Hard-loop dynamics of non-abelian plasma instabilities

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I discuss recent advances in the understanding of non-equilibrium gauge field dynamics in plasmas which have particle distributions which are locally anisotropic in momentum space. In contrast to locally isotropic plasmas such anisotropic plasmas have a spectrum of soft unstable modes which are characterized by exponential growth of transverse (chromo)-magnetic fields at short times. The long-time behavior of such instabilities depends on whether or not the gauge group is abelian or non-abelian. Here I will report on recent numerical simulations which attempt to determine the long-time behavior of an anisotropic non-abelian plasma within hard-loop effective theory.

1. Introduction

One of the mysteries emerging from the RHIC ultrarelativistic heavy-ion collision experiments is that the matter produced in the collisions seems to be well-described by hydrodynamic models. In order to apply hydrodynamical models the chief requirement is that the stress-energy tensor be isotropic in momentum space. Additionally, current hydrodynamic codes also assume that they can use an equilibrium equation of state to describe the time evolution of the produced matter. Therefore, the success of these models suggests that the bulk matter produced is *isotropic* and *thermal* at very early times, $t < 1$ fm/c. Estimates of the isotropization and thermalization times from perturbation theory [1], however, indicate that the time scale for thermalization is more on the order of $t \sim 2 - 3$ fm/c. This contradiction has led some to conclude that perturbation theory should be abandoned and replaced by some other (as of yet unspecified) calculational framework. However, it has been proven recently that previous perturbative estimates of the isotropization and equilibration times had overlooked an important aspect of nonequilibrium gauge field dynamics, namely the possibility of *plasma instabilities*.

One of the chief obstacles to thermalization in ultrarelativistic heavy-ion collisions is the intrinsic expansion of the matter produced. If the matter expands too quickly then there will not be time enough for its constituents to interact before flying apart into non-interacting particles and therefore the system will not reach thermal equilibrium. In a heavy-ion collision the expansion which is most relevant is the longitudinal expansion of the matter since at early times it's much larger than the radial expansion. In the absence of interactions the longitudinal expansion causes the system to quickly become much colder in the longitudinal direction than in the transverse (radial) direction, $< p_L > \ll < p_T >$. We can then ask how long it would take for interactions to restore isotropy in the p_T-p_L

plane. In the bottom-up scenario [1] isotropy is obtained by hard collisions between the high-momentum modes which interact via an isotropically screened gauge interaction. The bottom-up scenario assumed that the underlying soft gauge modes responsible for the screening were the same in an anisotropic plasma as in an isotropic one. In fact, this turns out to be incorrect and in anisotropic plasmas the most important collective mode corresponds to an instability to transverse magnetic field fluctuations [2]. Recent works have shown that the presence of these instabilities is generic for distributions which possess a momentum-space anisotropy [3,4] and have obtained the full hard-loop action in the presence of an anisotropy [5].

Here I will discuss numerical results obtained within the last year which address the question of the long-time behavior of the instability evolution [6–9] within the hard-loop framework. This question is non-trivial in QCD due to the presence of non-linear interactions between the gauge degrees of freedom. These non-linear interactions become important when the vector potential amplitudes become $<A>_\mathrm{soft} \sim p_\mathrm{soft}/g \sim (gp_\mathrm{hard})/g$, where p_hard is the characteristic momentum of the hard particles. In QED there is no such complication and the fields grow exponentially until $<A>_\mathrm{hard} \sim p_\mathrm{hard}/g$ at which point the hard particles undergo large-angle scattering in the soft background field invalidating the assumptions underpinning the hard-loop effective action. Initial numerical toy models indicated that non-abelian theories in the presence of instabilities would "abelianize" and fields would saturate at $<A>_\mathrm{hard}$ [6]. This picture was largely confirmed by simulations of the full hard-loop gauge dynamics which assumed that the soft gauge fields depended only on the direction parallel to the anisotropy vector and time [7]. However, recent numerical studies have now included the transverse dependence of the gauge field and it seems that the result is then that the gauge field's dynamics changes its behavior from exponential to linear growth when its amplitude reaches the soft scale, $<A>_\mathrm{soft} \sim p_\mathrm{hard}$ [8,9]. This linear growth regime is characterized by a cascade of the energy pumped into the soft scale by the instability to higher momentum plasmon-like modes [10]. Below I will briefly describe the setup which is used by these numerical simulations and then discuss questions which remain in the study of non-abelian plasma instabilities.

