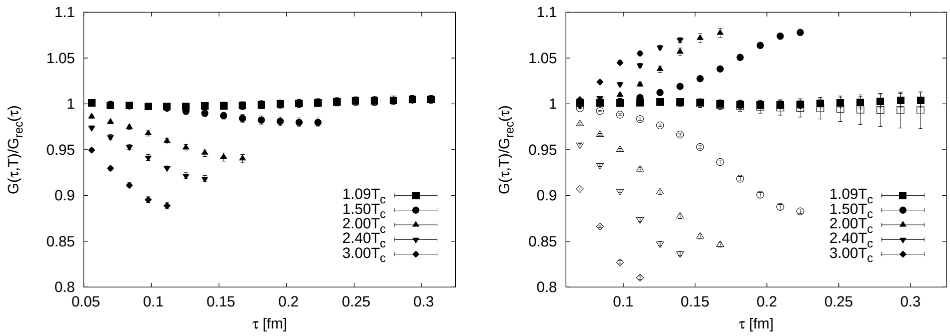


Figure 1. The ratio $G^h/G^h_{\rm rec}$ for pseudo-scalar (left) and vector (right) channels at $\mathbf{k}=0$. For the vector case we show both the sum over spatial components (filled symbols) and all four components (open symbols).

$G^{ii}(\mathbf{k}=0,\tau,T)$ at $\tau=1/(2T)$ due to the low energy part of the spectral function $\rho^{ii}_{\rm low}$. If $\rho^{ii}_{\rm low}$ was the only contribution to the spectral function and $\eta=0$ the correlator would be constant. The question is how to determine the small curvature in $G^{ii}(\mathbf{k}=0,\tau,T)$, arising from finite value of $\eta$, from the curvature arising from the resonance and continuum contributions. This can be done by introducing a small chemical potential for the heavy quark, $\mu \ll M$. Since the transport contribution is proportional to $\chi_s$, the small chemical potential will enhance it by factor of $\cosh(\mu/T)$ [17]. The small charm chemical potential will not affect the resonance and continuum contributions to the spectral function to leading order in the heavy quark density, $\sim e^{-(M-\mu)/T}$. Thus we expect that $\delta G^{ii} \equiv G^{ii}(\tau,T,\mu) - G^{ii}(\tau,T,0) \simeq (\cosh(\mu/T)-1)\int_0^\infty d\omega\, \rho^{ii}_{\rm low}\,(\omega))|_{\mu=0}\, K(\omega,\tau,T)$ is largely insensitive to the high frequency behavior of the spectral function. Thus if numerical accuracy of about 0.5% can be achieved for the difference $\delta G^{ii}$, the curvature and thus the $\eta$, or equivalently $D$ can be estimated in lattice QCD.

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