Heavy Quark Transport in the Quark Gluon Plasma

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Heavy quarks provide a unique opportunity to probe the transport properties of the the Quark Gluon Plasma (QGP). Since the mass is large compared to the temperature, the time scale for heavy quark equilibration is $(M/T)\tau_{\rm R}$, where $\tau_{\rm R}$ is the typical relaxation time of the medium. Given an estimate of this fundamental time scale, one should be able to predict whether heavy quarks thermalize or not. The heavy quark spectrum (or more precisely the associated electron spectrum) is being measured in detail and motivates this work [2,3].

It is natural to propose a simple stochastic model for the interaction of the heavy quark with the surrounding medium [4,1,5]. Denoting the heavy quark momentum $\mathbf{p}$, we write down a stochastic equation of motion for the heavy quark in the rest frame of the medium

$$\frac{dp_L}{dt} = -\eta_{\rm D}(p)p^i + \xi_L \; , \qquad (1)$$

$$\frac{dp_T}{dt} = \xi_T \; . \qquad (2)$$

Here dp_L and dp_T are the momentum increments parallel and transverse to the direction of the heavy quark. ξ_L and ξ_T are random momentum kicks in the longitudinal and transverse directions which satisfy

$$\langle \xi^i_L(t)\, \xi^j_L(t') \rangle = \kappa_L(p)\, \hat{p}^i\hat{p}^j \delta(t-t') \; , \qquad (3)$$

$$\langle \xi^i_T(t)\, \xi^j_T(t') \rangle = \kappa_T(p)\, (\delta^{ij} - \hat{p}^i\hat{p}^j)\delta(t-t') \; . \qquad (4)$$

$\eta_D(p)p$ is the drag coefficient and records the momentum loss per unit Δt. $\kappa_T(p)$ and $\kappa_L(p)$ are the variances of the transverse and longitudinal momentum transfers per unit Δt. Under this stochastic process, an initial distribution of heavy quarks will approach the equilibrium distribution, $P(\mathbf{p}) \propto e^{-\frac{E_{\mathbf{p}}}{T}}$, if and only if the drag and longitudinal diffusion coefficients are related by the Einstein relation, $\eta_{\rm D}(p) = \kappa_L(p)/(2TE)$. This model equation is a Langevin equation with momentum dependent coefficients. The ambiguity in such equations has only recently been clarified [6].

The value of the drag coefficient at zero momentum is related the diffusion coefficient

*Based on work done in collaboration with Guy D. Moore [1]

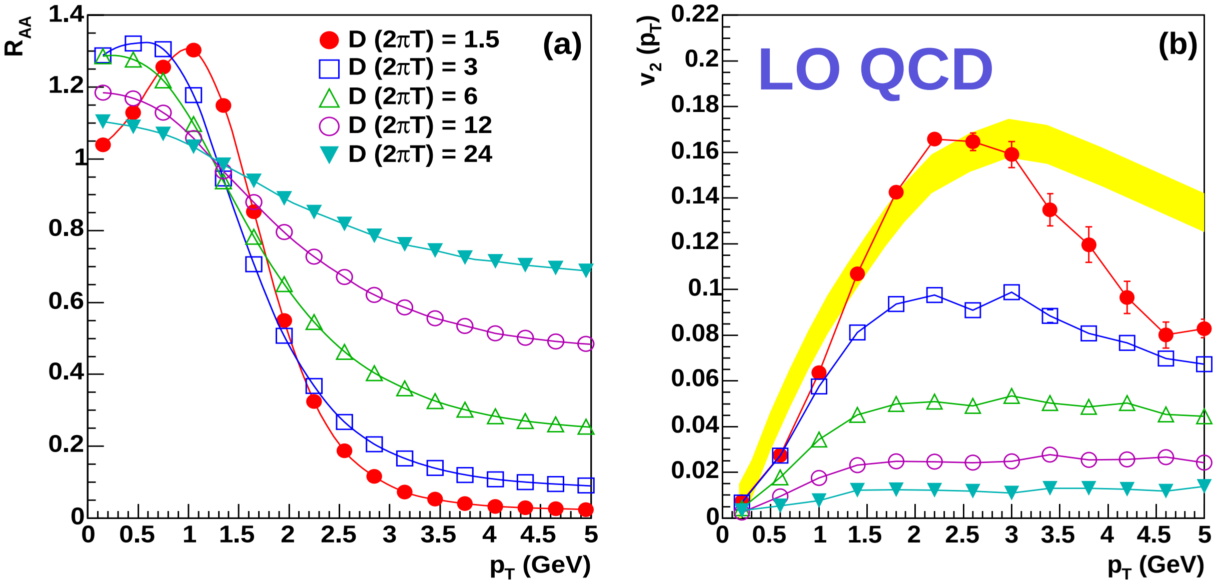

Figure 1. (a) The nuclear modification factor R_{AA} for charm quarks for representative values of the diffusion coefficient. (b) $v_2(p_T)$ for charm quarks for the same set of diffusion coefficients given in the legend in (a). In perturbation theory, $D \times (2\pi T) \approx 6\,(0.5/\alpha_s)^2$. The band estimates the light hadron elliptic flow for the same impact parameter, $b = 6.5\,\mathrm{fm}$, using STAR data.

