Universal scaling of the rapidity dependent elliptic flow and the perfect fluid at RHIC

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The pseudo-rapidity dependence of the elliptic flow at various excitation energies measured by the PHOBOS Collaboration in Au+Au collisions at RHIC is one of the surprising results that has not been explained before in terms of hydrodynamical models. Here we show that these data are in agreement with theoretical predictions based on perfect fluid hydrodynamics. We also show that these PHOBOS data satisfy the universal scaling relation predicted by the Buda-Lund hydrodynamical model, based on exact solutions of perfect fluid hydrodynamics.

1. Introduction

One of the unexpected results from experiments at the Relativistic Heavy Ion Collider (RHIC) is the relatively strong second harmonic moment of the transverse momentum distribution, referred to as the elliptic flow. Measurements of the elliptic flow by the PHENIX, PHOBOS and STAR collaborations (see refs. [1–4]) reveal rich details in terms of its dependence on particle type, transverse and longitudinal momentum variables, on the centrality and the bombarding energy of the collision. In the soft transverse momentum region, these measurements at mid-rapidity are reasonably well described by hydrodynamical models [5,6]. However, the dependence of the elliptic flow on the longitudinal momentum variable pseudo-rapidity and its excitation function has resisted descriptions in terms of hydrodynamical models.

Here we show that these data are consistent with the theoretical and analytic predictions that are based on perfect fluid hydrodynamics.

2. Results

Our tool in describing the pseudorapidity-dependent elliptic flow is the Buda-Lund hydrodynamical model. The Buda-Lund hydro model [7] is successful in describing the BRAHMS, PHENIX, PHOBOS and STAR data on identified single particle spectra and the transverse mass dependent Bose-Einstein or HBT radii as well as the pseudorapidity distribution of charged particles in Au + Au collisions both at $\sqrt{s_{\rm NN}} = 130$ GeV [8] and at $\sqrt{s_{\rm NN}} = 200$ GeV [9]. However the elliptic flow would be zero in an axially symmetric case, so we developed the ellipsoidal generalization of the model that describes an expanding ellipsoid with principal axes X, Y and Z. Their derivatives with respect to proper-time (expansion rates) are denoted by $\dot{X}$, $\dot{Y}$ and $\dot{Z}$.

The generalization goes back to the original one, if the transverse directed principal axes of the ellipsoid are equal, ie $X = Y$ (and also $\dot{X} = \dot{Y}$).

One can define a deviation from axial symmetric flow, a momentum-space eccentricity:

$$\epsilon_p = \frac{\dot{X}^2 - \dot{Y}^2}{\dot{X}^2 + \dot{Y}^2}. \tag{1}$$

The exact analytic solutions of hydrodynamics (see ref. [10–12]), which form the basis of the Buda-Lund hydro model, develop Hubble-flow for late times, ie $X \rightarrow_{\tau\rightarrow\infty} \dot{X}\tau$, so the momentum-space eccentricity ϵ_p nearly equals space-time eccentricity ϵ. Hence, in this paper we extract space-time eccentricity (ϵ) and average transverse flow (u_t) from the data, instead of $\dot{X}$ and $\dot{Y}$.

In the time dependent hydrodynamical solutions, these values evolve in time, however, it was show in ref. [13] that $\dot{X}$ and $\dot{Y}$, and so ϵ and u_t become constants of the motion in the late stages of the expansion.

The result for the elliptic flow (under certain conditions detailed in ref [14]) is the following simple universal scaling law:

$$v_2 = \frac{I_1(w)}{I_0(w)}. \tag{2}$$

The model predicts an *universal scaling:* every v_2 measurement is predicted to fall on the same *universal* scaling curve I_1/I_0 when plotted against w.

This means, that v_2 depends on any physical parameter (transverse or longitudinal momentum, center of mass energy, centrality, type of the colliding nucleus etc.) only trough the scaling paremeter w.

Here w is the scaling variable, defined by

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right), \tag{3}$$

and

$$T_{*,x} = T_0 + \overline{m}_t\,\dot{X}^2\frac{T_0}{T_0 + \overline{m}_t a^2}, \tag{4}$$

$$T_{*,y} = T_0 + \overline{m}_t\,\dot{Y}^2\frac{T_0}{T_0 + \overline{m}_t a^2}, \tag{5}$$

and

$$\overline{m}_t = m_t \cosh(\eta_s - y). \tag{6}$$

Here $a = \langle \Delta T/T \rangle_t$ represents the temperature gradient in the transverse direction, at the freeze-out, m_t is the transverse mass, T_0 the central temperature at the freeze-out, while η_s is the space-time rapidity of the saddle-point (point of maximal emittivity). This saddlepoint depends on the rapidity, the longitudinal expansion, the transverse mass and on the central freeze-out temperature:

$$\eta_s - y = \frac{y}{1 + \Delta\eta \frac{m_t}{T_0}}, \tag{7}$$

where $y = 0.5 \log(\frac{E+p_z}{E-p_z})$ is the rapidity and $\Delta\eta$ represents the elongation of the source expressed in units of space-time rapidity. See more details in ref. [14].

