Fermionic Quasiparticles in QCD at High Baryon Density

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We study fermionic quasi-particles in QCD at very high baryon density. In the normal quark matter phase unscreened magnetic gluon exchanges lead to non-Fermi liquid behavior. Non-Fermi liquid effects manifest themselves in low energy Green functions that depend on logarithms and fractional powers of energy. In the superfluid phase there is an energy gap for fermionic excitations. Quark mass effects can cause the energy gap to vanish. Gapless fermions in the color flavor locked phase cause an instability towards a state with a non-zero supercurrent.

1. Non-Fermi liquid behavior

At high baryon density the relevant degrees of freedom are particle and hole excitations which move with the Fermi velocity v. Since the momentum $p \sim v\mu$ is large, typical soft scatterings cannot change the momentum by very much and the velocity is approximately conserved. An effective field theory of particles and holes in QCD is given by [1]

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{1}{2p_F} D_\perp^2 \right) \psi_v + \mathcal{L}_{HDL} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots , \tag{1}$$

where $v_\mu = (1, \vec{v})$. The field ψ_v describes particles and holes with momenta $p = \mu\vec{v} + l$, where $l \ll \mu$. We will write $l = l_0 + l_\parallel + l_\perp$ with $\vec{l}_\parallel = \vec{v}(\vec{l} \cdot \vec{v})$ and $\vec{l}_\perp = \vec{l} - \vec{l}_\parallel$. At energies below the screening scale $g\mu$ hard dense loops have to be resummed. The generating functional for hard dense loops in gluon n-point functions is given by [2]

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}, \tag{2}$$

where $m^2 = N_f g^2 \mu^2/(4\pi^2)$ is the dynamical gluon mass and the sum over patches corresponds to an average over the direction of $\vec{v}$.

The hard dense loop action describes static screening of electric fields and dynamic screening of magnetic modes. Since there is no screening of static magnetic fields low energy gluon exchanges are dominated by magnetic modes. The resummed transverse gauge boson propagator is given by

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0|/|\vec{k}|}, \tag{3}$$

where $\eta = \frac{\pi}{2} m^2$ and we have assumed that $|k_0| < |\vec{k}|$. We observe that the gluon propagator becomes large in the regime $|\vec{k}| \sim (\eta k_0)^{1/3} \gg k_0$. This leads to an unusual scaling

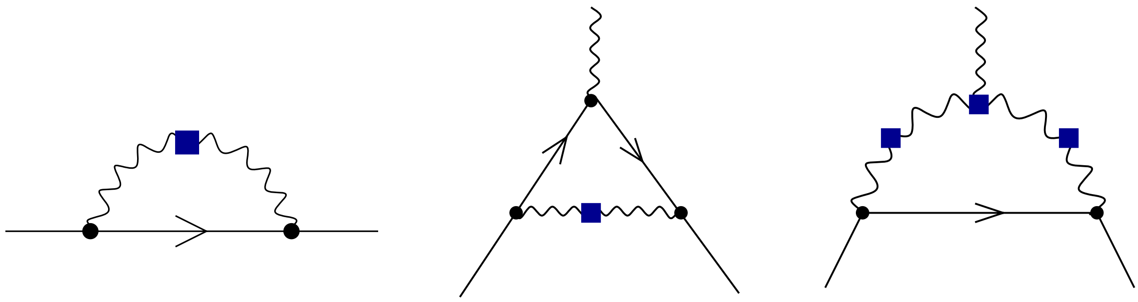

Figure 1. One loop corrections to the fermion propagator and the quark gluon vertex in dense QCD. The solid squares denote hard dense loop insertions.