of the heavy quark

$$D = \frac{T}{M\eta_{\rm D}(0)}\,. \tag{5}$$

By specifying the diffusion coefficient the model is constrained at small momentum. Typically D is of order of the light quark relaxation time and of order of the momentum diffusion length, $\eta/(e+p)$. (In perturbation theory these transport coefficients differ by a factor of order ~ 5.) Thus constraints on D imply constraints on the shear viscosity η.

Extrapolating the model from small momentum to high momentum requires additional assumptions and a model for how the drag and fluctuation coefficients change with momentum. Indeed, the assumption of the Langevin approach is that the energy transferred to a heavy quark in a typical collision is much smaller than the temperature [1]. At strong coupling this requirement is satisfied only in the non-relativistic limit. In weak coupling this requirement is satisfied to leading $\log(1/\alpha_s)$. The Boltzmann equation does not need to make this assumption [7]. Radiation involves coherent multiple scattering and can not be included in a stochastic model. Nevertheless, radiation can be neglected (in perturbation theory) until the heavy quark gamma factor is large

$$\gamma v \sim \frac{1}{\alpha_s}\frac{m_{\rm D}}{T} \sim 6\,. \tag{6}$$

The last estimate for α_s and $m_{\rm D}/T$ is based upon the radiative calculations presented at this conference [8]. Although radiative energy loss certainly becomes important at high momentum [8,9], collisional energy loss is studied here. Collisional energy loss of heavy

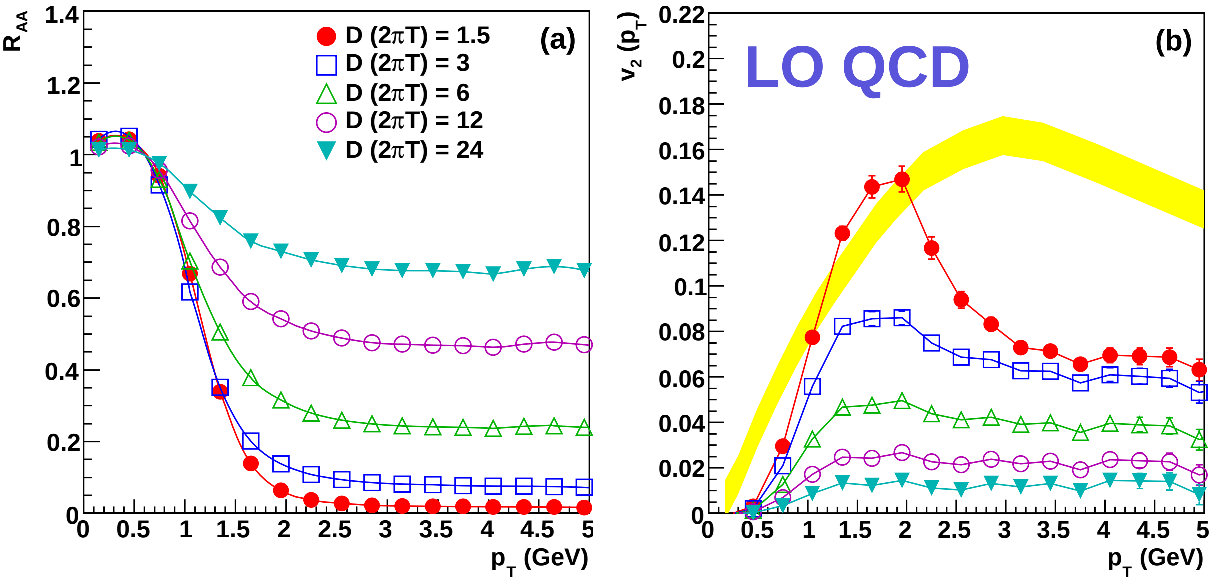

Figure 2. (a) The nuclear modification factor R_{AA} and (b) the elliptic flow $v_2(p_T)$ for non-photonic electrons from charm. The legend applies to both (a) and (b). See Fig. 1 for further explanation.

quarks was estimated using Hard Thermal Loop (HTL) perturbation theory in [10,1]. These collisional energy loss calculations find that the energy loss is roughly proportional to the momentum, $\frac{dp}{dt} \propto p$, in much of experimentally relevant range. We will take the momentum dependence of the drag coefficient from the HTL calculation.