Eq. 2 depends, for a given centrality class, on rapidity y and transverse mass m_t. Before comparing our result to the $v_2(\eta)$ data of PHOBOS, we thus performed a saddle point integration in the transverse momentum variable to end up with a formula that can be directly fitted to $v_2(\eta)$ of PHOBOS.

The fitting package is available at ref. [15], more about the results (eg. contour plots) are available at ref. [16].

We have found that the essential fit parameters are ϵ and $\Delta\eta$, and the quality of the fit is insensitive to the precise value of T_0, a, u_t and R_g. These parameters dominate the azimuthal-averaged single particle spectra as well as the HBT (Bose-Einstein) radii, however they only marginally influence v_2. In a broad region their precise value is irrelevant and does not influence the confidence level of the $v_2(\eta)$ fits. Hence we have fixed their values as given in the caption of table 1. We also removed points with large rapidity from the fits in case of lower center of mass energies.

Fits to PHOBOS data of ref. [1] are shown on the left panel of fig. 1. The right panel of fig. 1 demonstrates that these data points follow the theoretically predicted predicted scaling law.

	19.6 GeV	62.4 GeV	130 GeV	200 GeV
ϵ	0.294 ± 0.029	0.349 ± 0.008	0.376 ± 0.005	0.394 ± 0.006
$\Delta\eta$	1.70 ± 0.25	2.16 ± 0.05	2.46 ± 0.04	2.56 ± 0.04
χ^2/NDF	1.8418/11	20.1388/13	34.7935/15	27.4865/15
conf. level	99.995%	21.4036%	1.00341%	7.03121%

Table 1
Results of fits to PHOBOS data of ref. [1]. Remaining parameters were fixed as follows: $T_0 = 175$ MeV, $a = 1.19$ and $u_t = 1.64$.

3. Conclusions

In summary, we have shown that the excitation function of the pseudorapidity dependence of the elliptic flow in Au+Au collisions is well described with the formulas that are predicted by the Buda-Lund type of hydrodynamical calculations.

We have provided a quantitative proof of the validity of the perfect fluid picture of soft particle production in Au+Au collisions at RHIC but also show here that this perfect fluid extends far away from mid-rapidity.

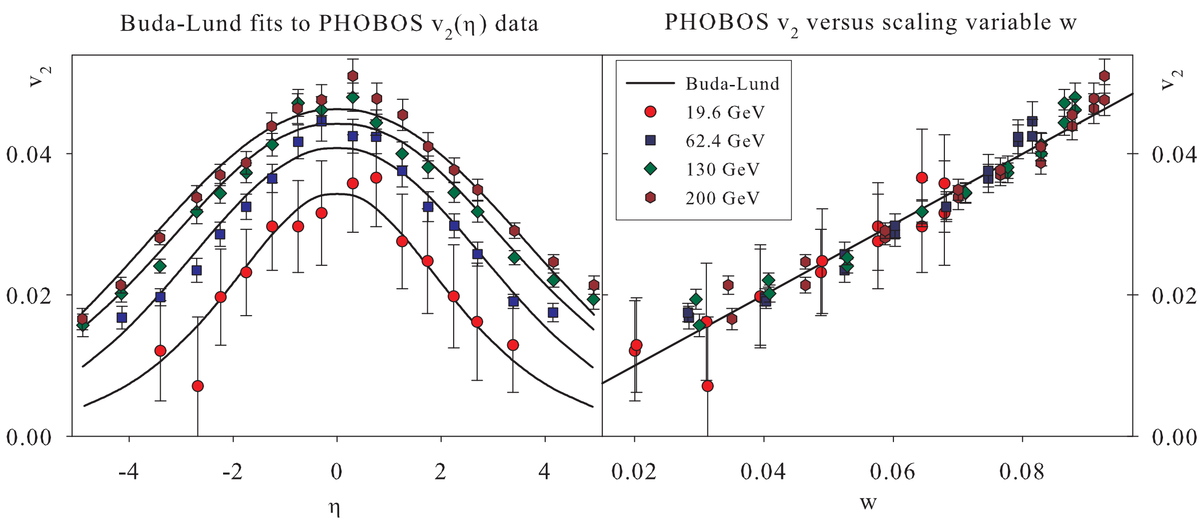

Figure 1. On the left: PHOBOS data on the pseudorapidity dependence of the elliptic flow [1], at various center of mass energies, with Buda-Lund fits. On the right: elliptic flow versus scaling variable w is plotted. The data points show the predicted scaling behavior

The universal scaling of PHOBOS $v_2(\eta)$, expressed by eq. 2, and illustrated by fig. 1 provides a successful quantitative as well as qualitative test for the appearence of a perfect fluid in Au+Au collisions at various colliding energies at RHIC.

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