behavior of Green functions in the low energy limit. Consider a generic Feynman diagram and scale all energies by a factor s. Because of the functional form of the gluon propagator in the Landau damped regime gluon momenta scale as $|\vec{k}| \sim s^{1/3}$. This implies that the gluon momenta are much larger than the gluon energies. The quark dispersion relation is $k_0 \simeq k_{||} + k_\perp^2/(2p_F)$. The only way a quark can emit a very spacelike gluon and remain close to the Fermi surface is if the gluon momentum is transverse to the Fermi velocity. We find

$$k_0 \sim s, \quad k_{||} \sim s^{2/3}, \quad k_\perp \sim s^{1/3}, \tag{4}$$

and $k_0 \ll k_{||} \ll k_\perp$. These scaling relations have many interesting consequences. As an example we consider the fermion self energy, see Fig. 1. The one-loop self energy is

$$\Sigma(p) \sim \int dk_0 \int dk_\perp^2 \, \frac{k_\perp}{k_\perp^2 + i\eta k_0} \int dk_{||} \, \frac{\Theta(p_0 + k_0)}{k_{||} + p_{||} - (k_\perp^2 + p_\perp^2)/(2p_F) + i\epsilon} \sim p_0 \log(p_0). \tag{5}$$

A more careful calculation gives [3]

$$\Sigma(\omega) = \frac{g^2}{9\pi^2}\left[\omega \log\left(\frac{4\sqrt{2}m}{\pi|\omega|}\right) + \omega + i\frac{\pi}{2}|\omega|\right]. \tag{6}$$

There are no corrections of the form $g^{2n}\log^n(\omega)$ [4]. Higher order corrections involve fractional powers $(\omega/m)^{1/3}$. Equ. (6) shows that cold quark matter is not a Fermi liquid. The Fermi velocity vanishes on the Fermi surface and the specific heat scales as $T\log(T)$.

2. CFL Phase

If the temperature is sufficiently low quark matter is expected to become a color superconductor and quarks acquire a gap due to pairing near the Fermi surface. In the following we will concentrate on the color flavor locked (CFL) phase which is the ground state of three flavor quark matter at very high baryon density. Our starting point is the effective theory of the CFL phase derived in [5,6]. The effective lagrangian contains Goldstone boson fields Σ and baryon fields N. The meson fields arise from chiral symmetry breaking in the CFL phase. The leading terms in the effective theory is

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger - v_\pi^2\vec{\nabla}\Sigma\vec{\nabla}\Sigma^\dagger\right). \tag{7}$$

Baryon fields originate from quark-hadron complementarity. The effective lagrangian is

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \{\mathcal{A}_\mu, N\}\right) - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 [\mathcal{A}_\mu, N]\right) \\ &\quad + \frac{\Delta}{2}\left\{\left(\mathrm{Tr}(N_L N_L) - [\mathrm{Tr}(N_L)]^2\right) - \left(\mathrm{Tr}(N_R N_R) - [\mathrm{Tr}(N_R)]^2\right) + h.c.\right\}, \qquad (8)\end{aligned}$$

where $N_{L,R}$ are left and right handed baryon fields in the adjoint representation of flavor $SU(3)$, $v^\mu = (1, \vec{v})$ is the Fermi velocity, and Δ is the superfluid gap. We can think of N as being composed of a quark and a diquark field, $N_L \sim q_L\langle q_L q_L\rangle$. The interaction of the baryon field with the Goldstone bosons is dictated by chiral symmetry. The covariant derivative is given by $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$. The vector and axial-vector currents are

$$\mathcal{V}_\mu = -\frac{i}{2}\left\{\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right\}, \qquad \mathcal{A}_\mu = -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi, \qquad (9)$$

where ξ is defined by $\xi^2 = \Sigma$. The low energy constants f_π, v_π, D, F can be calculated in perturbative QCD. Symmetry arguments can be used to determine the leading mass terms in the effective lagrangian. Bedaque and Schäfer observed that $X_L = MM^\dagger/(2p_F)$ and $X_R = M^\dagger M/(2p_F)$ act as effective chemical potentials and enter the theory like the temporal components of left and right handed flavor gauge fields [7]. We can make the effective lagrangian invariant under this symmetry by introducing the covariant derivatives

$$\begin{aligned} D_0 N &= \partial_0 N + i[\Gamma_0, N], \quad \Gamma_0 = -\frac{i}{2}\left\{\xi\left(\partial_0 + iX_R\right)\xi^\dagger + \xi^\dagger\left(\partial_0 + iX_L\right)\xi\right\}, \qquad (10) \\ \nabla_0\Sigma &= \partial_0\Sigma + iX_L\Sigma - i\Sigma X_R. \qquad (11)\end{aligned}$$

Using equ. (8-10) we can calculate the dependence of the gap in the fermion spectrum on the strange quark mass. For $m_s = 0$ there are 8 quasi-particles with gap Δ and one quasi-particle with gap 2Δ. As m_s increases some of the gaps decrease. The gap of the lowest mode is approximately given by $\Delta = \Delta_0 - 3\mu_s/4$ where $\mu_s = m_s^2/(2p_F)$ and Δ_0 is the gap in the chiral limit.