2. Discretized Hard-Loop Dynamics

At weak gauge coupling g, there is a separation of scales in hard momenta $|\mathbf{p}| = p^0$ of (ultrarelativistic) plasma constituents, and soft momenta $\sim g|\mathbf{p}|$ pertaining to collective dynamics. The effective field theory for the soft modes that is generated by integrating out the hard plasma modes at one-loop order and in the approximation that the amplitudes of the soft gauge fields obey $A_\mu \ll |\mathbf{p}|/g$ is that of gauge-covariant collisionless Boltzmann-Vlasov equations [11]. In equilibrium, the corresponding (nonlocal) effective action is the so-called hard-thermal-loop effective action which has a simple generalization to plasmas with anisotropic momentum distributions [5]. The resulting equations of motion are

$$
\begin{aligned}
D_\nu(A)F^{\nu\mu} &= -g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2|\mathbf{p}|} p^\mu \frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})\,, \\
F_{\mu\nu}(A)v^\nu &= [v \cdot D(A)]\, W_\mu(x;\mathbf{v})\,,
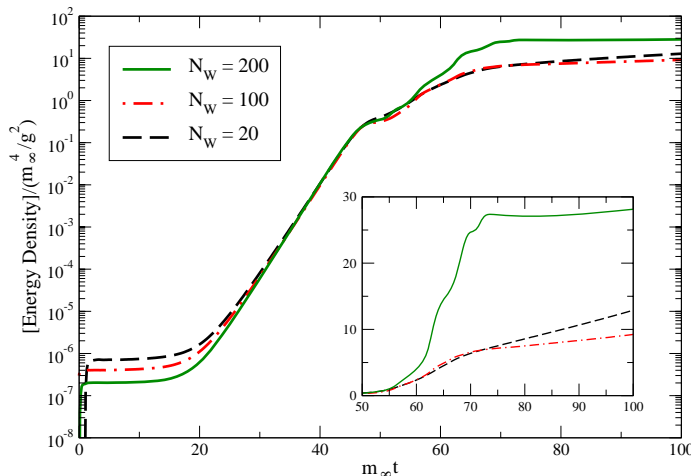\end{aligned}
\tag{1}
$$

Figure 1. Comparison of the energy transferred from hard to soft scales, $|\mathcal{E}(\mathrm{HL})|$, for 3+1-dimensional simulations with $N_w = 20, 100, 200$ on $96^3, 88^3, 69^3$ lattices. Inset shows late-time behavior on a linear scale.

where f is a weighted sum of the quark and gluon distribution functions [5] and $v^\mu \equiv p^\mu/|\mathbf{p}| = (1, \mathbf{v})$.

At the expense of introducing a continuous set of auxiliary fields $W_\beta(x; \mathbf{v})$ the effective field equations are local. These equations of motion are then discretized in space-time and $\mathbf{v}$, and solved numerically. The discretization in $\mathbf{v}$-space corresponds to including only a finite set of the auxiliary fields $W_\beta(x; \mathbf{v}_i)$ with $1 \leq i \leq N_W$. For details on the precise discretizations used see Refs. [8,9].

3. Results and Discussion

During the process of instability growth the soft gauge fields get the energy for their growth from the hard particles. In an abelian plasma this energy grows exponentially until the energy in the soft field is of the same order of magnitude as the energy remaining in the hard particles. As mentioned above in a non-abelian plasma one must rely on numerical simulations due to the presence of strong gauge field self-interactions. In Fig. (1) I have plotted the time dependence of the energy extracted from the hard particles obtained in a 3+1 dimensional simulation of an anisotropic plasma initialized with very weak random color noise [9]. As can be seen from this figure at $m_\infty t \sim 60$ there is a change from exponential to linear growth with the late-time linear slope decreasing as N_W is increased.

The first conclusion that can be drawn from this result is that within non-abelian plasmas instabilities will be less efficient at isotropizing the plasma than in abelian plasmas. However, from a theoretical perspective "saturation" at the soft scale implies that one can still apply the hard-loop effective theory self-consistently to understand the behavior of the system at late times. Looking forward, I note that the latest simulations [8,9] have only presented results for distributions with a finite $\mathcal{O}(1-10)$ anisotropy and these seem to imply that in this case the induced instabilities will not have a significant effect on the hard particles. This means, however, that due to the continued expansion of the system

that the anisotropy will increase. It is therefore important to understand the behavior of the system for more extreme anisotropies. Additionally, it would be very interesting to study the hard-loop dynamics in an expanding system. Naively, one expects this to change the growth from $\exp(\tau)$ to $\exp(\sqrt{\tau})$ at short times but there is no clear expectation of what will happen in the linear regime. The short-time picture has been confirmed by early simulations of instability development in an expanding system of classical fields [14]. It would therefore be interesting to incorporate expansion in collisionless Boltzmann-Vlasov transport in the hard-loop regime and study the late-time behavior in this case.

I note in closing that the application of this framework to phenomenologically interesting couplings is suspect since the results obtained strictly only apply at very weak couplings; however, the success of hard-thermal-loop perturbation theory at couplings as large as $g \sim 2$ [12] suggests that the nonequilibrium hard-loop theory might also apply at these large couplings. For going to even larger couplings perhaps colored particle-in-cell simulations [13] could be used if they are extended to full 3+1 dynamics.

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