To fully calculate the spectrum of heavy quarks in the heavy ion reaction we need to describe how the medium evolves. We will adopt the hydrodynamic model of [11] and calculate how the initial spectrum charm quarks is changed when subjected to the Langevin update rule in the local rest frame of the medium. The resulting R_{AA} and $v_2(p_T)$ for charm quarks are shown in Fig. 1.

To make contact with the experimental data on non-photonic electrons we must fragment the charm quarks into hadrons and decay the resulting hadrons into electrons. These steps will be summarized in a separate publication [12]. The resulting electrons are shown in Fig. 2. Currently, the data show a modification factor in the measured decay electrons of approximately $0.3 - 0.4$ for $p_T \sim 3.0\,\mathrm{GeV}$. It is premature to compare Fig. 2 directly to the data since the bottom quark contribution has not been included. However, the R_{AA} data seem to support a diffusion coefficient of order $D \times (2\pi T) \approx 3 - 6$. This is a small diffusion coefficient which may ultimately be compared to the lattice [13]. With this value of the diffusion coefficient it is hard to explain the large $v_2(p_T)$ observed by the STAR and PHENIX collaborations [2,3]. Thus it seems that some physics is not captured in this stochastic model – coalescence perhaps [5].

For the smallest diffusion coefficient (the closed red circles), Fig. 1 and Fig. 2 show a definite transition around $p_T \approx 2.0\,\mathrm{GeV}$. To illuminate the dynamics in this region we show two correlation functions. Fig. 3(a) shows the initial angle-final angle correlation, $P(\Delta\phi)$. We see that above $p_T \sim 2.0\,\mathrm{GeV}$ the heavy quark becomes increasingly correlated with its initial direction. The deflection angle $\Delta\phi$ is of order 50^o for $p_T \sim 2.0\,\mathrm{GeV}$.

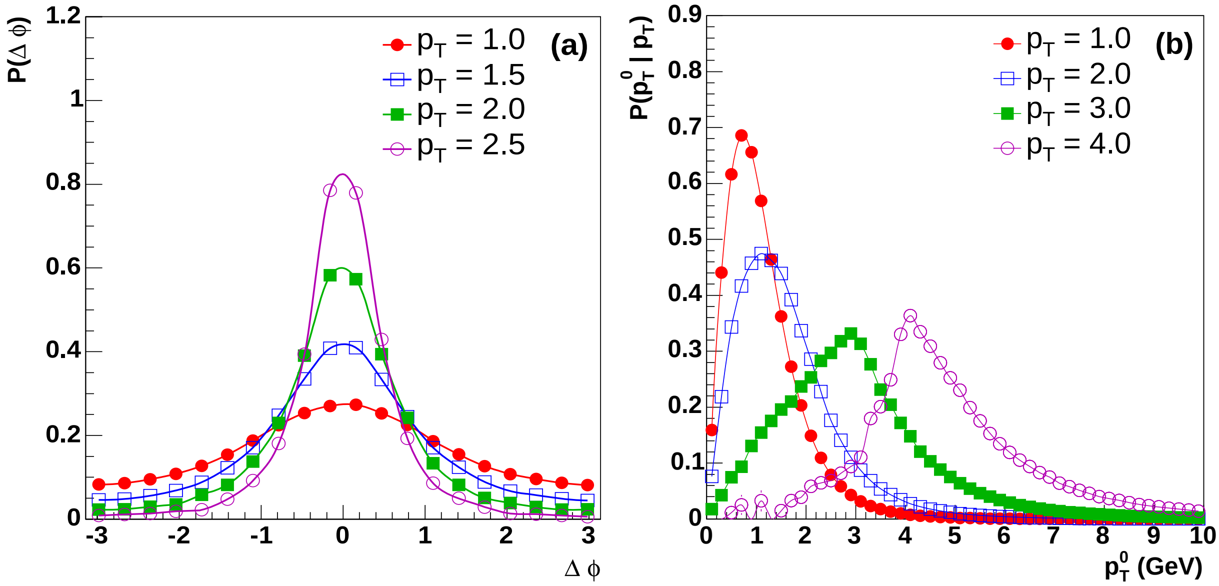

Figure 3. Thermalization of Charm Quarks: (a) The probability, $P(\Delta\phi)$, that a charm quark will by deflected by an angle $\Delta\phi$ from its original direction, for various values of p_T. (b) The probability, $P(p_T^0|p_T)$ that a charm quark with final transverse momentum p_T started with transverse momentum p_T^0, for various values of p_T.

Fig. 3(b) shows the probability that a quark with final momentum p_T originated with momentum p_T^0. Approximately 30% of charm quarks with $p_T \sim 4.0\,\mathrm{GeV}$ are accelerated to their final momentum by the medium. This percentage increases as p_T is decreased. In summary, the changes seen in this momentum region reflect a transition from a thermal to a kinetic regime.

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