For $\mu_s > 4\Delta_0/3$ the system contains gapless fermions interacting with light or even massless Goldstone bosons. This situation is superficially similar to the normal phase discussed in Sect. 1, but this is not the case. The gapless CFL phase is unstable with respect to the formation of non-zero currents [8]. This can be seen by considering the dispersion relation in the presence of a hypercharge or isospin current. The dispersion relation of the lowest mode is [9,10]

$$\omega_l = \Delta_0 + \frac{l^2}{2\Delta_0} - \frac{3}{4}\mu_s - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K, \qquad (12)$$

where l is the momentum relative to the Fermi surface and $\jmath_K$ is the current. The energy relative to the CFL phase is the kinetic energy of the current plus the energy of occupied gapless modes

$$\mathcal{E} = \frac{1}{2}v_\pi^2 f_\pi^2 \vec{\jmath}_K^{\,2} + \frac{\mu^2}{\pi^2}\int dl \int \frac{d\Omega}{4\pi}\,\omega_l\theta(-\omega_l). \qquad (13)$$

The energy functional can develop a minimum at non-zero $\jmath_K$ because the current lowers the energy of the fermions near one of the pole caps on the Fermi surface. Introducing the dimensionless variables $x = \jmath_K/(a\Delta)$ and $h = (3\mu_s - 4\Delta)/(a\Delta)$ we can write

$$\mathcal{E} = Cf_h(x), \quad f_h(x) = x^2 - \frac{1}{x}\left[(h+x)^{5/2}\Theta(h+x) - (h-x)^{5/2}\Theta(h-x)\right], \qquad (14)$$

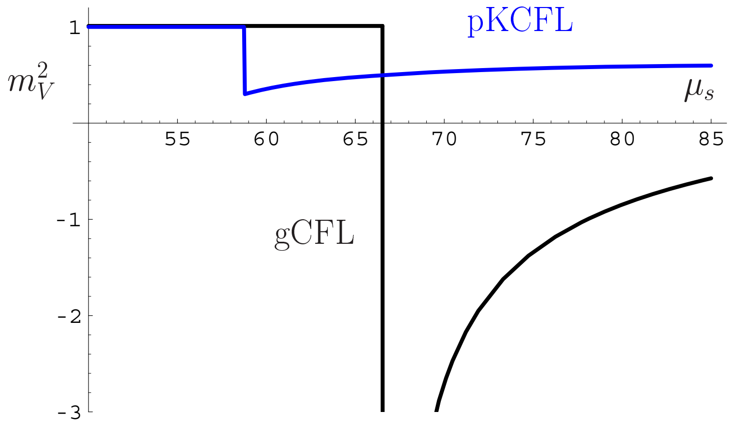

Figure 2. Screening mass in the CFL phase as a function of the effective chemical potential $\mu_s = m_s^2/(2p_F)$. The screening mass is defined as the second derivative of the energy density with respect to an isospin or hypercharge current. The blue and black curves show the result in the gapless CFL phase with (pKCFL) and without (gCFL) a supercurrent.

where C and a are numerical constants. The functional equ. (14) was analyzed in [11,9,10]. There is a critical chemical potential $\mu_s = (4/3 + ah_{crit}/3)\Delta$ above which the groundstate contains a non-zero supercurrent $\jmath_K$. This current is canceled by a backflow of gapless fermions. The screening mass $m_V^2 = (\partial^2\mathcal{E})/(\partial \jmath_K^2)$ is shown in Fig. 2. Without the supercurent an instability occurs for $\mu_s = 4\Delta/3$, but the instability is resolved by a non-zero current. The new phase is analogous to p-wave pion condensates at lower densities because the current is carried by Goldstone kaons in the CFL phase and the instability is caused by the p-wave interaction between kaons and fermions.

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